Accounting for Plant-Level Misallocation*

Virgiliu Midrigan
New York University

Daniel Yi Xu†
New York University

April 2009

Abstract

We use panel data for Korean Manufacturing plants to document substantial dispersion in the average product of capital, three times greater than dispersion in the average product of other factors. If one interprets this as evidence of misallocation (dispersion in the marginal product of capital), aggregate productivity losses are substantial, about 40 percent. We evaluate the ability of a model of industry dynamics in which firms face non-convex capital adjustment costs, financing frictions, and uninsurable investment risk to account for the dispersion in the marginal product of capital. We show that the frictions necessary to reconcile the model's predictions with the data are large and account for the bulk of within-plant time-series variance in the average product of capital. They are incapable, however, of sustaining the large and persistent differences in the marginal product of capital in the cross-section and thus account for a small fraction (less than 10%) of the misallocation in the data.

Keywords: Productivity. Misallocation. Adjustment costs. Borrowing constraints. Uninsured investment risk.

---

*We thank Vivian Yue for many useful discussions during an early stage of this project. We have also benefited from conversations with Paco Buera, Andres Rodriguez-Claire, Mark Gertler, Joe Kaboski, Tim Kehoe, Bob Lucas and Yongseok Shin. Matthias Lux provided excellent research assistance.

†virgiliu.midrigan@nyu.edu, daniel.xu@nyu.edu, 19 W 4th St, 6th Floor, New York, NY 10012.

1. Introduction

Plant and firm-level data show substantial dispersion in the average (revenue) product of capital and labor, even for plants in narrowly defined industries[1]. Examined through the lens of a neoclassical production function, this dispersion is often interpreted as evidence of misallocation: inefficient dispersion in the marginal product of factors of production across establishments. The extent of misallocation is enormous: Hsieh and Klenow (2008) calculate that reallocating factors of production to their most productive use and thereby equating their marginal product across plants would raise aggregate productivity by almost 100% in India and China, and by around 30-40% in the US.

The degree of misallocation is strongly correlated with a country's GDP per capita and thus total factor productivity. Bartelsman et. al (2008) report that the variance of the average product of labor is equal to around 0.30 in US, UK and France, around 0.6 in Portugal, Taiwan and Korea, and is as high as 1.0 in Brazil, Venezuela and several East European countries. One conjecture is therefore that part of the differences in total factor productivity in the aggregate is an outcome of differences in policy and other distortions that prevent the allocation of factors of production to their most efficient use[2]. The distortions emphasized in recent work include inefficient government policies[3], credit constraints[4], frictions that distort factor mobility[5], lack of insurance against the risk associated with entrepreneurial activity[6] and measurement problems[7].

Our goal in this paper to quantitatively assess the role of frictions that distort investment decisions in accounting for the observed dispersion in the average product of capital. The focus on capital is motivated by our observation, using data from a survey of Korean manufacturing plants, that dispersion in the average product of capital is substantially larger (three times greater) than dispersion in the average product of other factors of production (labor and intermediate goods). A calculation similar to that of Hsieh and Klenow (2008) shows that reallocating capital *alone* to the most efficient plants would raise productivity and therefore output by 40% in our data.

[1]Bartelsman and Doms (2000) and Tybout (2000) review the evidence; Hsieh and Klenow (2008), Bartelsman, Haltiwanger and Scarpetta (2008) are two important recent contributions.

[2]Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Hsieh and Klenow (2008).

[3]Restuccia and Rogerson (2008), Hsieh and Klenow (2008), Guner, Ventura, Xu (2008).

[4]Banerjee and Duflo (2005), Buera and Shin (2007) and Buera, Kaboski and Shin (2009).

[5]Hopenhayn and Rogerson (1993), Lagos (2006).

[6]Banerjee and Duflo (2005) and references therein. See also Angeletos (2008).

[7]Bartelsman et. al (2008).

We study a model of firm dynamics, similar to that of Hopenhayn (1992), which we extend to allow for three distortions that have been argued important in shaping plant-level investment: a) non-convex capital adjustment costs, b) borrowing frictions, and c) a consumption-smoothing motive for the firm owner in the presence of limited insurance against investment risk[8]. We use this framework to assess the role of these frictions, as well as to interpret, through the lens of the model, several salient relationships between investment and the average product of capital and plant characteristics.

We start by documenting a number of facts in our data from the Korean Annual Manufacturing Survey on the extent to which capital is inefficiently allocated across plants. We show that the average product of capital is highly dispersed across plants and that most of this variation is driven by persistent differences across plants within narrowly defined (5-digit) industries. Although individual plants do exhibit large fluctuations in their average product of capital over time, these fluctuations are much smaller than cross-sectional differences and account for less than 1/5 of the overall dispersion. We also document several salient features of the data that we use to evaluate the predictions of our model: the investment-capital ratio is sharply increasing with a plant's revenue (and profits), the average product of capital declines with age (the oldest plants are 30% less productive than new entrants), declines with a plant's stock of capital, and increases with a plant's revenue and operating profits.

We then study the quantitative implications of the three frictions on capital acummulation. We study the role of each separately, by first considering the role of non-convex capital adjustment costs (a fixed cost of installing new capital, as well as a partial irreversibility), then turning to a model in which limited enforcement of contracts restricts a plant's ability to borrow externally, and finally solving for the optimal investment decision of an entrepreneur with concave preferences whose ability to smooth consumption fluctuations is limited to the use of a single risk-free security[9]. We pin down the size of these frictions by requiring the model to account for salient facts about plant-level investment dynamics (most plants do not invest in any given period, and when they do so they typically invest a large amount),

[8]Domms and Dunne (1998), Caballero, Engel Haltiwanger (1995), Cooper and Haltiwanger (2007) document that plant-level investment is lumpy and infrequent. Cooley and Quadrini (2001), Whited (2006), Hennesy and Whited (2007), Gilchrist and Sim (2007) document an important role for financing constraints in determining plant-level investment. See also Banerjee and Duflo (2005, 2008), Wang (2008) and Kaboski and Townsend (2009) for micro-level evidence on the role of credit constraints. Castro, Clementi and MacDonald (2004, 2008) and Angeletos (2008) study the role of uninsurable idiosyncratic investment risk. See also Moskowitz and Vissing-Jorgensen (2002) for evidence that enterpreneurial risk is poorly diversified.

[9]We assume risk-neutral entrepreneurs in the first two economies to isolate the role of limited insurance.

as well as the relationship between investment and the average product of capital and plant characteristics. The latter, we show, vary with the strength of financing frictions assumed in the model.

We briefly summarize our findings. We find, as in earlier work, an important role for capital adjustment costs, especially irreversibilities, in shaping plant-level investment. Moreover, these frictions generate sizable, but short-lived, fluctuations in a plant's average product of capital over time. Inaction is incapable however of generating persistent differences in the average product of capital across plants. Intuitively, plants find it optimal to pay the adjustment costs in response to persistent and large deviations of their stock of capital from the optimum. Most inaction is accounted for by plants that have little to lose by not adjusting their stock of capital and it thus contributes little to distorting the economy's capital allocation. Overall, we find that non-convex capital adjustment costs generate 2% aggregate productivity losses from misallocation, much smaller than the 40% losses in the data[10].

As for financing frictions, we again find that severe borrowing constraints are necessary to render the model's predictions in line with the data, especially the relationship between investment and the average product of capital and plant size. Once again, however, the model generates small differences in the average product of capital across establishments. Financing frictions, through their effect on misallocation, lower the economy's aggregate productivity by 3%. Intuitively, financing frictions cannot prevent a highly productive plant from accumulating capital for too long: it is exactly its high productivity that allows the plant to accumulate internal funds and eventually grow out of its borrowing constraint. Thus, although financing frictions can generate fairly large time-series fluctuations in a plant's average product of capital (as large as in the data), they cannot generate differences in the average product of capital across plants that last for a prolonged period of time. We note that this is not an impossibility result: we do present parametrizations of the model capable of generating substantial misallocation from borrowing frictions[11]. Rather, ours is a result specific to the dataset we study: parametrizations of the model consistent with the

[10]Our paper is also related to that of Hall (2004) who finds small (convex) capital and labor adjustment costs in 2-digit US manufacturing industries. Our focus on non-convex adjustment costs at the plant level leads us to conclude in favor of substantial costs of adjusting capital. We share however his conclusion that these costs generate small wedges in the plant's optimality conditions.

[11]Buera, Kaboski and Shin (2009) also find important aggregate consequences of financing frictions (aggregate productivity losses as large as 25%) for economies with low external finance to GDP ratios.

relationship between the average product of capital and plant size in the data predict small aggregate productivity losses from borrowing frictions.

Finally, we find that inability to insure consumption fluctuations adds little (less than 1/10th of a percent) aggregate productivity losses from misallocation. By accumulating a buffer stock of the risk-free asset entrepreneurs are able to finance both consumption and investment and avoid distorting the allocation of capital. This is true even if we impose a no-borrowing constraint or lower the rate of return on the risk-free security to a level substantially lower than the rate of time-preference. Only by eliminating the risk-free security altogether (so that investment in capital is the only means of saving for the entrepreneur), can the model economy generate sizable dispersion in the average product of capital (10% aggregate productivity losses). This increase in misallocation comes however at the expense of the model's ability to account for the variability of investment in the data: the standard deviation of the investment-capital ratio is three times smaller than in the data.

A final comment is in order. The aggregate productivity losses (2-3%) that capital accumulation frictions are found to generate are by no means trivial. In fact, these numbers are comparable to those obtained by Hopenhayn and Rogerson (1993) who state: "*We believe that the finding of such large welfare costs stands as one of the most important findings of our study. Policies that interfere with the job creation/destruction process are apparently quite costly.*" Rather, these losses are small relative to those one infers using data on the dispersion in the average product of capital and interpreting it through the lens of a neoclassical production function. Establishing the source of this dispersion (technology differences across plants, distortionary taxes, differences in the user cost of capital other than those induced by financing frictions, differences in markups) remains an important question for future work.

This paper proceeds as follows. Section 2 presents the data and documents the extent of 'misallocation' of capital across plants. Section 3 studies an economy with plant entry and exit and non-convex capital adjustment costs. Section 4 studies a variation of this economy with financing frictions. Section 5 studies an economy with a consumption-smoothing motive for the entrepreneur and limited insurance. Section 6 concludes. An appendix shows that a number of measurement concerns (mis-measurement of the capital stock and revenue, variable capital utilization, departures from a Cobb-Douglas production function) account for little of the measured dispersion in the marginal product of capital. In contrast, the fact that plants rent a substantial portion of their capital stock accounts for one-third of the measured misallocation.

2. Data

In this section we briefly discuss the source of the plant-level data we use and present a number of facts about the extent to which factors of production (and in particular capital) are misallocated. In particular, we first describe the assumption we make on the production technology, discuss the extent to which the average product of capital differs across plants, and then relate this dispersion to a measure of aggregate productivity losses.

A. Data Description

The data we use is from the Korean Annual Manufacturing Survey, which is collected by the Korean National Statistical Office. The survey is conducted every year from 1991 to 1998, except for the year of Industrial Census (1993) for which we supplement the data using the Census data (which covers all establishments). The survey covers all manufacturing plants with five or more workers.

The survey reports information about each plant's total revenue, number of employees, total wage bill, payments for intermediate goods (materials), as well as energy use. The survey also reports the book value of a plant's capital stock, as well as purchases, retirement/sales, and depreciation for land, buildings, machinery and equipment. This information allows us to construct a measure of plant-level capital using a perpetual inventory method[12]. We follow earlier work and focus on buildings, machinery and equipment (and not land) as our measure of capital stock. We construct this measure according:

$$\begin{aligned} I_t &= PUR_t - RET_t \\ K_{t+1} &= K_t - DEP_t + I_t \end{aligned}$$

where I_t is investment and DEP_t is the depreciation of capital stock. We use a plant's book value of capital to initialize each series. All data (investment, retirements, depreciation, initial capital stock) is deflated using price deflators for capital for Korea's Manufacturing Sector available from the OECD STAN Industrial Database. We also deflate plant revenue and intermediate inputs using revenue and materials deflators from the same data source.

[12] See e.g. Caballero et al. (1995)

We drop observations that are clearly an outcome of measurement error: observations with negative reported revenue, depreciation, capital book value, and capital purchases. Because our focus is on measuring dispersion in the average product of capital, we also exclude extreme observations for which this ratio is either in the highest or lowest 1 percentile when reporting statistics related to revenue and/or capital. Our final sample consists of 592996 plant-year observations over an eight year period from 1991 to 1998.

B. Technology

We next describe the assumptions we make on the production technology. These assumptions are useful in order to organize and interpret the features of the data we are about to document.

We assume that plants produce output Y using inputs of capital, K, labor, L and intermediate goods, M, with a Cobb-Douglas production function[13]:

$$Y = \exp(x) L^{\gamma} M^{\beta} K^{1-\gamma-\beta}.$$

Plants face a downward-sloping demand curve with constant elasticity, $\eta < 0$[14]:

$$P = Y^{\frac{1}{\eta}}$$

We assume, here, and throughout the rest of the paper, that plants face no frictions in their choice of labor and intermediate inputs. Optimization over these variable inputs then yields a revenue function

$$R = PY = A(x) K^{\alpha}$$

where

$$\alpha = \frac{1 + \frac{1}{\eta} - \left(1 + \frac{1}{\eta}\right)(\gamma + \beta)}{1 - \left(1 + \frac{1}{\eta}\right)(\gamma + \beta)}$$

[13] In the Appendix we study the role of departing from Cobb-Douglas and work with a more general CES specification.

[14] An alternative interpretation is that perfectly competitive plants face a production function that exhibits decreasing returns to scale, with a degree of returns to scale equal to $1 + \frac{1}{\eta}$.

and A depends on the plant's productivity[15], as well as the price of labor and intermediate goods. The parameter that determines the curvature of the revenue function, α, is a function of both the markup, $\left(1+\frac{1}{\eta}\right)^{-1}$, as well as the revenue share of materials and labor.

Given these assumptions, the marginal revenue product of capital is equal to

$$R'(K) = \alpha \frac{R}{K}$$

and thus proportional to the revenue/capital ratio.

We next consider an efficient benchmark against which to contrast our data. Assume that the economy is populated by N plants that differ in their productivity, A_i, and face the technology described above. Consider the problem of allocating the economy's stock of capital, K, to these plants in order to maximize total revenue in this economy[16]:

$$\max_{K_i} \sum_{i=1}^{N} A_i K_i^{\alpha}$$

subject to

$$\sum_{i=1}^{N} K_i \leq K.$$

Clearly, the solution of this problem entails equalizing the marginal (and average) revenue product of capital across plants:

$$\frac{R_i}{K_i} = \frac{R_j}{K_j}$$

which in turn implies allocating each plant a share of the total capital stock according to its

[15]We refer to A as productivity for simplicity, although we do recognize that variation in A can arise from variation in idiosyncratic preference or factor price shocks.

[16]Identical results obtain if the objective is to maximize consumer's welfare, as opposed to firm revenues, where consumer's utility is defined over consumption of individual goods: $c = \left(\sum_{i=1}^{N} c_i^{1+\frac{1}{\eta}}\right)^{\frac{\eta}{1+\eta}}$. The difference between these problems is in the markup (and total amount of output) chosen by a monopolist vs. a planner, not in the allocation of capital across plants.

productivity:

$$K_i = K\left(\frac{A_i}{A}\right)^{\frac{1}{1-\alpha}} \text{ where } A = \left(\sum_i A_i^{\frac{1}{1-\alpha}}\right)^{1-\alpha}.$$

We next ask: how close is the Korean Manufacturing data to this benchmark?

C. Dispersion in average product of capital

Table 1 reports moments of the distribution of the (log) revenue-to-capital ratio in the data. We pool all plant-year observations together when reporting these statistics. The data shows enormous dispersion in this ratio, even though we have excluded the top and bottom 1% outliers. The variance of $\ln \frac{R_i}{K_i}$ is equal to 1.23 and the interquartile range is equal to 1.49. Interpreted through the lens of the production technology we have described above, the data suggests that the ratio of the user cost of capital of a plant in the 75th percentile to that of a plant in the 25th percentile is equal to 4.5 ($\approx \exp(1.49)$). We also report the dispersion in the average product of capital for large plants, defined as the largest plants that together account for 90% and 80% of the revenue in our sample (roughly 20% and 10% of the plants, respectively). The variance of the log average product of capital is only slightly lower, at 1.11 and 1.06.

We also report in Table 1a the dispersion in the average revenue product of the other factors of production. The variance of the revenue-labor ratio is equal to 0.35, while that of revenue to materials use is 0.40, roughly one-third of that of the average product of capital. This motivates our focus in the rest of this paper on frictions that distort capital acummulation.

To put the dispersion numbers in perspective, we also report the variance of capital and revenue in Table 1b. Both revenue and capital are more dispersed than the average product of capital, roughly twice and three times more volatile. The correlation of revenue and a plant's stock of capital is thus high, around 75%.

We next decompose the dispersion in the log average product of capital into year, industry and plant-specific effects. We show that most of this dispersion arises from large differences in this ratio across plants that persist over time. To see this, Table 2 shows that year and industry (5-digit)-specific fixed effects account for 0.35% and 14% of the total variance, respectively. In contrast, plant-specific dummies (we report this statistic only for plants that produce during all eight years of our sample: 1991-1998) account for 72% of the variance of the average product of capital.

Another way to see that persistent difference across plants account for most of the dispersion in the average product of capital is to compute the correlation of $\ln \frac{R}{K}$ with its lagged value. We do this by pooling together all plants in sample during 1991-1998. This correlation is high and equal to 0.86. In contrast, a similar correlation computed using data demeaned of plant-specific fixed effects is much lower, at 0.39. Finally, the median (across plants) time-series variance of $\ln \frac{R_i}{K_i}$ is equal to 0.16. This within-plant variation is by no means small. Figure 1 shows an example of a plant in the data with a time-series variance of the average product of capital of 0.16. Notice how in the first year the plant's revenue almost triples ($\exp(1-0)$), and then declines almost immediately. It's stock of capital however is virtually unchanged, and as a result the plant experiences an almost three-fold increase average product of capital. Although these within-plant fluctuations are large, they are transitory and dwarfed by the large and persistent cross-sectional differences across plants.

To summarize, we have documented large and persistent differences in the average product of capital across plants. The bulk of these differences are not accounted for by industry or time-variation. Rather, differences in the average product of capital are large for plants in a given industry.

D. Losses from misallocation

We next put these numbers in perspective and compute aggregate productivity losses due to misallocation. To do so, we postulate, as in Restuccia and Rogerson (2008) and Hsieh and Klenow (2008), that plant-year specific wedges (e.g. distortionary taxes/subsidies) account for the observed dispersion in the marginal revenue product of capital. That is, we back out these wedges from:

$$w_{it} = \alpha \frac{R_{it}}{K_{it}}.$$

Aggregating across plants, total revenue is

$$\begin{aligned} R_t &= \sum_i A_{it} K_{it}^{\alpha} = A_t K_t^{\alpha} \text{ where} \\ A_t &= \frac{\sum_i A_{it}^{\frac{1}{1-\alpha}} w_{it}^{-\frac{\alpha}{1-\alpha}}}{\left[\sum_i \left(\frac{A_{it}}{w_{it}}\right)^{\frac{1}{1-\alpha}}\right]^{\alpha}} \end{aligned}$$

Absent these wedges, an economy with the same amount of capital would produce

$$R_t^e = A_t^e K_t^\alpha$$

where

$$A_t^e = \left(\sum_i A_{it}^{\frac{1}{1-\alpha}} \right)^{1-\alpha}$$

We next report $\Delta \log A = \log\left(A_t / A_t^e\right)$, i.e., the losses in revenue-productivity associated with dispersion in the marginal revenue product of capital. To do so, we need a measure of α and plant-level productivity A_{it}. Before we describe how we compute these, we note briefly that when w_{it} and A_{it} are joint log-normally distributed with a correlation of $\sigma_{a,w}$ and mean and variance of w equal to 0 and σ_w^2, respectively, we have

$$\log\left(A_t / A_t^e\right) = -\frac{1}{2} \frac{\alpha}{1-\alpha} \sigma_w^2$$

In other words these losses are proportional to the variance in the marginal revenue product of capital and increase with α.

We construct a measure of A_{it} by using data on revenue and capital, together with an estimate of α, the curvature of the revenue function. Denoting the natural log of a variable with lower-case letters, we have

$$\hat{a}_{it} = r_{it} - \hat{\alpha} k_{it}$$

Recall that α is related to the primitive parameters of the model via[17]:

$$\alpha = \frac{1 + \frac{1}{\eta} - \left(1 + \frac{1}{\eta}\right)(\gamma + \beta)}{1 - \left(1 + \frac{1}{\eta}\right)(\gamma + \beta)}$$

where $\left(1 + \frac{1}{\eta}\right)(\gamma + \beta)$ is the sum of the labor and materials share in revenue and we compute

[17]We present, in an Appendix, several additional estimates of α, and redo the calculations below for each of these estimates. Two of the estimates we obtain using an extension of Hall's instrumental variable (GMM) method yield almost identical results to those below ($\alpha = 0.44$), while an Olley-Pakes-type method yields $\alpha = 0.34$.

$1+\frac{1}{\eta}$ using

$$\left(1+\frac{1}{\eta}\right)^{-1}=\frac{R}{wL+pM+uK}$$

where u is the Jorgenson user cost of capital. We construct a measure of the user cost of capital for Korea using data from Hyun, Kwack and Lee (2006) who report data on corporate income tax rates, investment tax credit, and depreciation allowance, together with an interest rate series for Korea. This calculation gives a median revenue-to-cost ratio across all plants of

$$\left(1+\frac{1}{\eta}\right)^{-1}=1.205$$

i.e., a markup of 20.5%[18]. In turn, this value of η implies $\alpha = 0.45$, a number close to that estimated by Gilchrist and Sim (2007) using data for publicly listed Korean and a different estimation technique. Notice also that the returns to scale parameter, $\nu = \left(1+\frac{1}{\eta}\right)$, is equal to 0.83, and thus also in line with numbers used in earlier work[19].

Table 3 reports the correlation between plant-level productivity and capital derived for our measure of α. We find that capital and productivity are correlated at the firm level, but imperfectly so: $\rho\left(a_{it}, k_{it}\right) = 0.33$. Aggregate productivity losses are accordingly, very large: reallocating capital to the most efficient plants would raise aggregate productivity by almost 40%. These numbers underscore the importance of understanding the source of dispersion in the measured marginal product of capital. Clearly, the welfare implications are very different depending on whether this dispersion reflects plant-specific distortionary taxes or simply differences in the technology under which a plant operates.

E. Additional features of the data

Given our focus on the average product of capital, we next present several additional features of the data. In particular, we focus on the relationship between a) the investment-capital ratio and b) the (log) of the average product of capital on one hand (vertical axis), and a plant's size, revenue, operating profits, and the capital stock on the other hand (on the

[18] Our results are not sensitive to reasonable perturbations in the user cost number we use because of the the predominant share of labor and materials. For exampple, increasing the user cost by 1/5th only lowers our estimate of the markup by 0.015, to 0.19.

[19] See e.g. Basu and Fernald (1995, 1997). Atkeson and Kehoe (2007) cite this and other evidence and argue for a span-of-control parameter ($1+\frac{1}{\eta}$ using our notation) equal to 0.85.

horizontal axis). For all dependent variables other than age, we assign plants to deciles based on the plant's characteristic (e.g. revenue) and compute the average investment-capital ratio and average product of capital for plants in each decile.

Figure 2 illustrates that the investment-capital ratio declines with age: newly entering plants invest 5% more on average than 20-year old plants[20]. The investment-capital ratio is U-shaped in plant's capital stock, with the small plants investing 15%, medium-size plants investing 11% and the largest plants investing 15%. Finally, the investment-capital ratio is sharply increasing in a plant's revenue (and operating profits, which we define as the difference between revenue and the expenditure on materials and labor). The smallest firms invest 7% of the value of the existing capital stock, while the highest revenue firms invest 22%.

Figure 3 shows that the average product of capital declines with age (the oldest plants are 30% less productive on average than new entrants), declines sharply with a plant's capital stock (almost a 2 log-point difference between the average product of capital for firms in the lowest and highest capital stock decile), and increases with revenue and operating profits (a 0.6 log-point difference). We will use these features of the data to evaluate the models.

3. Capital adjustment costs

We next study the role of adjustment costs in generating the dispersion in the average product of capital. Figure 4 reports the distribution of the investment-capital ratio in our data, for all plant-year observations (left panel) as well as for those plants that do adjust their capital stock. As the figure illustrates, roughly 60% of the plants do not buy or sell capital in a given period. Even those that do show considerable skewness and preponderance of positive investment rates, suggesting that investment spikes and irreversibility are an important feature of plant-level investment dynamics[21].

We first document several features of the investment data that have been argued useful in measuring the size of capital adjustment frictions. We then formulate and calibrate to our data a partial equilibrium model of plant dynamics in which plants are subject to transitory

[20]We truncate the data as few plants in the data survive for more than 20 years and the data is much noisier.

[21]See also Doms and Dunne (1997) for plant-level evidence for US. Abel and Eberly (1994, 1996), Bertola and Caballero, Caballero and Leahy (1996), Caballero and Engel (1999), Veracierto (2002), Thomas (2002), Khan and Thomas (2007), Bachmann-Caballero-Engel (2007) study economies with non-convex adjustment costs.

and persistent idiosyncratic productivity shocks and three types of capital adjustment frictions: a) a one-period lag between the ordering and installation of capital, b) a fixed cost of installing new capital, and c) a partial irreversibility in the form of a lower selling price of capital. We also allow for plant exit and entry. Exit takes place in our model because of a fixed operating cost that plants face every period: unprofitable ones choose to exit in order to avoid this cost, as in Hopenhayn (1992). We allow for the entry-exit margin in our model because turnover is important in the data: 18 percent of plants exit in any given year (8.4 percent of plants with more than 50 workers)[22], and failure to account for selection may lead us to overstate the persistence of productivity shocks. Moreover, we would like to evaluate our model's ability to account for the relationship between age and investment and the average product of capital we document above.

Table 4 reports several moments of the distribution of $\frac{I}{K}$ across plants in the data. We report both unweighted statistics, as well as statistics that weigh each plant-year observation by its revenue. We do so because in the data a large share of revenue (80%) is accounted for by a small number of plants (slightly less than 10%), but, as we show below, the patterns of investment are not too dissimilar for large and small plants.

The table shows that a) the distribution of $\frac{I}{K}$ is skewed (the skewness is equal to 5 and the difference between the mean and median is large: 0.13 vs. 0, respectively); b) few plants sell capital (the fraction of episodes with negative investment is equal to 4%); c) the bulk of plants invest little in any given period (67% of observations are between 0 and 3%). We also report two additional moments that will be useful in calibrating the model: the mean investment-capital ratio for negative investment episodes is equal to -0.13, and the correlation of $\frac{I}{K}$ with its lagged value is equal to 0.08. As Cooper and Haltiwanger (2007) show, the latter statistic is useful to discriminate between fixed installation costs (which induce negative serial correlation in investment) and a partial irrevesibility, both of which are capable of generating episodes of inaction. As the right column of Table 4 shows, these features of the data are similar for large plants, although the weighted moments show somewhat less inaction[23] (32% vs. 63%), more frequenct sales of capital (7% vs. 4%), and less skewness (3.49 vs. 5.01). The latter numbers are similar to those reported in Cooper and Haltiwanger (2007) for the

[22]These numbers are reported for 1991-1996 only. The fraction of plants that exit is much higher (30%) in 1997-98, the years of the Asian financial crisis.

[23]We define episodes of "inaction" as episodes in which the investment-capital ratio is greater or equal to 0 and less than a quarter of the mean investment-capital ratio.

US Annual Survey of Manufacturing which samples mostly large plants.

A. Model Economy

The economy is inhabited by a continuum of plants, each of which has access to a production technology that produces profits (revenue net of the cost of materials and labor, see the previous section for a full description of the technology) using

$$\Pi = \exp(a + z) K^{\alpha}$$

where Π is profits, K the capital stock, a is a persistent productivity component, and z is a purely transitory component of the plant's productivity. We assume that $z \sim N(0, \sigma_z^2)$ and that the incumbents' persistent productivity component, a, follows an AR(1) process with innovations $\varepsilon \sim N(0, \sigma_\varepsilon^2)$:

$$a = \rho a_{-1} + \varepsilon.$$

Both z and ε are iid across firms and time. We first describe the problem of incumbent plants and then the entry decisions.

To continue operating in the industry, an incumbent plant must pay a fixed operating cost ξ at the end of each period. This fixed cost buys the right to operate next period and must be paid prior to the realization of next period's productivity shocks. We assume that the fixed cost ξ is random and iid, drawn from a distribution $G(\xi) = \left(\frac{\xi - 0}{\xi_{\max} - 0}\right)^{\theta}$ and revealed at the beginning of the period (before the exit decision is made). The support of this distribution is $[0, \xi_{\max}]$ and θ governs its shape.

The plant enters the period taking as given its existing stock of capital K, productivity, a and z, as well as the size of the fixed cost, ξ, it must pay in order to continue operating again next period. The plant chooses whether to pay the fixed cost and continue, or exit the industry. If the plant continues, it decides how much capital to sell or purchase (if anything). We normalize the purchase price of capital to 1. The selling price of capital is equal to P_s and is assumed less then or equal to 1. Sales or purchases of capital entail payment of a fixed installation cost κ.

We next present the plant's dynamic program. Let

$$V(K, a, z, \xi) = \max(V^a, V^n, V^e)$$

denote the value of a plant with initial capital K, productivity a and z and operating fixed cost ξ. This value is the envelope of the values associated with the three options the plant has: continue and adjust its capital stock, V^a; continue and not adjust its capital stock, V^n; exit, V^e. These values are given by:

$$\begin{aligned} V^a &= \max_{K'} \exp(a+z) K^\alpha - \kappa - \xi - \underbrace{(K' - (1-\delta)K)}_{I} \times \\ &\quad (1 \times (I > 0) + P_s (I < 0)) + \beta \int\limits_{a' \times z' \times \xi'} V(K', a', z', \xi') dF(a', z', \xi' | a) \end{aligned} \tag{1}$$

$$V^n = \exp(a+z) K^\alpha - \xi + \beta \int\limits_{a' \times z' \times \xi'} V((1-\delta)K, a', z', \xi') dF(a', z', \xi' | a) \tag{2}$$

$$V^e = \exp(a+z) K^\alpha + P_s (1-\delta) K \tag{3}$$

We assume that the plant owners discount profits at rate β and are risk-neutral. An exiting plant earns profits and sells the undepreciated portion of its capital stock. We assume that the fixed cost is paid one-period in advance in order to isolate the role adjustment costs have in generating dispersion in the average product of capital. The alternative approach of liquidating the exiting plant at the beginning of the period would make it optimal for firms that have a high likelihood of exiting to under-invest, as the returns to investment are lower in event of exit. Given that under-investment is solely an artefact of our time-to-build assumption, we opted to impose a similar time-to-build assumption on investment in the fixed operating cost. We show below that our model accounts well for the negative relationship between the average product of capital and age whereas the alternative timing assumption would not because younger firms are more likely to exit and would find it optimal to hold too litle capital.

Consider next a plant's entry decision. Entry requires payment of a fixed cost, ϕ_e. The plant then draws its productivity a from $N\left(0, \frac{\sigma_a^2}{1-\rho^2}\right)$, and purchases $K(a)$ units of capital. Capital becomes productive with a one-period delay, at which point the plant learns ε, z, ξ and proceeds like any other incumbent. The value of the potential entrant is thus given by:

$$V^o = -\phi_e + \int_a \left[\max_{K(a)} -K(a) + \beta \int_{a' \times z' \times \xi'} V(K, a', z', \xi') dF(a', z', \xi' | a) \right] d\Phi(a)$$

As above, we make these assumptions (that a is revealed prior to the plant's purchase of K and is then subject to an additional shock) to ensure that the entry-exit margin does not distort the allocation of capital for entering vs. incumbent firms. We assume that the fixed entering cost is such that $V^o = 0$, a necessary condition for an invariant measure of firms in an economy with positive entry.

B. Decision Rules

The exit decision is characterized by an adjustment cutoff $\bar{\xi}(K, a)$: the highest realization of the operating fixed cost draw that makes an incumbent plant indifferent between continuing and exiting. In particular, $\bar{\xi}(K, a, z)$ satisfies

$$\max \left[V^a(K, a, z, \bar{\xi}), V^n(K, a, z, \bar{\xi}) \right] = \exp(a + z) K^\alpha + P_s (1 - \delta) K$$

One can easily show by inspecting the value functions that, when $\kappa = 0$, our timing assumptions and the assumption that z is iid imply that the exit cutoff $\bar{\xi}(K, a, z)$ is independent of z, increasing in a as long as $\rho > 0$ and (weakly) increasing in K as long as $P_s < 1$.

We next describe the optimal investment policy of the plant: $K'(K, a)$. Figure 4 presents this decision rule for a plant that faces a) no fixed installation costs, b) no partial irreversibility and c) both frictions (left, right, and lower panels of the figure) in the productivity, a, space. We contrast these decision rules with those that are optimal in the absence of adjustment frictions. Notice that both the fixed cost and the irreversibility generate an inaction region in which $K' = (1 - \delta) K$. The fixed cost generates inaction because the plant finds it optimal in some states to exercise the option to not invest and save the fixed installation cost, and the irreversibility because of the discontinuity in the price schedule faced by the plant. The difference between the two decision rules is that $K'(K, a)$ is continuous in a in the presence of irreversibilites only, and exhibits a jump outside the inaction region in the presence of fixed costs.

C. Parametrization

We next describe the choice of parameter values we use. We assign values to two parameters: the discount factor, β, is assumed equal to 0.925, consistent with the real corporate bond rate of 8% in Korea during our sample period. We also assign a value $\alpha = 0.45$ to the curvature of the profit function, a number necessary to rationalize the revenue-cost ratio given the factor shares we observe in the data.

The rest of the parameters are jointly calibrated using a simulated method of moments-type procedure that minimizes the distance between moments in our data and the model-simulated data. We compute moments in the model by constructing an 8-year panel of plants, the same length as in the data, for an economy that has converged to the invariant distribution. As in the data, we drop from all calculations observations with an average product of capital in the bottom and top 1 percentile. The parameters we calibrate are ρ – the parameter governing the persistence of productivity shocks; σ_a and σ_z – the standard deviation of productivity shocks; δ – the rate at which capital depreciates; P_s– the selling price of capital; κ– the fixed cost of installing capital; $\xi_{\max}$ – the upper bound of the fixed operating cost, and θ – the parameter determining the curvature of the distribution of fixed operating costs.

The 14 moments we use to pin down these eight parameters are listed in Table 5. These are moments of the distribution of log revenue[24]: variance and the first, and second-order autocorrelation (the three jointly are informative about ρ, σ_a^2 and σ_z^2); the exit hazards of plants (we condition on the age of the plant and compute separate hazards for plants aged 1-5, 6-10, and 11-20), which are clearly informative about the parameters of the fixed cost distribution, as well as the moments of the distribution of the investment-capital ratio we have discussed above. The latter help pin down the size of the adjustment frictions, as well

[24] Revenue is proportional to profits under the assumptions we make on the production technology. We use data on revenue rather than profits in order to minimize the role of measurement error (the cost of goods sold is more difficult to measure because of the various accounting practices employed to value inventories). All the statistics we report are similar whether we use data on profits or revenue (the correlation between the two is 0.92).

as the rate at which capital depreciates (mean $\frac{I}{K}$).

The second column of Table 5 reports moments in the model (column titled "Benchmark") and contrasts them with those in the data. The model accounts well for the main regularities in the data, including the adjustment hazards, as well as the distribution of the investment-capital ratio. The last column reports statistics for an economy with time-to-build alone ($P_s = 1$, $\kappa = 0$ and all parameters identical to those in the Benchmark): clearly, the economies with and without adjustment costs differ substantially in the distribution of investment-capital ratios they predict, suggesting that adjustment cost play an important role in our model in shaping plant-level investment dynamics.

Figure 6 (upper panel) reports the exit hazards in the age, revenue, capital, and profit space. Although we have only targeted the age-hazard profile, the model accounts well for the other profiles as well. The lower panel of Figure 5 reports the growth rates of a plant's sales as a function of plant characteristics. Notice that the model accounts well for the scale (as measured by revenue and profits) and age-dependence of growth rates. Also notice that a model with time-to-build alone replicates most of these facts as well. Finally, the main discrepancy between the model and the data is the lack (in the data) of the negative relationship (driven by mean-reversion in productivity) between growth and the size of a plant's stock of capital. We return to this discrepancy below.

We report the parameter values that achieve this fit in Table 6. The transitory component of productivity is quite volatile, $\sigma_z = 0.30$, a result driven by the requirement that the model simultaneously accounts for the lower first-order serial correlation of log revenue, 0.942 with the higher ratio of the second to first-order serial correlation: 0.972. The resulting AR(1) coefficient for the persistent productivity component is equal to 0.949. Also notice that adjustment costs on capital are non-trivial. The selling price of capital, P_s, is equal to 0.81, while the the fixed installation cost is equal to 0.89% of the mean investment. Finally, the distribution of the fixed costs of operating has wide support (the upper bound is 14.7 times the mean per-period profits), but most of the mass is concentrated in the neighborhood of 0 as θ is equal to 0.089. The distribution of fixed costs is thus one in which most firms face small fixed costs, while a few draw extraordinarily large costs. The latter feature is necessary in order to account for the fact that a non-trivial proportion of older (and thus larger) plants exit in the data.

D. Misallocation

We next ask, to what extent do capital adjustment frictions prevent optimal reallocation of capital in our model economy? As above, we measure misallocation by computing the variance of the (log) average product of capital. We report these statistics, in the model and in the data, in Table 7. We also report statistics for the economy with no adjustment costs.

Table 7 shows that, despite the considerable inaction in the Benchmark model, it generates little dispersion in the average product of capital. The economy with no adjustment costs and time-to-build alone predicts $var(r/k) = 0.13$. Adding the capital adjustment costs raises the variance of the average product of capital to 0.17, i.e. roughly 1/7th of the dispersion in the data. Similarly, the Benchmark model predicts too little autocorrelation in r/k : 0.34 vs. 0.87 in the data.

Adjustment frictions do account to a large extent for the within-plant time-series variance of the average product of capital (as measured by the median (for plants in sample during all eight years) time-series variance of a given plant's revenue-capital ratio): 0.14 vs. 0.16. Interestingly, this dispersion is driven to a large extent by the assumption that new capital is installed with a one-period delay as the model with no adjustment costs alone predicts the bulk of this dispersion: 0.13. Notice also that while in the data fluctuations in the average product of capital are somewhat persistent (serial correlation is 0.39), the model with adjustment costs predicts much more transitory deviations (the autocorrelation is 0.17).

The final row of Table 7 reports the aggregate productivity losses due to misallocation. As reported above, these losses are a staggering 39.5% in the data, and equal to -6.7% in the economy with adjustment costs (-4.9% in the economy with time-to-build alone).

We conclude thus that capital adjustment costs account for little of the observed dispersion of the average product of capital: their marginal contribution is to raise the variance of the average product of capital by 0.04 and generate additional TFP losses of 1.8% relative to what a model with time-to-build alone predicts.

This result is driven by the fact that large differences in the marginal product of capital from its user cost entail large losses, large enough to justify payment of the adjustment costs. Thus adjustment costs generate only small and temporary gaps in the plant's actual stock of capital from its desired level. The large and more persistent deviations of the type observed in the data are difficult to rationalize using adjustment frictions alone. At work here is a selection effect at play in models with adjustment frictions of this type. Although few plants invest in any given period, those that do so are exactly those that need to invest. The many plants that do not invest have a stock of capital sufficiently close to the desired one, making inaction optimal and thus contributing little to the dispersion in the marginal product of capital.

E. Additional Implications

We next evaluate the model's implications vis-a-vis the additional features of the data that we discussed: the relationship between the investment-capital ratio and average product of capital and plant characteristics. We report statistics computed for the benchmark model with adjustment costs, as well as for an economy with no capital adjustment frictions other than time-to-build, all computed in the same fashion as the statistics in the data.

First, notice in Figure 7 (lower panel) that the model with adjustment costs accounts well for the negative relationship between age and the average product of capital: as in the data, newly entering plants have an average product of capital that is 0.3 log-points higher than that of plants that are 20 years old. Notice that this property is specific to the model with adjustment costs, and is thus driven by the positive correlation between the stock of capital and age, which, combined with the partial irreversibility makes it suboptimal for older plants to sell capital. The reason these age-induced differences in the average product of capital have little effect on the unconditional variance of r/k is the high turnover in both the model and the data which imparts considerable skewness to the age distribution of plants: most plants are young in both the model and the data.

The other panels of Figure 7 illustrate however that the model misses important features of the data. First, the investment-capital ratio, reported in the upper panel of Figure 7, is strongly positively correlated with revenue and operating profits in the data. The models, in contrast, predict a negative correlation because of the mean reversion in productivity: plants that are more productive today and thus have higher revenue expect a decrease in productivity next period and hence invest less. Similarly, the investment-capital ratio is roughly independent of the capital stock in the data, but strongly negatively correlated with it in the model, again due to the mean-reversion in productivity.

Second, the model predicts a virtually flat relationship between the average product of capital and revenue/operating profits. In contrast, the average product of capital is strongly negatively correlated with the stock of capital in the data, and virtually flat in the model.

These types of relationships in the data are typically interpreted as evidence of borrowing frictions that drive a wedge between the cost of internal and external funds. Borrowing frictions have the potential to account for these features of the data because plants with higher revenue have more funds to finance investment (thus implying higher investment-revenue sensitivities) and higher average product of capital (because the financing frictions prevent plants from fully responding to good productivity shocks). We next ask, to what extent can a model with financing frictions accounts for the facts in Figure 7 generate capital misallocation?

4. Role of financing frictions

One challenge we face in measuring financing frictions is that we have only plant and not firm identifiers[25]. As a result we lack balance sheet information[26] and other financial information in our data. A careful assessment of the size of financing frictions requires assessing the costs of equity issuance, bankruptcy costs, as well as details of the tax system, a task impossible given the limitation of our data. Rather than build a rich model which incorporates many important details of corporate finance[27], we chose instead to study a parsimonious model in which firms can borrow (but not save) using a one-period non-state-contingent debt contract, and in which limited enforcement of debt contracts imposes a constraint on the plant's ability to borrow.

The difference between the model of this section and the model we study above lies in our assumption here that firms cannot issue equity. In fact, a non-negative dividends constraint alone makes, quantitatively, little difference to our original results. Given the markup implied by the curvature of the profit function we use, a plant's period-by-period profits are sufficient for most firms to finance all desired investment. To allow finance a non-trivial role, we thus require that firms maintain a minimum payout ratio of $1-\phi$. Investment in new capital must thus be financed using the ϕ fraction of profits the plant receives in any given period or using debt.

[25]Gilchrist and Sim (2007) and Kim and Rousseau (2008) use a micro-level dataset for listed firms in Korea that allows them to more precisely measure financing constraints. Their sample size is however much smaller and includes mostly large firms, so we chose not to evaluate our model's implications vis-a-vis the statistics reported in these papers.

[26]Except for debt and asset information in the first two years of the sample, although interpreting this data through the lens of our single-plant firms model is difficult because we have plant and not firm-level information.

[27]See Hennessy and Whited (2007) for a study of Compustat firms.

We assume that plants have the option to default on their debt. If they do so they lose a fraction λ of their capital stock. Plants therefore are borrowing constraint and cannot issue debt that exceeds a fraction λ of its capital stock. In addition, we assume no adjustment costs (the sole friction we allow is time-to-build) and shut down the exit-entry margin (by assuming away the fixed costs of production). We do so in order to isolate the role of financing frictions, but also because we have established above that an economy that does have these features 1) generates little additional dispersion in the average product of capital and 2) accounts well for the relationship between age and the average product of capital. Recall that the latter feature of the model plays little role because of the preponderance of younger firms.

Our assumption that firms can only borrow, but not save, is clearly restrictive. Firms in this economy have the incentive to save for precautionary reasons due to the possibility of a binding borrowing constraint. Given that capital is their only means of saving, unproductive firms will choose to invest by more than they would if they had access to an additional savings technology. This economy will thus generate a larger degree of misallocation than an economy in which firms can save. This is the case both because of the dual role of capital as a factor of production and means of savings, but also because firms are precluded from accumulating enough assets to reduce the need for external finance. What we argue below, however, is that financing frictions, even when very severe, generate very little dispersion in the marginal product of capital across firms. Allowing an additional savings technology would thus only strengthen our argument[28].

A. Model

The key features of the model, the production technology and the specification of the two productivity shocks, are identical to those of the economy studied above. We abstract from capital adjustment costs and the exit/entry margin. We flesh out next the additional details of the economy with financing frictions.

Firms start the period taking as given their debt obligations, denoted by B, as well as their productivity, a and z, and the existing capital stock. They produce and earn profits $\Pi = \exp(a + z)K^{\alpha}$ of which a fraction $(1 - \phi)$ is paid as dividends[29]. The plant then invests

[28]We do allow plants to save in the model of the next section, and make this argument explicit.

[29]We implictly assume here that firms face no frictions in financing labor and intermediate good expenditures. Thus financing frictions only distort the investment margin.

by purchasing $K' - (1 - \delta) K$ units of capital. It finances this purchase using the fraction of profits it retains, net of the bond payments, as well as new bond issues. We assume that the price at which the plant issues bonds is equal to $q = \frac{1}{1+r} = \beta$, the rate at which the plant discounts profits. Let B' denote the quantity of bonds the plant issues. Then the 'no equity issuance' constraint says:

$$I \leq \frac{1}{1+r}\phi \exp(a + z)K^{\alpha} - B + \frac{1}{1+r}B'$$

This is an economy in which debt contracts cannot be enforced. That is, firms can walk away from their debt obligations. If they do so, they lose a fraction λ of their undepreciated capital stock $(1 - \delta) K$. Given that this is the only punishment for default, and given that the defaulting plant is assumed to re-gain access to financial markets immediately, the plant will choose to default as long as

$$B > \lambda (1 - \delta) K$$

This then implies an upper bound on the plant's ability to borrow and no default occurs in equilibrium.

The plant's dynamic program is therefore (we reduce the dimensionality of the state-space by defining

$$W = \phi \exp(a + z)K^{\alpha} + (1 - \delta) K - B,$$

where W is the plant's net worth after dividend issuance):

$$V(W, a) = \max_{K', B'} \left(W - K' + \frac{1}{1+r}B' \right) +$$
$$\beta \int (V(W', a') + (1 - \phi) \exp(a' + z') K'^{\alpha}) \, dF(a', z'|a)$$

subject to:

$$0 \leq B' \leq \lambda (1 - \delta) K'$$

$$K' \geq 0$$

$$W - K' + \frac{1}{1+r} B' \geq 0$$

$$W' = \phi \exp (a' + z') K'^{\alpha} + (1 - \delta) K' - B'$$

B. Decision Rules

The first order condition for bond issues is:

$$1 + \mu - \gamma(1 + r) = \beta(1 + r) \int [1 + \mu(W', a')] \, dF(a', z'|a)$$

where μ is the multiplier on the 'no equity issuance constraint' and γ is the multiplier on the borrowing constraint.

Inspecting the problem above it is straightforward to show that given a choice of capital stock K', the plant's bond issues satisfy

$$B' = \min [\max (0, K' - W) , \ \lambda (1 - \delta) K']$$

The min operator reflects the borrowing constraint, while the max operator simply reflects the fact that the plant either does not borrow (if it has sufficient internal funds to finance its investment) or borrows exactly the amount necessary to satisfy the 'no equity issuance' constraint. The reason the plant's debt never exceeds this amount is because of the assumption that $\beta (1 + r) = 1$ which makes it suboptimal to borrow simply to raise dividend payments. This follows because of risk-neutrality (the benefit to raising dividends by issuing an additional unit of debt is equal to 1) and the possibility of the borrowing constraint binding next period, which implies that the marginal cost of borrowing is greater than 1:

$$\beta(1 + r) \int [1 + \mu(W', a')] \, dF(a', z'|a) > 1$$

where $\mu(W', a') \geq 0$ is the multiplier on the 'no equity issuance constraint' at state (W', a').

Notice finally that choices of K' for which $(1+r)(K' - W) > \lambda(1-\delta) K'$ are not in the plant's budget set. In this case the plant cannot simultaneously satisfy the two constraints: the maximum amount it can borrow is not sufficient to finance its capital expenditures. This then gives an upper bound on the maximum stock of capital the plant can carry into the next period: $K' < \frac{(1+r)W}{1+r-\lambda(1-\delta)}$.

The first order conditions for capital accumulation is:

$$1 + \mu - \gamma\lambda(1-\delta) = \beta \int \alpha \left[1 + \phi\mu(W', a')\right] \exp(a' + z') K'^{\alpha-1} dF(a', z'|a) + (1-\delta)$$

Notice how the returns to capital depend on ϕ as the higher ϕ is, the higher a plant's net worth (after paying dividends) which relaxes next period's 'no equity issuance' constraint.

Figure 8 illustrates the capital decision rules in the a space for two plants that differ in their net worth. We also contrast these decision rules with those in the frictionless economy. The a space divides into three regions, reflecting the constraints that bind for those productivity levels. The first region is one with very low productivity shocks that make it optimal for the plant to invest little. None of the constraints thus bind in this region, although notice that the plant carries more capital into the next period than it would in the frictionless environment (precautionary savings motive) and its capital stock is less sensitive to changes in a (as an increase in a allows the plant to come closer to its frictionless capital stock since the $\mu(W', a')$ constraint is less likely to bind).

The second region (for intermediate values of a) is one in which K' is initially flat in W and then rises. This is the region in which the no-equity issuance constraint binds, and the plant must borrow to finance its investment. The flat region arises from a discontinous jump in the cost of investing at the point at which investment must be financed by issuing bonds (similar to that in the economy with partial irreversibility). To see the source of this discontinuity, notice that if the plant finances investment with internal funds, the cost of an additional unit of capital is equal to 1, while if the plant finances by borrowing, the cost is equal to $\beta(1+r) \int [1 + \mu(W', a')] \, dF(a', z'|a) > 1$ because of the non-zero probability of the borrowing constraint binding next period (repaying the bond is costly in states of the world in which the plant is constrained in the availability of internal funds). Finally, the third region is one in which both the borrowing constraint and no equity-issuance constraint bind and the plant chooses the upper bound on its capital stock: $K' = \frac{(1+r)W}{1+r-\lambda(1-\delta)}$.

C. Parametrization

The discussion above suggests that the role of borrowing constraints in this economy is to reduce the elasticity of a plant's capital stock to its productivity (relative to the frictionless optimum) and thus prevent optimal allocation of capital. It also suggests that borrowing frictions operate in this setup in a similar fashion as adjustment frictions on capital do: by increasing the elasticity of the average product of capital with respect to a plant's revenue and profits. The more constrained firms are, the less sensitive is a plant's capital stock to its productivity (compare the two firms in Figure 8), and thus the more sensitive its average product of capital to revenue and profits. This suggests an important dimension of the data the model must account for and motivates our strategy to use the elasticity of the average product of capital to revenue to discipline the strength of borrowing frictions.

As above, we assign values to the parameters of our model that are difficult to pin down using the data we have: we set $\alpha = 0.45$, and $\beta = 0.925$. We pin down the parameters that govern the process for productivity shocks, ρ, σ_ε and σ_z to match the variance, first and second-order covariance of log revenue and the rate of which capital depreciates, δ, to match the investment-to-capital ratio in the data. The two parameters that determine the strength of the borrowing frictions, ϕ and λ, are chosen to account for three features of our data that, we have shown, a model without financing frictions cannot account for: the elasticity of $\frac{I}{K}$ with respect to a plant's profits (0.032 as measured by the slope of a regression of $\frac{I}{K}$ on $\log(R - pM - wL)$ in the data[30]), the elasticity of $\frac{I}{K}$ with respect to the capital stock (0.001), and finally the elasticity of the average product of capital with respect to revenue (0.113 as measured by the slope of a regression of $\log \frac{R}{K}$ on $\log(R)$ in the data). [31]

A debate in the corporate finance literature, originating with Fazzari, Hubbard, Petersen (1988) focuses on the ability of invesment-cash flow sensitivities to uncover the size of financing frictions. A number of recent authors, including Erickson and Whited (2000), Gomes (2001), Cooper and Ejarque (2001), Kaplan and Zingales (1997), have criticized the approach of using investment-cash flow regressions to measure financing constraints. We are

[30]Standard errors for all these estimates are very small (of the order of 10^{-4}) given the large number of observations we have.

[31]Profits are proportional to sales (revenue) in the model, and strongly correlated in the data. The elasticity of $\frac{I}{K}$ with respect to revenues is equal to 0.035 and that of $\log(\frac{R}{K})$ to $\log R$ is equal to 0.071. Thus targeting this additional set of moments makes little difference to our results.

aware of this criticism: the positive correlation between profits and the investment-capital ratio in the data can indeed be driven by measurement error, failure to account for q, or can be accounted for by other departures from the model above (e.g. allowing for serially correlation in productivity *growth rates*[32]). This motivates us to target a broader set of moments in addition to cash-flow sensitivities in the model. We nevertheless do not interpret our exercise as one of measuring financial frictions. Rather, the goal is to study the properties of an economy with financing frictions large enough to account for the relationship between plant characteristics and the investment-capital ratio and average product of capital.

Table 8 (column labeled "Benchmark") reports the parameter values that give the best fit, while Table 9 reports the moments in the model and in the data. Notice that the model requires fairly stringent financing frictions in order to account for how the investment-capital ratio and the average product of capital varies with a plant's capital stock and profits. In particular, the minimum dividend payout ratio is equal to $1 - \phi = 0.55$, and firms are able to borrow up to only a fraction $\lambda = 0.10$ of their capital stock. Notice in Table 9 that the fit of the model is imperfect as the model cannot simultaneously account for all targets used to pin down the strength of borrowing frictions. In particular, the investment cash flows sensivities are too low in the model relative to the data (both by roughly 0.01), while the average product of capital is somewhat too sensitive to a plant's profits in the model (the elasticity in the model is 0.135 vs. 0.113 in the data).

The second column (labeled "Large FF" (large financing frictions)) of Tables 8 and 9 report parameter values and moments of an alternative parameterization of this economy in which we exclude the elasticity of the average product of capital to revenue from the set of moments used in calibration. In this economy the investment sensitivies are very close to those in the data. This fit comes at the expense of the model's ability to account for the elasticity of the average product of capital with respect to profits. This elasticity is now double relative to the data. Given our focus on the model's ability to account for the dispersion in the average product of capital and the arguments in earlier work against using investment sensitivities alone to gauge the extent of the borrowing frictions, we prefer the "Benchmark" estimates which reconcile the tension in simultaneously accounting for both targets in the data. Notice also, in Table 8, the sense in which financing frictions in the latter

[32] Which would however preclude the model from accounting for the scale dependence in establishment growth rates we document above.

economy are "large": firms must pay 95% of their profits as dividends, and can only borrow up to a quarter of their capital stock.

The last columns of Table 8 and 9 report parameter values and moments in a frictionless economy with time-to-build only. In this economy the elasticity of investment to profits is equal to 0, and that with respect to capital is much lower than in the data (-0.026 vs. 0.001), while the elasticity of the average product of capital to profits is one-half of its value in the data.

Figures 9 and 10 summarize this discussion by comparing how the investment-capital ratio and average product of capital vary across deciles of the profit and capital distribution. Notice that the Benchmark economy fits the $\log \frac{R}{K}$–$\log (R)$ profile almost perfectly, except at the last decile of the distribution (the most profitable plants), while somewhat missing the investment sensitivities. In contrast, the economy with larger frictions does poorly at accounting for how the average product of capital varies with revenue. Finally, notice in the figures that none of the models is capable of accounting for the steep negative relationship between the average product of capital and a plant's capital stock. We return to this issue below.

Interestingly, if we compute the investment elasticities used to pin down the size of financing frictions only for the largest plants in our data (the slightly fewer than 20 % of plants which together account for 90% of revenue), we obtain an elasticity of $\frac{I}{K}$ to $\ln (K)$ equal to -0.03 and $\frac{I}{K}$ to $\ln(\Pi)$ equal to 0.01. These statistics are much closer to those in the economy without financing frictions, suggesting that financing frictions play a smaller role for the largest plants. Recall however from Table 1 that the dispersion in the average product of capital for these plants is also very high and equal to 1.11 (vs. 1.23 for all plants). To us this is additional evidence that financing frictions play little role in accounting for the dispersion in the average product of capital in the data.

D. Misallocation

We next show that the financing frictions we pin down above, although arguably large, account for little of the unconditional dispersion in the average product of capital we observe in the data. To see this, Table 10 reports the variance and autocorrelation of the average product of capital. Notice that the variance of $\log \frac{R}{K}$ is equal to 0.20, and thus only 0.08 points higher than that in a frictionless economy with time-to-build alone. Financing frictions thus only account for 7% (0.08/1.23) of the unconditional dispersion of the average product of capital in the data. Importantly, this economy accounts almost fully for the within-plant time series dispersion and autocorrelation of the average product of capital (the variance is 0.17 in the model vs. 0.16 in the data, and the autocorrelation is 0.29 in the model and 0.39 in the data). The last row of the table reports the aggregate productivity losses in this economy: equalizing the marginal product of the capital across plants would raise TFP in this economy by 7.5%. Comparing the "Benchmark" and "Frictionless" columns of Table 10 implies that borrowing constraints generate additional 3% aggregate productivity losses, much smaller than those we have computed in the data.

Notice that we are not suggesting that financing frictions *cannot* generate substantial TFP losses through misallocation. Consider the "Large Financial Frictions" column, which is the economy calibrated to match the investment to capital and profit sensitivies alone. This economy accounts for 1/3 of the dispersion in the average product of capital in the data and generates much larger (almost 14 percent) aggregate productivity losses, reflecting the much more stringent borrowing frictions. Recall however that the average product of capital is much more sensitive in this economy to sales than it is in the data. Thus the dispersion in $\log \frac{R}{K}$ predicted by the model simply reflects this counterfactually high elasticity. Moreover, this economy also generates too much within-plant variance and persistence in the average product of capital. This again suggests that this economy's ability to generate additional unconditional dispersion in $\log \frac{R}{K}$ comes at the cost of missing important features of the data.

E. 1997-1998 Financial Crisis

The 1997-1998 Korean financial crisis provides an interesting case study against which to evaluate our model's predictions. The crisis started in January of 1997 with the bankruptcy of Hanbo steel, one of the largest chaebols, followed by the failure of another steel producer (Samni group), as well as a number of other chaebols and business groups (including Kia motors, the third largest automakers in July 1997)[33]. Figure 11 (left panel) reports the investment/capital ratio (left axis) as well as the dispersion of the average product of capital for each of the years 1991-1998 in our sample. Given that our model abstracts from entry and exit, we compute the latter statistics only for a balanced panel of approximately 15,000

[33]See Adelman and Nak (1998) for a detailed description of the crisis.

plants that are in sample throughout this period (a total of 121,000 plant × year observations that together account for 60% of revenue in the entire sample).

Notice in the Figure that the investment-capital ratio declines sharply, from 0.14 to 0.11 in 1997 and to a further 0.07 in 1998. The industry also experiences a fairly large increase in the dispersion of the average product of capital in 1998, the first year for which the investment decisions during the years of the crisis affect the plant's measured stock of capital.[34] The variance of the average product of capital is equal to 0.90 on average during this period (lower than reported earlier because of our focus here on a balanced panel), and increases by 0.24 (0.97 to 1.21) from 1997 to 1998.

The right panel of Figure 11 reports the aggregate (revenue) productivity of capital (measured as $A_t = \frac{\sum_i R_{it}}{\left(\sum_i K_{it}\right)^{\alpha}}$ with $\alpha = 0.45$) where R_{it} is a plant's (real) revenue and K_{it} is its (real) capital stock), together with the efficient productivity level A_t^e that would prevail absent dispersion in the average product of capital. The Figure shows that both measures of productivity decline sharply during 1998, the year that follows the October devaluation. Importantly, while the actual productivity declines by 0.19 log-points, the efficient productivity level does so by only 0.14 log-points. The increase in misallocation documented thus accounts for one quarter (0.05/0.19) of the productivity growth during this year, a sizable amount.

We next ask, to what extent can financing frictions account for the 5% drop in capital productivity due to misallocation in the data? To do so, we start from the stationary distribution of the Benchmark economy with $\phi = 0.45$ and $\lambda = 0.10$ and tighten the plant's borrowing constraint by permanently lowering λ, i.e., the share of its capital stock the plant can use as collateral, to 0.01 (we choose this number to generate the 0.03 drop in the average investment/capital ratio we observe in 1997 in the data). We recompute plant's decision rules and find an increase in the variance of $\log\left(\frac{R}{K}\right)$ to 0.21 (from 0.20) and an increase in TFP losses to 7.8% (from 7.5%) in the first year after imposing the more stringent borrowing constraint. Thus financing frictions (although severe) alone account for a small fraction of the observed increase in misallocation in 1998.

Notice that this result is not driven by our understating of the effect of borrowing frictions during the crisis. An alternative extreme experiment in which we do not allow plants to invest at all for one period increases the variance of the average product of capital

[34]Recall that we construct our measure of capital stock according to $K_t = I_{t-1} + K_{t-1} - DEP_{t-1}$.

in the subsequent year to only 0.25 and the TFP losses to 9.6%. Thus one period alone is simply insufficient (given the volatility of innovations to the productivity process) to generate much misallocation in our model even if firms do not invest at all.

F. Discussion

The intuition for why financing frictions, although arguably very severe, play little role in raising the dispersion of the average product of capital in the model economy we study, is as follows. Although the unconditional dispersion of plant productivity ($a_{it}+z_{it}$) is high, both in the model (its variance is equal to 0.83) and in the data (0.98, computed as $r_{it}-\alpha k_{it}$ with $\alpha=0.45$), the variance of *changes* in a plant's productivity is much lower (0.27 in the model vs. 0.26 in the data), much of it reflecting the transitory component to which firms cannot respond at all because of our time-to-build assumption. Thus, the variance of changes in a_{it}, the persistent component to which firms do respond, is equal to only 0.07. Consider now a plant that starts in the lower tail of the a distribution and that eventually becomes productive and transits to the upper tail of the a distribution. Given the low variance of changes in a, the transition from a low productivity state to a high productivity state is gradual, giving the plant ample time to accumulate sufficient capital to grow out of its borrowing constraint. If the transition from a low a to a high a were more rapid, financing frictions would be more potent, as some firms that start with a low productivity and a low capital stock would become highly productive but constrained and unable to finance their investment.

A similar argument applies to the model with adjustment costs as well: even though few firms adjust in any given period, the extent of inaction in the data is far too low (40% do invest in any given period), thus giving firms enough time to respond to persistent shocks to their productivity. Recall that both types of frictions we have studied almost fully account for the time-series dispersion in the average product of capital in the data. The latter is much lower than the unconditional distribution, reflecting again that year-to-year changes in a plant's productivity and revenue are not too large.

Leptokurtic productivity shocks

The discussion above suggests that the nature of productivity shocks may determine whether firms can finance their desired investment in response to increases in productivity.

In particular, we have assumed log-normal productivity shocks, an assumption that makes it difficult for the models considered above to account for two features of the data.

First, the size distribution of firms shows much more inequality then the models: e.g., the Benchmark economy with finance frictions produces a Gini coefficient of only 0.69 and 90% of all sales are accounted for by the largest 44% of the plants, whereas the Gini coefficient in the data is equal to 0.88 and 90% of all sales is accounted for by only 19.5% of the largest plants. Second, the distribution of changes in a plant's productivity (which we compute in the data using $A_{it} = r_{it} - \alpha k_{it}$ with $\alpha = 0.45$) is fat-tailed, with a kurtosis equal to 8.2[35] (and close to 3 in the model).

We next consider an economy in which shocks to the persistent productivity component, ε_{it} are drawn from a distribution with fat-tailed shocks. In particular, letting $\tilde{\varepsilon}_{it} \sim N(0,1)$, we assume

$$\varepsilon_{it} = \sigma_{\varepsilon}\tilde{\varepsilon}_{it} \exp\left(\kappa \frac{\tilde{\varepsilon}_{it}^2}{2}\right)$$

where κ determines the kurtosis of the distribution, and σ its variance[36]. We assume that transitory productivity shocks are log-normal and maintain the assumption of an AR(1) process for $a_{it.}$. We keep the parameters governing the strength of borrowing frictions equal to their values in the Benchmark experiment above, $\phi = 0.45$ and $\lambda = 0.10$, and recalibrate $\sigma_z, \rho, \sigma_{\varepsilon}$ as well as κ to match the variance and first and second auto-correlation of revenue, as well as a Gini coefficient of 0.88 as in the data[37].

The next-to-last column of Table 10 reports the results and contrasts them to those in our Benchmark economy with finance frictions. Clearly the economy with fat-tailed productivity shocks generates a larger dispersion in the average product of capital (0.38) and larger aggregate productivity losses from misallocation (12.3%). Notice however that in this economy the elasticity of the average product of capital to a plant's revenue increases to 0.24, i.e., more than double its value in the data. We thus conclude that modifying the process for productivity shocks does not alter our conclusions: an economy capable of accounting for the relationship between the average product of capital and plant size (as measured by sales)

[35]The variance/kurtosis of ΔA_{it} is not very sensitive to the exact choice of α in the data: as we change α from 0.25 to 0.65, the variance changes from 0.25 to 0.27, while the kurtosis changes from 7.7 to 8.5.

[36]This is the so-called h distribution studied by Tukey (1977). See Headrick et. al (2008) for a detailed discussion.

[37]An alternative would be to pin down κ by matching the kurtosis of changes in A_{it} in the data. We prefer the approach reported here as the latter statistic depends, in the model, on the assumption one makes regarding the distribution of z_{it} shocks.

predicts small aggregate productivity losses from misallocation.

5. Uninsurable Investment Risk

We finally study the role of departing from the assumption of risk-neutral plant owners. We assume constant relative risk aversion (CRRA) preferences over consumption: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Plant owners' sole source of income are cash flows generated by the firm. We also allow trade in risk-free securities at a price $\frac{1}{1+r}$, and assume $\beta(1+r) < 1$ in order to prevent unbounded acummulation of assets. We also assume a (loose) borrowing constraint: the plant's debt cannot exceed its stock of capital. We later consider a more stringent no borrowing constraint.

The problem of the owner is to choose how much to consume, invest in the two assets (capital and the risk-free bond) in order to

$$V = \max_{c_t, K_{t+1}, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{aligned} c_t + K_{t+1} + \frac{1}{1+r} B_{t+1} &= B_t + \exp(a_t + z_t) K_t + (1-\delta) K_t = w_t \\ B_{t+1} &\geq -(1-\delta) K_{t+1} \end{aligned}$$

The Euler equation that governs capital accumulation now reads:

$$u'(c(w)) \geq \beta E_a u'(c(w')) [\alpha \exp(a' + z\prime) K'^{\alpha} + (1-\delta)]$$

and hence differences in a firm's net worth (w) induce differences in the user cost of capital.

Figure 12 (upper-left panel) reports the optimal decision rule for two plants that differ in their net worth. For low values of productivity the firm is able to finance its investment out of its own funds, and in fact, somewhat over-invests relative to the frictionless optimum for precautionary motives (recall that $\beta(1+r) < 1$). When productivity is sufficiently high, the plant owner is constrained and cannot borrow further ($\delta > 0$ and $r > 0$). As a result, the owner responds to further increases in its productivity by lowering next period's stock of capital in order to finance an increase consumption (as long as productivity shocks are persistent due to the preference to smooth consumption).

Table 11 reports the dispersion in the average product of capital in this economy. We assign the same parameter values here as those in the frictionless economy with no exit and entry (last column of Table 8). As we show below, adding a consumption smoothing motive alters little the prediction of that model, and so these parameter values account well for the variance and autocorrelation of revenue, as well as the mean investment-capital ratio in this economy as well. We consider two alternative values for the elasticity of intertemporal substitution, $\gamma = 2$ and $\gamma = 5$, as well as two values for the interest rate, $r = \frac{.97}{\beta} - 1$ and $r = 0$. We also impose a no-borrowing constraint in experiments reported in the last two columns of the table.

None of these experiments generate important departures from the economy with risk-neutral entrepreneurs and no frictions. The second column of Table 11 reports results for $\gamma = 2$ and $r = \frac{.97}{\beta} - 1$. The effect of the consumption-smoothing motive is to raise aggregate productivity losses by 1/10th of a percent: plant owners simply invest in the risk-free bond (the average holdings of the risk free bond are three times larger than their capital stock on average) and capital allocations are thus close to the frictionless (linear) case. This result is reminiscent of that of Krusell and Smith (1996) who also find that a risk-free bond is a very good instrument for smoothing consumption and that decision rules in an economy with uninsured idiosyncratic risk are similar to those in a complete markets economy. Magnifying the consumption-smoothing motive (by raising the value of γ to 5 in the third column of Table 11) or imposing a no-borrowing constraint (fourth column of Table 11) does little to alter these conclusions: agents simply hold even greater holdings of the risk-free asset and avoid distorting the investment margin. Only when the risk-free rate is sufficiently lower relative to the rate of time-preference, ($r = 0$ and $\beta = .925$) does the no-borrowing constraint play a visibile role (fifth column of Table 11). Even in this case however the increase in aggregate productivity losses relative to the frictionless economy is 1/2 of a percent.

We finally consider an extreme parametrization in which we assume away the risk-free savings technology altogether. The lower panel of Figure 12 shows that in this case productivity and the capital stock are negatively correlated conditional on a plant's net worth. Higher productivity leads to an incentive to raise consumption, which, in this environment, can only be financed by reducing the stock of capital. As a result the consumption-smoothing motive distorts the investment margin significantly. Table 12 shows that in this economy the variance of the average product of capital is equal to approximatively 0.50, thus roughly 40% of its value in the data. The model also produces almost 18% aggregate productivity losses and (not reported) a correlation between plant productivity and its capital stock of 0.60 (much lower than the 0.9 predicted by earlier parameterizations we have considered, but still higher than the 0.33 correlation in the data).

These latter results are clearly an outcome of our extreme assumption of absence of an additional savings technology. This economy also produces counter-factual implications vis-a-vis additional features of the data. Notice for example, in Table 12, that these economy predicts a much less volatile investment-capital ratio than in the data, especially when the elasticity of intertemporal substitution is low $(\gamma = 5)$. This counterfactual implication is a direct consequence of the plant's inability to respond to its productivity disturbances, and thus questions this mechanism's ability to account for the dispersion in the average product of capital. We thus conclude that uninsurable investment risk accounts for little of the differences in the average product of capital we document in the data.

6. Conclusions

We document large dispersion in the average product of capital across plants in Korean manufacturing. This dispersion mostly reflects persistent differences across plants, rather than time-series or industry-specific variation. Interpreted through the lens of a neoclassical production function, this is evidence of misallocation that reduces aggregate productivity by as much as 40%. We show that lack of insurance opportunities, financing frictions and capital adjustment costs account for little of the cross-sectional dispersion in the average product of capital, although they do account for most of the within-plant time series dispersion.

At work throughout all of these results are the strong forces of optimization. Large wedges in the plant's Euler equations for capital accumulation entail large losses, sufficiently large to make it optimal for firms to respond quickly (by paying the adjustment costs and/or accumulating enough capital to grow out of the borrowing constraints) and thus render differences across firms in the marginal product of capital small and short-lived. We thus conclude that it is difficult to rationalize large and persistent differences in the plant's marginal product of capital by frictions that distort capital accumulation. Policy distortions or other market failures (e.g. imperfect competition and price discrimination in the market for investment goods) that generate differences in a plant's return or cost of capital are thus a more plausible source of the misallocation we document in the data. We emphasize again that these distortions must, to some extent, be specific to capital accumulation: the dispersion in the marginal product of capital in our data is three times[38] larger than that for other factors of production.

We emphasize that ours is not an impossibility result: we do not argue that frictions on capital accumulation *cannot* generate large dispersion in the marginal product of capital. Indeed, we present several parametrizations capable of generating a substantially larger degree of misallocation than in our Benchmark economy. Rather, our conclusion is that these alternative parametrizations miss important facts about the relationship between plant size and its marginal product of capital, or the variability of plant-level investment, that we document in our data. Our results are thus specific to the relatively more developed industry we study: Korean manufacturing during 1991-1998. Moreover, our focus is on characterizing the stationary distribution of productivity and the average product of capital across plants in an economy that has converged to its steady state. We thus abstract from transitional dynamics as well as from studying the role financing frictions play in accounting for sectoral differences in productivity or preventing plants from adopting technologies that involve high initial setup costs. Buera and Shin (2008), Castro, Clementi and MacDonald (2008) and Buera, Kaboski and Shin (2009) are several recent contributions that find an important role for financing frictions and risk in generating misallocation during transitions as well as in accounting for sectoral differences in scale, productivity and investment rates.

[38]Twice greater when one accounts for the fact that smaller plants tend to rent a substantial portion of the capital stock. The appendix discussed this and other measurement issues.

References

[1] Abel, Andrew, and Janice Eberly, 1994. "A Unified Model of Investment Under Uncertainty," *American Economic Review*, 84: 1369-1384.

[2] Abel, Andrew, and Janice Eberly, 1996. " Optimal Investment with Costly Reversibility,"*American Economic Review*, 84: 1369-1384.

[3] Adelman, Irma and Song Byung Nak, 1998. "The Korean Financial Crisis of 1997-1998," mimeo.

[4] Angeletos, Marios, 2008, "Uninsured Idiosyncratic Investment Risk and Aggregate Saving," forthcoming, Review of Economic Dynamics

[5] Atkeson, Andrew and Patrick Kehoe, 2007, "Modeling the transition to a new economy: lessons from two technological revolutions," American Economic Review, 97(1): 64-88.

[6] Bachmann, Ruediger, Ricardo J. Caballero and Eduardo M.R.A. Engel, 2008. "Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model," mimeo.

[7] Banerjee, Abhijit V. and Esther Duflo, 2005. "Growth Theory through the Lens of Development Economics,"in: Philippe Aghion & Steven Durlauf (ed.), Handbook of Economic Growth, edition 1, volume 1, chapter 7, pages 473-552.

[8] Banerjee, Abhijit V. and Esther Duflo, 2008. "Do Firms Want to Borrow More? Testing Credit Constraints Using a Directed Lending Program," mimeo.

[9] Bartelsman, Eric J. and Mark Doms, 2000. "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38(3), 569-594.

[10] Bartelsman, Eric J., John Haltiwanger and Stefano Scarpetta, 2008. "Cross Country Differences in Productivity: The Role of Allocative Efficiency," mimeo.

[11] Basu, Susanto and John Fernald, 1995, "Are apparent productive spillovers a figment of specification error?" *Journal of Monetary Economics*, 36: 165-188.

[12] Basu, Susanto and John Fernald, 1997, "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, 105(2): 249-283.

[13] Bertola, Giuseppe and Ricardo Caballero, 1994. "Irreversibility and Aggregate Investment," *Review of Economic Studies*, 61: 223-246.

[14] Buera, Francisco J. and Yongseok Shin, 2008. "Financial Frictions and the Persistence of History: A Quantitative Explanation," mimeo.

[15] Buera, Francisco J., Joseph P. Kaboski and Yongseok Shin, 2009. "Finance and Development: A Tale of Two Sectors," mimeo.

[16] Caballero, Ricardo J., Eduardo M.R.A. Engel and John C. Haltiwanger, 1995. "Plant-Level Adjustment and Aggregate Investment Dynamcis," *Brookings Papers on Economic Activity*, 26(2), 1-54.

[17] Caballero, Rucardi and Eduardo Engel, "Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach," *Econometrica*, 64(4): 783-826.

[18] Cabarello, Ricardo J. and John Leahy, 1996. Fixed Costs: the Demise of Marginal q," *NBER Working Papers*, no. 5508.

[19] Castro, Rui, Gian Luca Clementi, and Glenn MacDonald, 2004. "Investor Protection, Optimal Incentives, and Economic Growth," *Quarterly Journal of Economics*, 119(3): 1131-1175.

[20] Castro, Rui, Gian Luca Clementi, and Glenn MacDonald. 2008. "Legal Institutions, Sectoral Heterogeneity, and Economic Development," mimeo.

[21] Cooley, Thomas and Vincenzo Quadrini, 2001. "Financial Markets and Firm Dynamics," *American Economic Review*, 91(5): 1286-1310.

[22] Cooper, Russell and Joao Ejarque, 2001. "Exhuming Q: Market Power vs. Capital Market Imperfections," *NBER Working Papers*, no. 8182.

[23] Cooper, Russell and John Haltiwanger, 2006. "On the Nature of Capital Adjustment Costs," *Review of Economic Studies*, 73, 611-634.

[24] Doms, Mark and Timothy Dunne, 1998. "Capital Adjustment Patterns in Manufacturing Plants," *Review of Economic Dynamics*, 1(2), 409-429.

[25] Erickson, Timothy and Toni M. Whited, 2000. "Measurement Error and the Relationship between Investment and q," *Journal of Political Economy*, 108(5), 1027-1057.

[26] Fazzari, Steven, R. Glenn Hubbard and Bruce C. Petersen, 1988. "Investment, Financing Decisions, and Tax Policy," *American Economic Review*, 78(2), 200-205.

[27] Galindo, Arturo, Fabio Schiantarelli, Andrew Weiss, 2005, "Does Financial Liberalization Improve the Allocation of Investment? Micro-evidence from developing countries," *Journal of Development Economics*, 83: 562-587.

[28] Gilchrist, Simon and Jae W. Sim, 2007. "Investment During the Korean Financial Crisis: A Structural Econometric Analysis," *NBER Working Papers*, no. 13315.

[29] Gomes, Joao F., 2001. "Financing Investment," *American Economic Review*, 91(5), 1263-1285.

[30] Guner, Nezih, Gustavo Ventura and Yi Xu, 2008. "Macroeconomic Implications of Size Dependent Policies," *Review of Economic Dynamics*, 11(4), 721-744.

[31] Hall, Robert E., 2004. "Measuring Factor Adjustment Costs," *Quarterly Journal of Economics*, 119(3), 899-927.

[32] Headrick, Todd C., Rhonda K. Kowalchuk and Yanyan Sheng, 2008. "Parametric Probability Densities and Distribution Functions for Tukey g-and-h Transformations and their Use for Fitting Data," *Applied Mathematical Sciences*, 9(2), 449-462.

[33] Hennessy, Christopher A. and Toni M. Whited, 2007. "How Costly is External Financing? Evidence from a Structural Estimation," *Journal of Finance*, 62(4), 1705-1745.

[34] Hopenhayn, Hugo, 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60(5), 1127-1150.

[35] Hopenhayn, Hugo and Richard Rogerson, 1993. "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, 101(5), 915-938.

[36] Hsieh, Chang-Tai and Peter Klenow, 2009. "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, forthcoming.

[37] Kaboski, Joseph P. and Robert M. Townsend, 2009. "The Impacts of Credit on Village Economies," mimeo.

[38] Kahn, Aubhik and Julia K. Thomas, 2008. "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 76(2), 395-436.

[39] Kaplan, Steven N. and Luigi Zingales, 1997. "Do Investment-Cash Flow Sensitivities Provide Useful Measures of Financing Constraints?," *Quarterly Journal of Economics*, 112(1), 169-215.

[40] Kim, Jong Hun and Peter L. Rousseau, 2008. "A flight to Q? Firm Investment and Financing in Korea before and after the 1997 Financial Crisis," *Journal of Banking & Finance*, 32(7), 1416-1429.

[41] Lagos, Ricardo, 2006. "A Model of TFP," *Review of Economic Studies*, 73(4), 983-1007.

[42] Levinsohn, James and Amil Petrin, 2003. "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studes*, 70(2), 317-341.

[43] Moskowitz, Tobias and Annette Vissing-Jorgensen, 2002. "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?" *American Economic Review*, 92: 745-778.

[44] Olley, G. Steven and Ariel Pakes, 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64(6), 1263-1297.

[45] Restuccia, Diego and Richard Rogerson, 2008. "Policy Distortions and Aggregate Productivity with Heterogeneous Plants," *Review of Economic Dynamics*, 11(4), 707-720.

[46] Thomas, Julia K., 2002. "Is Lumpy Investment Relevant for the Business Cycle?," *Journal of Political Economy*, 110(3), 508-534.

[47] Tybout, James R., 2000. "Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?," *Journal of Economic Literature*, 38(1), 11-44.

[48] Veracierto, Marcelo, 2002. "Plant-Level Irreversible Investment and Equilibrium Business Cycles," *American Economic Review*, 92(1): 181-197.

[49] Wang, Shing-Yi, 2008. "Credit Constraints, Job Mobility, and Entrepreneurship: Evidence from a Property Reform in Cina," mimeo.

[50] Whited, Toni M., 2006. "External Finance Constraints and the Intertemporal Pattern of Intermittent Investment," *Journal of Financial Economics*, 81, 467-502.

Table 1: Moments of the distribution of $\ln \frac{R_i}{K_i}$

	All plants	Top 90 % revenue	Top 80% revenue
variance	1.23	1.11	1.06
interquartile range	1.49	1.47	1.30
plant-year obs.	592996	133280	49464

Table 1a: Dispersion of marginal product of labor and materials

$\text{var}\left(\ln \frac{R_i}{K_i}\right)$	1.23
$\text{var}\left(\ln \frac{R_i}{L_i}\right)$	0.35
$\text{var}\left(\ln \frac{R_i}{M_i}\right)$	0.40
plant-year obs.	592996

Table 1b: Variance and correlation of $\ln R_i$ and $\ln K_i$

$\text{var}\left(\ln \frac{R_i}{K_i}\right)$	1.23
var $(\ln\ R_i)$	2.04
var $(\ln\ K_i)$	2.82
corr $(\ln R_i, \ln K_i)$	0.77
plant-year obs.	592996

Table 2: Decomposing variance $\ln\left(\frac{R_{it}}{K_{it}}\right)$

year dummies	0.0035
5-digit industry dummies	0.14
plant dummies (survive all years)	0.72
Conditional on plant in sample 1991-1998:	
autocorrelation (pooled)	0.86
median within-plant variance	0.16
median within-plant autoc.	0.39

Table 3: Losses from misallocation

	$corr\,(a_{it}, k_{it})$	$\Delta \log A$
$\alpha = 0.45$	0.33	-0.39

Table 4: Distribution of investment-capital ratio

	Unweighted	Revenue-weighted
mean	0.13	0.25
median	0.00	0.11
std. dev.	0.38	0.44
skeweness	5.01	3.49
fraction ≤ 0	0.04	0.07
fraction $\leq$ mean /4	0.67	0.39
mean if $IK \leq 0$	-0.13	-0.12
autocorr.	0.08	0.09

Table 5: Moments in economy with K adjustment costs

	Data	Benchmark	No costs
var r	2.06	2.06	1.88
autocorr. r	0.942	0.943	0.933
corr r_2/corr r_1	0.972	0.973	0.948
hazard exit 1-5	0.23	0.23	0.23
hazard exit 6-10	0.19	0.19	0.19
hazard exit 11-20	0.14	0.14	0.15
moments of I/K :			
mean	0.13	0.13	0.12
median	0.00	0.00	0.03
std. dev	0.38	0.33	0.44
skeweness	5.01	3.20	1.28
fraction ≤ 0	0.04	0.04	0.46
fraction $\leq$ mean /4	0.67	0.74	0.50
mean if $IK \leq 0$	-0.13	-0.16	-0.24
autocorr.	0.08	0.14	-0.02

Table 6: Parameter values, economy with K adj. costs

α	0.45
β	0.925
δ	0.055
ρ	0.949
σ_ϵ	0.235
σ_z	0.30
P_s	0.81
κ, % mean I	0.89
ξ_{max}, rel. to mean π	14.7
θ	0.089

Table 7: Misallocation, economy with K adj. costs

	Data	Benchmark	No costs
var r/k	1.23	0.17	0.13
autocorr. r/k	0.87	0.34	-0.01
median var r/k_i	0.16	0.14	0.13
autocorr. demeaned r/k_i	0.39	0.17	-0.09
$\Delta \log(A)$, $\times$ 100	-39.5	-6.7	-4.9

Table 8: Parameter values, economy with finance frictions

	Benchmark	Large FF	Fat tails	Frictionless
α	0.45	0.45	0.45	0.45
β	0.925	0.925	0.925	0.925
δ	0.107	0.140	0.13	0.087
ρ	0.951	0.947	0.932	0.972
σ_ε	0.268	0.310	0.217	0.185
σ_z	0.317	0.317	0.29	0.317
ϕ	0.45	0.05	0.45	-
λ	0.10	0.25	0.10	-

Table 9: Moments in economy with finance frictions

	Data	Benchmark	Large FF	Fat tails	Frictionless
var r	2.06	2.07	2.09	2.01	2.05
autocorr. r	0.942	0.940	0.935	0.939	0.942
corr r_2/corr r_1	0.972	0.971	0.974	0.969	0.972
mean I/K	0.13	0.13	0.13	0.13	0.13
I/K wrt. $\log(\Pi)$	0.032	0.023	0.029	0.025	0.000
I/K wrt. $\log(K)$	0.001	-0.012	0.000	-0.008	-0.026
$\log\left(\frac{R}{K}\right)$ wrt. $\log(R)$	0.113	0.135	0.235	0.244	0.057

Table 10: Misallocation, economy with finance frictions

	Data	Benchmark	Large FF	Fat tails	Frictionless
var r/k	1.23	0.20	0.42	0.38	0.12
autocorr. r/k	0.87	0.32	0.65	0.64	0.00
median var r/k_i	0.16	0.17	0.20	0.18	0.12
autocorr. demeaned r/k_i	0.39	0.29	0.67	0.63	-0.01
$\Delta \log(A)$, $\times$ 100	-39.5	-7.5	-13.7	-12.3	-4.6

Table 11: Misallocation, economy with uninsured investment risk

	Data	$\gamma = 2$ $r = \frac{.97}{\beta} - 1$	$\gamma = 5$ $r = \frac{.97}{\beta} - 1$	$\gamma = 2$ $r = \frac{.97}{\beta} - 1$ no debt	$\gamma = 2$ $r = 0$ no debt	Frictionless
var r/k	1.23	0.12	0.12	0.13	0.22	0.12
med var r/k_i	0.16	0.12	0.12	0.12	0.14	0.12
mean B/K	-	3.1	9.0	5.8	0.7	-
sd.dev. (I/K)	0.38	0.31	0.30	0.26	0.18	0.33
$\Delta \log(A)$, $\times$ 100	-39.5	-4.7	-4.7	-4.7	-5.1	-4.6

Table 12: Misallocation, economy with uninsured investment risk and no risk-free bond

	Data	$\gamma = 2$	$\gamma = 5$	Frictionless
var r/k	1.23	0.49	0.52	0.12
med var r/k_i	0.16	0.22	0.20	0.12
sd.dev. (I/K)	0.38	0.16	0.10	0.33
$\Delta \log(A)$, $\times$ 100	-39.5	-17.6	-18.1	-4.6

Figure 1: Example of plant

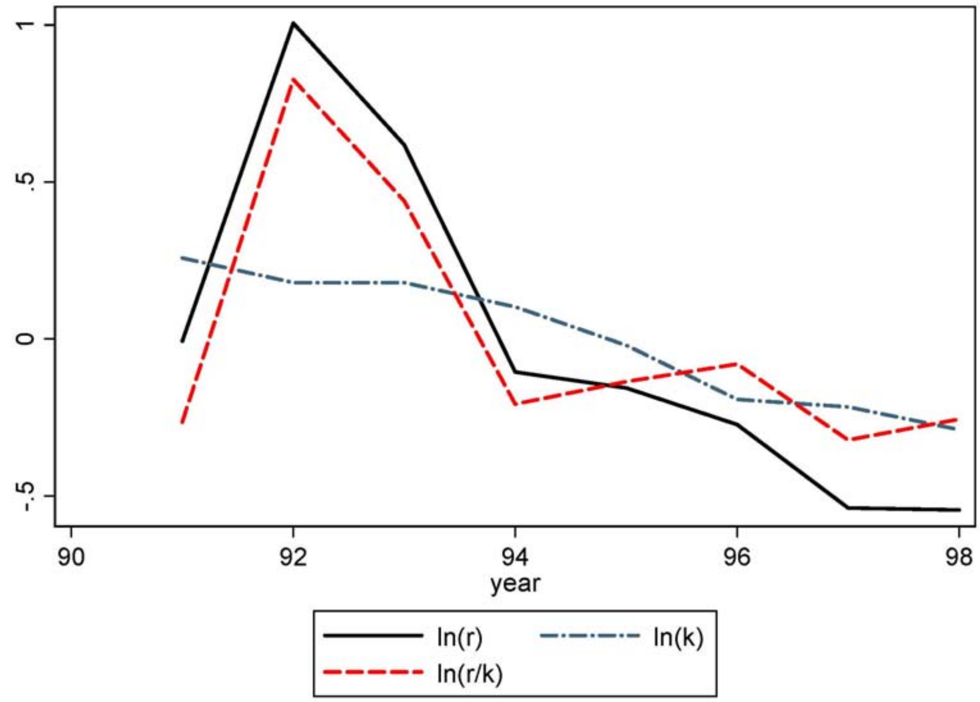


Figure 2: Investment-Capital Ratio

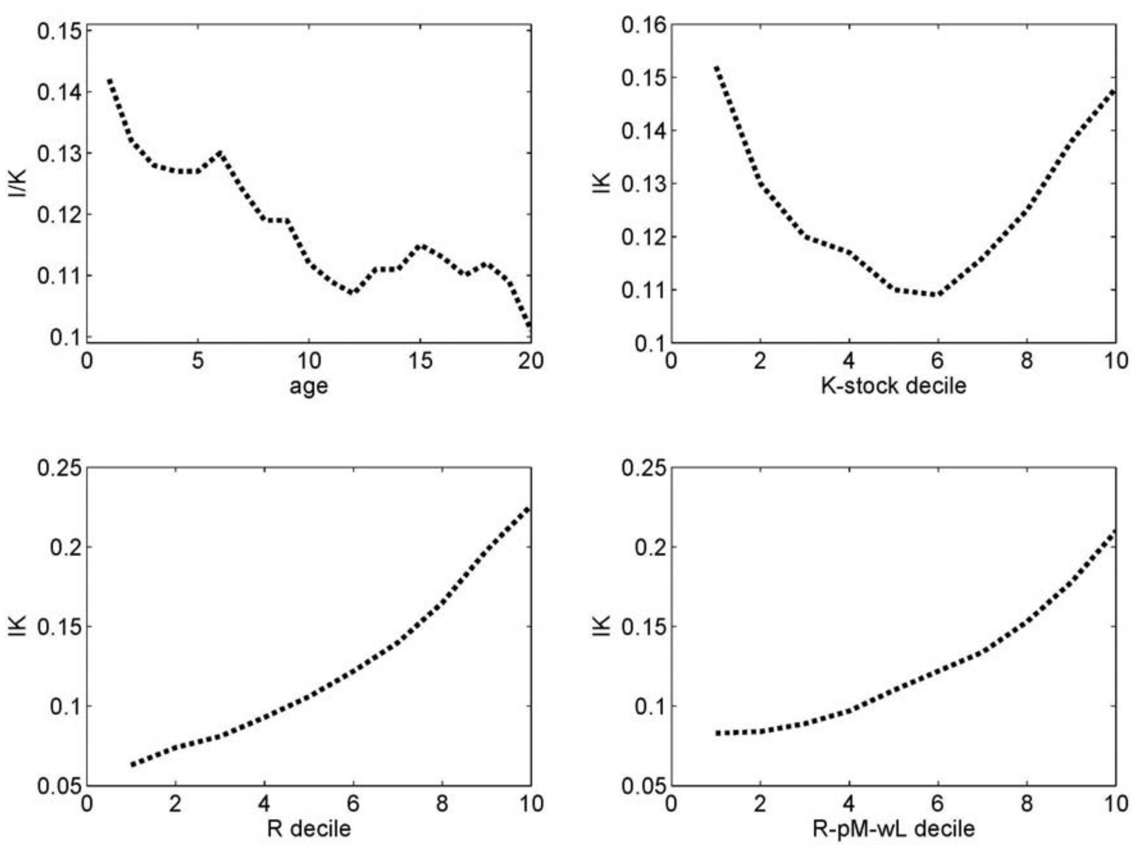


Figure 3: Average product of capital

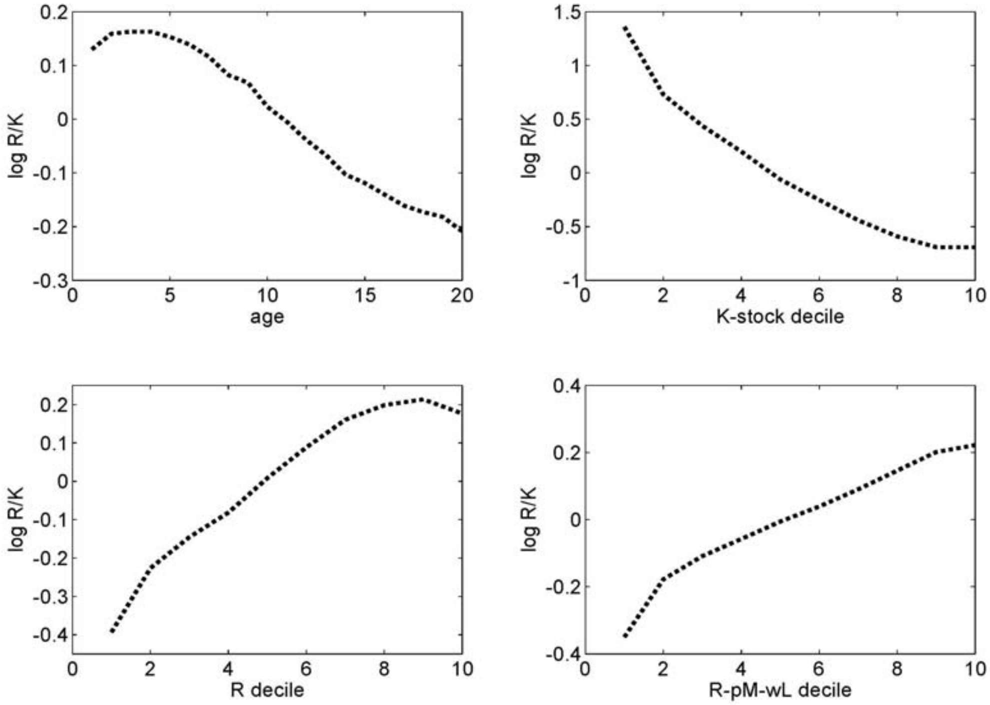


Figure 4: Distribution of plant-level I/K

a. unconditional

b. Conditional on |I/K|>0

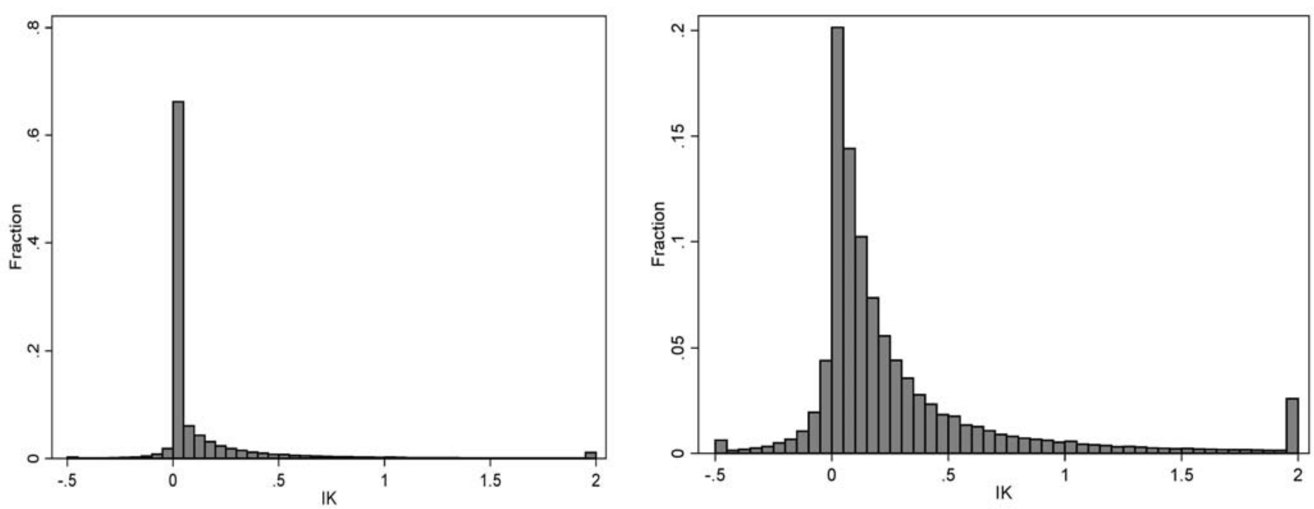


Figure 5: Decision rules: economy with adjustment costs

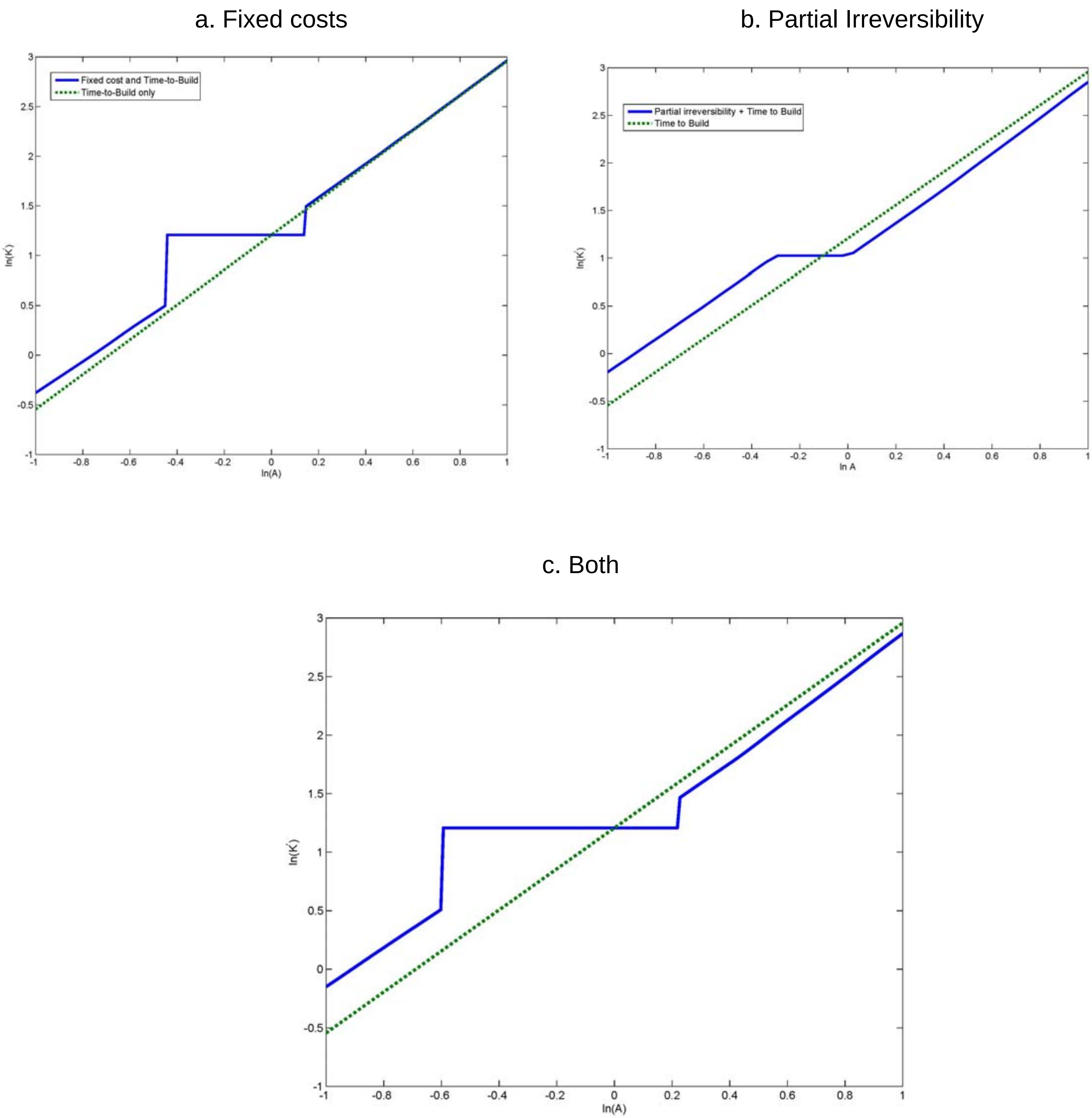
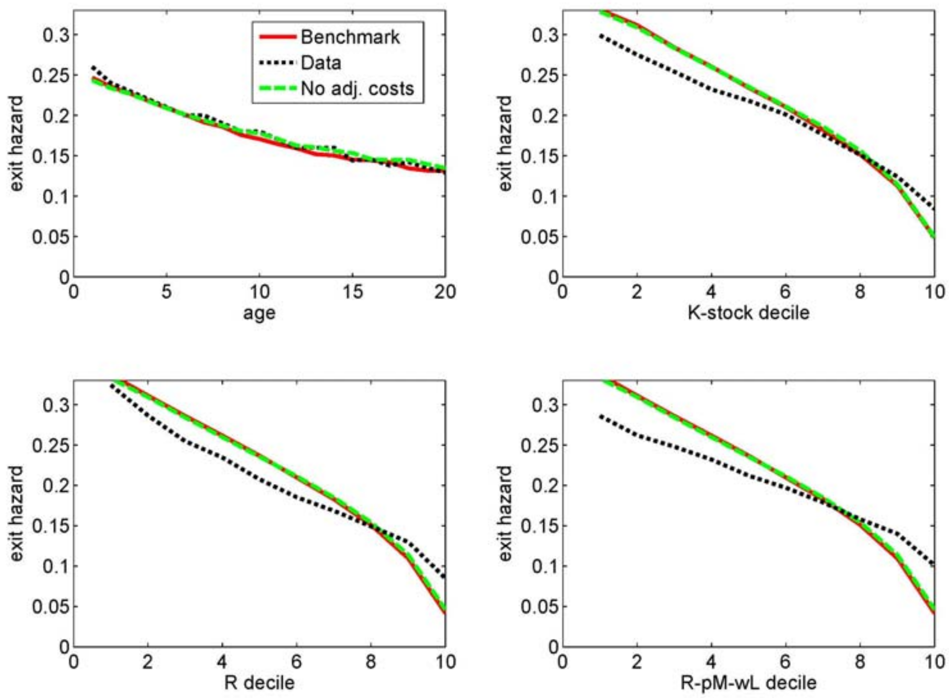


Figure 6: Exit Hazards and Plant Growth

a. Exit Hazards



b. Growth rate of sales

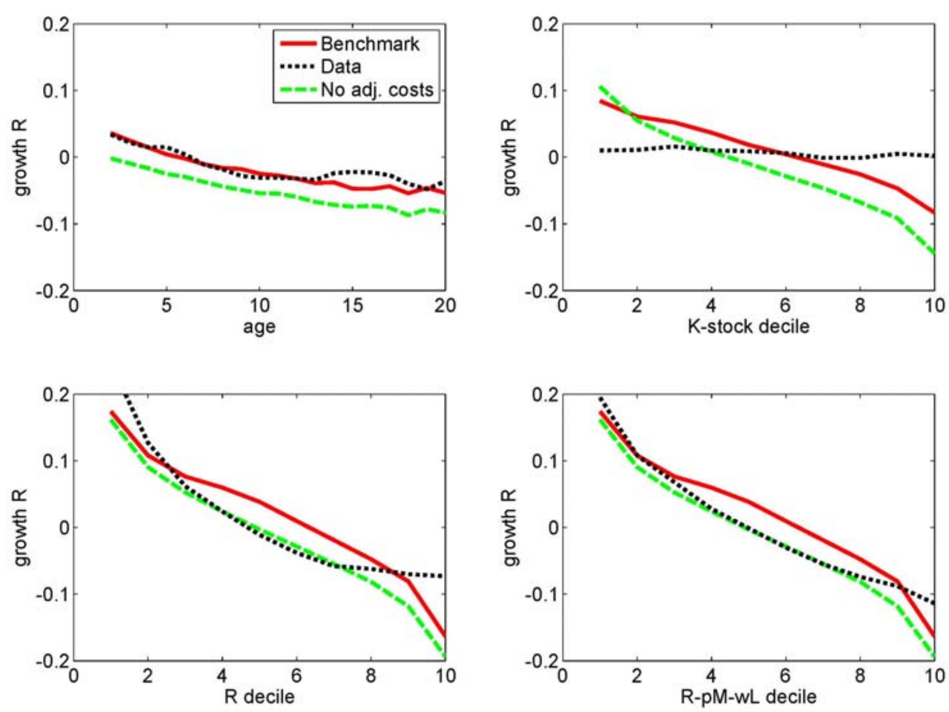
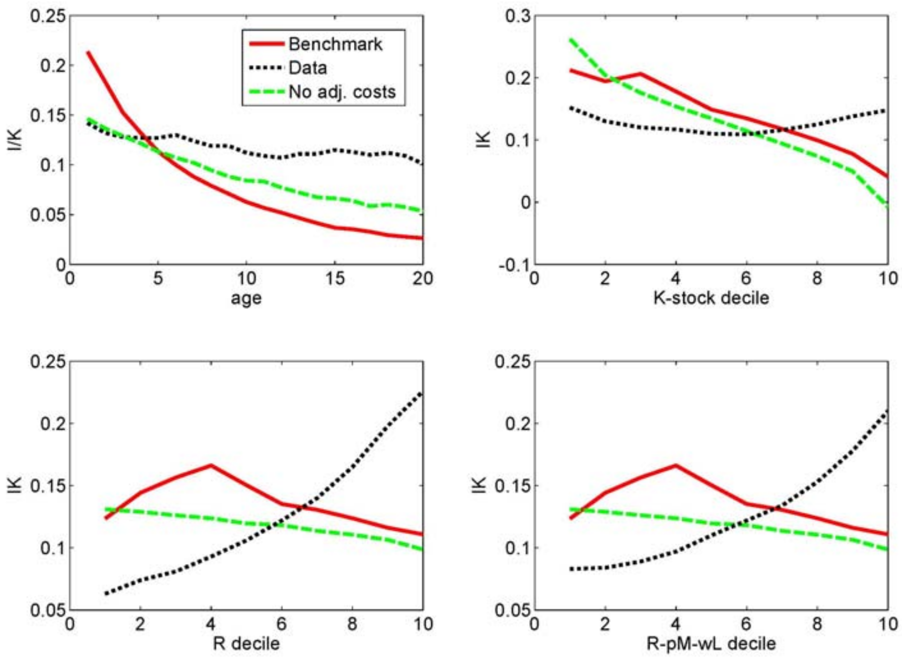


Figure 7: Investment and Average Product of capital

a. Investment-Capital Ratio



b. Average Product of Capital

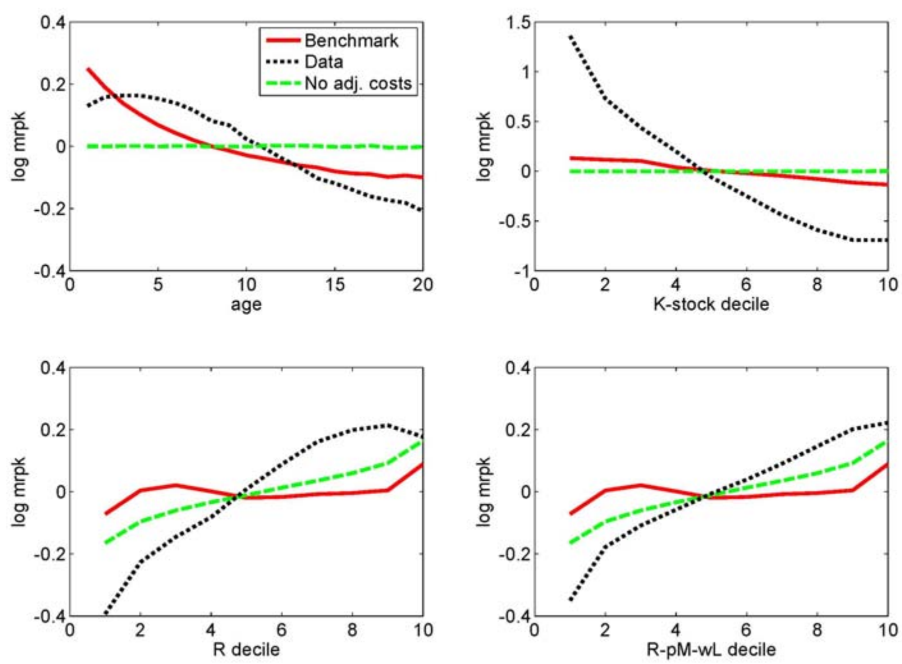


Figure 8: Decision rules in economy with Finance Frictions

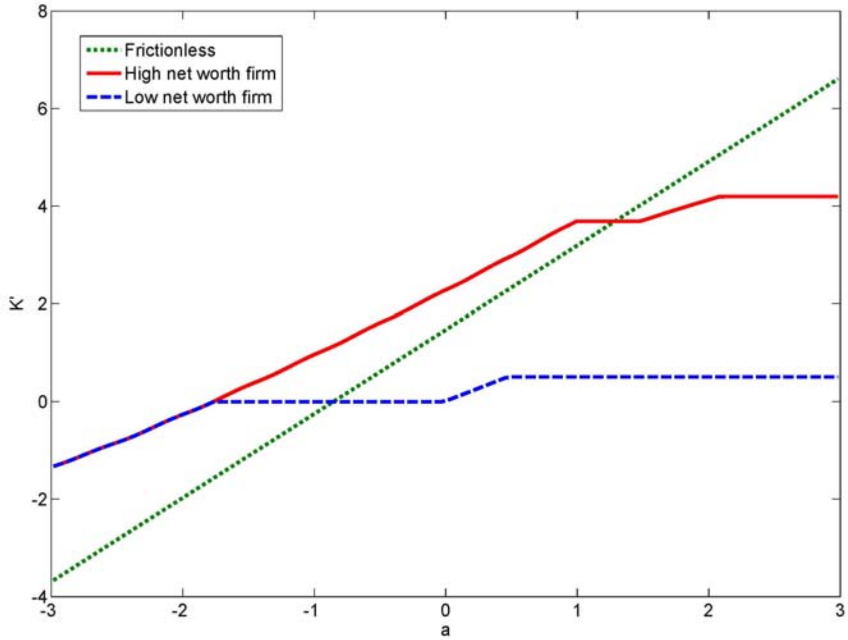


Figure 9: Benchmark Financing Frictions

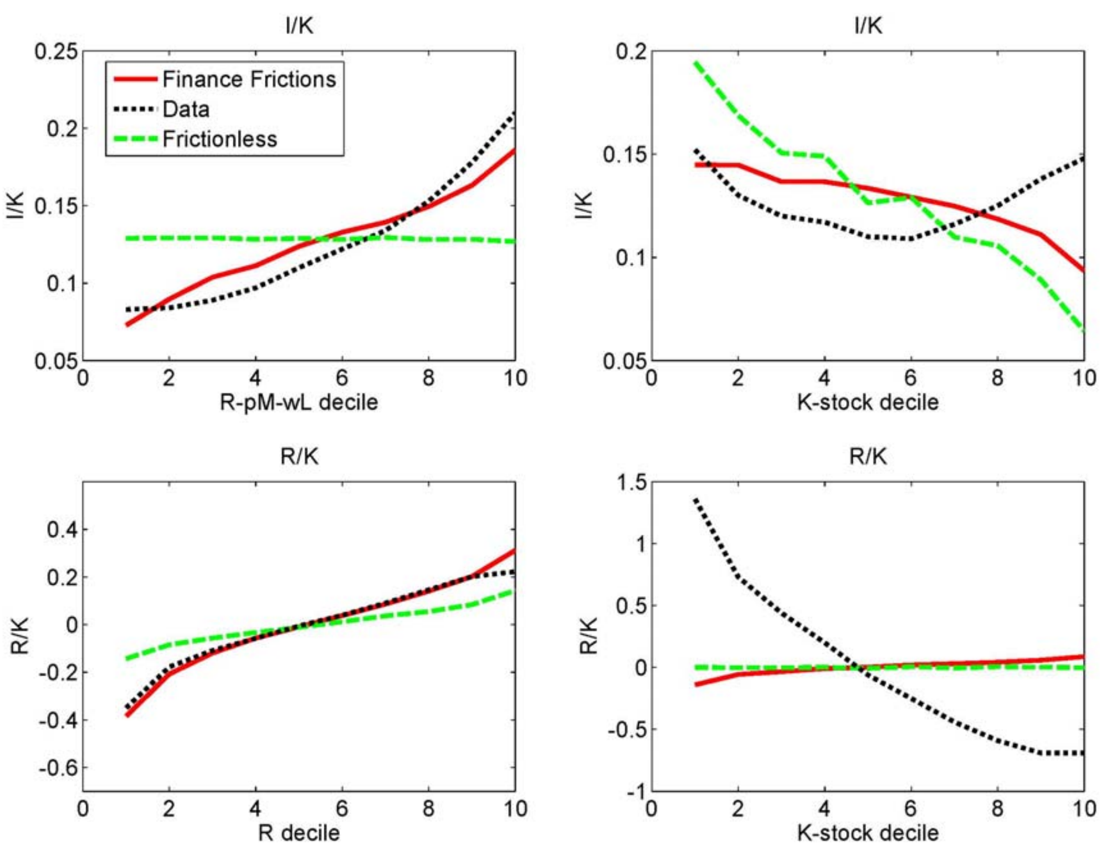


Figure 10: Large Finance Frictions

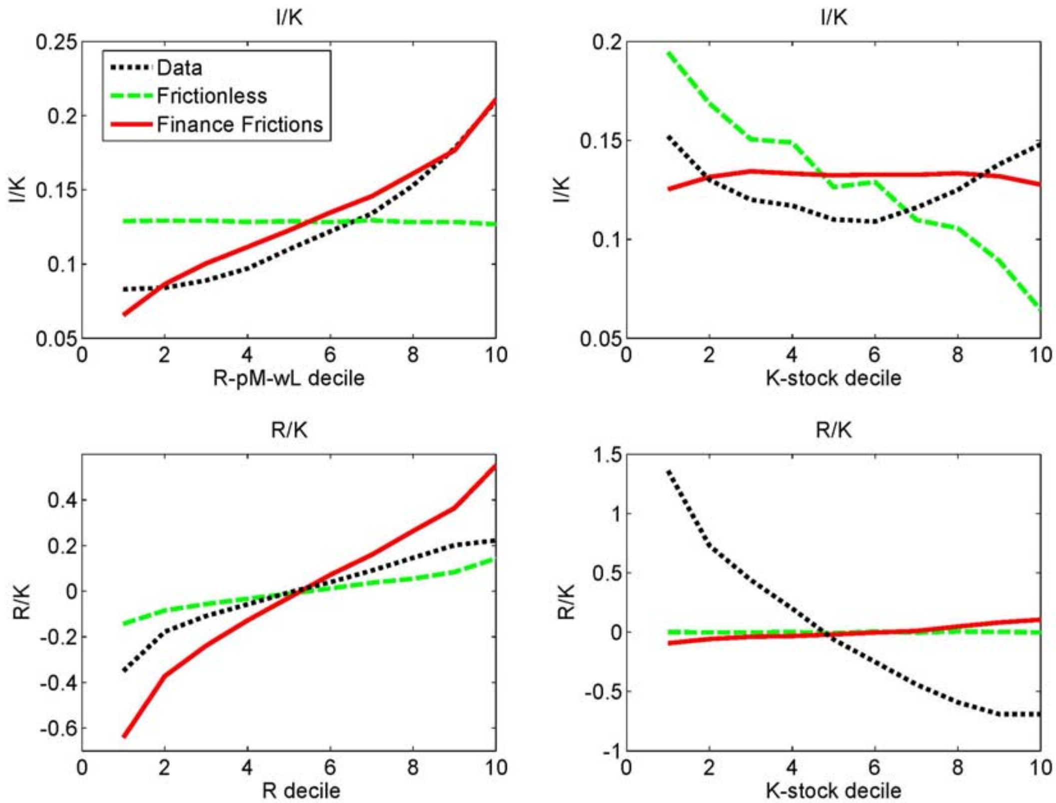


Figure 11: Korean Financial Crisis

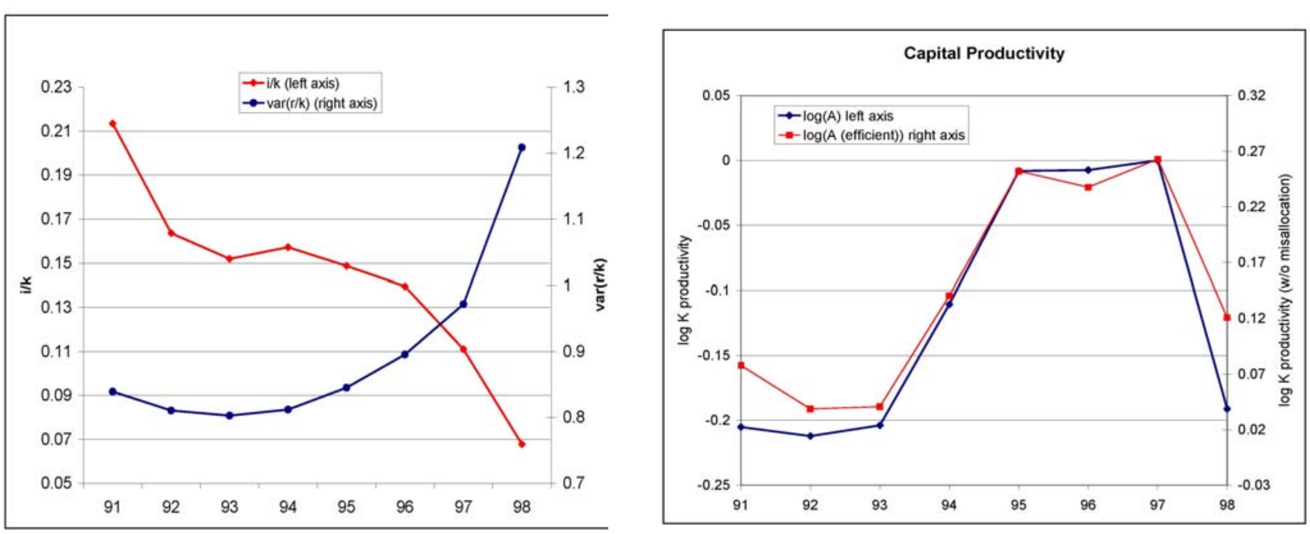
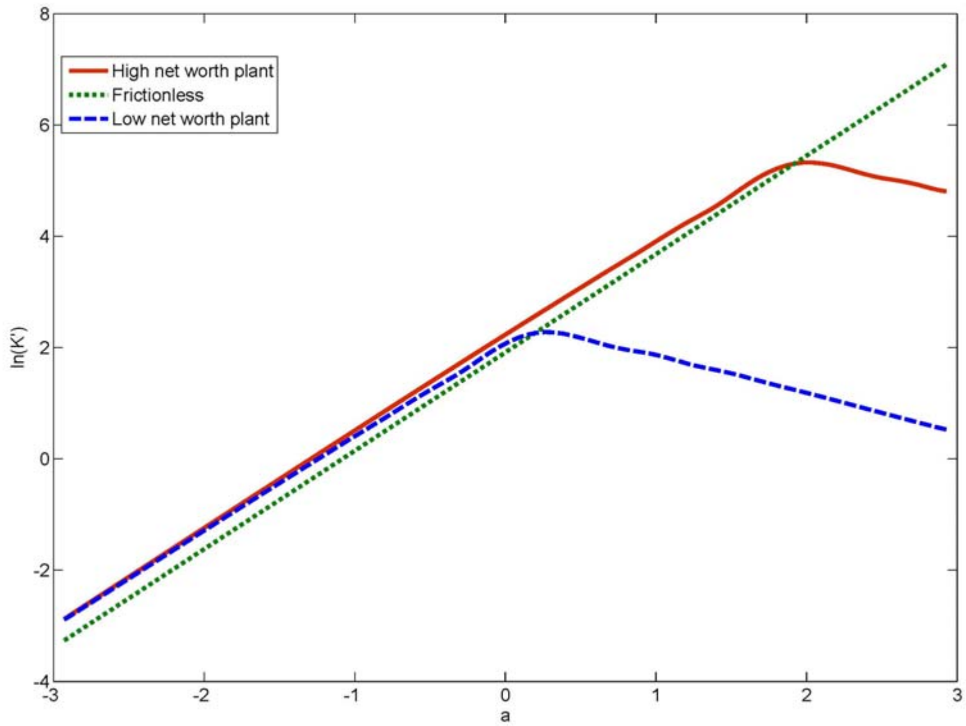
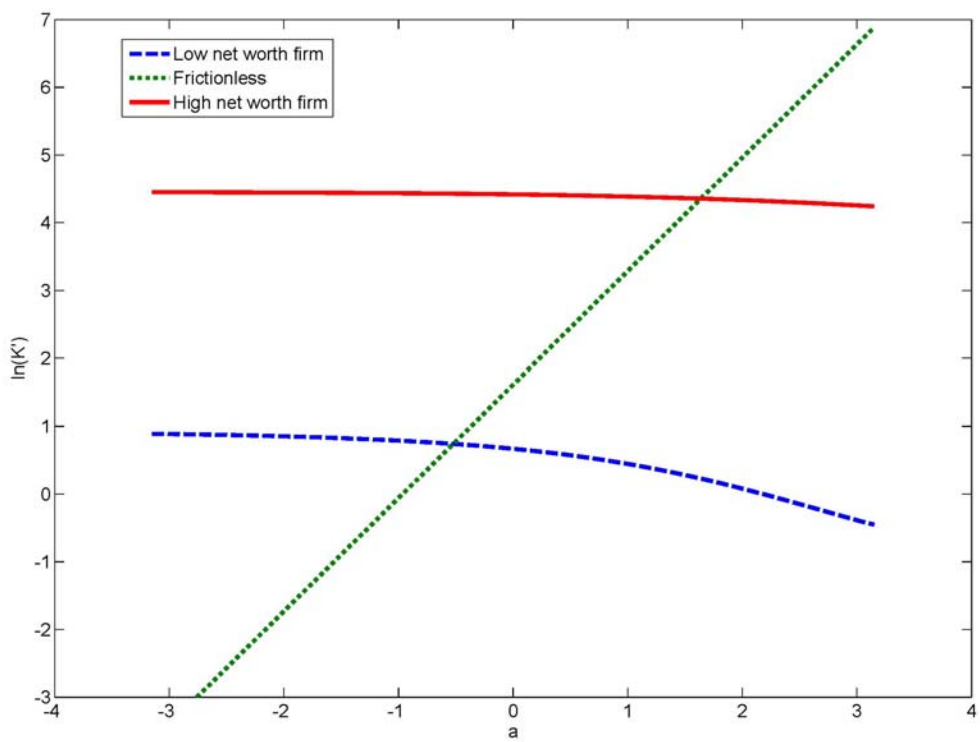


Figure 12: Decision rules with consumption-smoothing motive

a. Economy with risk-free bond



b. No risk-free bond



Appendix for "Accounting for Plant-level Misallocation"

Virgiliu Midrigan and Daniel Yi Xu

April 2009

Measurement

We ask: to what extent can measurement problems account for the large dispersion of marginal product of capital that we observe in the data? Although measurement problems are undoubtedly important, we are skeptical that they account for the bulk of the dispersion in the average product of capital. The data in Bartelsman et. al (2008) shows that the dispersion in the average product of labor is strongly correlated with country characteristics: GDP per capita, the degree of financial development, degree of labor market rigidities etc. This robust relationship is also present in the time-series. Bartelsman et. al (2008) report a sharp decrease in the degree of misallocation in the 1990s in Eastern European transition economies. Galindo et. al. (2007) show that the efficiency of investment increases following financial liberalizations. We document in this paper a 25% increase in the dispersion of the average product of capital of surviving plants in the aftermath of the 1997 Korean financial crisis. Finally, Hsieh and Klenow (2008) show that state-owned plants in China and India have substantially lower marginal product of capital and labor than other plants.

We focus here on three potential measurement problems: departures from a Cobb-Douglas production function (an assumption that allowed our interpretation of averages as marginals); problems in measuring a plant's stock of physical capital; the possibility of measurement error and/or unobserved (to the plant) shocks to its productivity.

A. Departures from Cobb-Douglas

We allow here for the possibility that the elasticity of substitution between capital and other factors of production is less then unity (asumming a more general CES production technology), as well as for the possibility that part of the capital stock is a fixed 'overhead' requirement that does not vary with a plant's output[39]. We argue that these two departures do not alter our interpretation of the dispersion of the average product of capital as evidence of misallocation.

[39]We use a specification here that has also been employed by Banerjee and Duflo (2005) and Bartelsman et. al. (2008).

CES Production Function

We first investigate the implication of generalizing to a CES production function. In this case, one plant's revenue function is:

$$R = \left(\exp(x)\left[(1-\gamma-\beta)K^{\frac{\theta-1}{\theta}} + \gamma L^{\frac{\theta-1}{\theta}} + \beta M^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}\right)^{\frac{1+\eta}{\eta}}$$

where θ determines elasticity of substitution of different inputs and γ and β determine each factor's share. One can show that, absent frictions on the adjustment of capital (including time-to-build), labor and materials, the optimality conditions that determine the choice of these inputs reduce to

$$R'(K) = \frac{1+\eta}{\eta} \times \frac{R(K)}{K} = u$$

where $u = \frac{r+\delta}{1+r}$, $r = \frac{1}{\beta} - 1$ is the user cost of capital, constant across all plants. Thus efficiency once again requires equalization of both the average and marginal product of capital across plants.

A caveat is in order however: the equality between the average and the marginal product of capital only holds here if all optimality conditions (including for capital accumulation) are satisfied. More generally, for any other stock of capital K, the marginal product of capital is related to the average product according to

$$R'(K) = \exp(x)^{\frac{\theta-1}{\theta}} \left(\frac{R}{K}\right)^{1-\frac{1}{\frac{\theta}{\theta-1}\frac{1+\eta}{\eta}}} K^{\frac{\frac{1}{\eta}}{\frac{\theta}{\theta-1}\frac{1+\eta}{\eta}}}$$

and may thus differ across firms.

Fixed cost

Assume next a production function of the form

$$R = \left(\exp(x)L^{\gamma}M^{\beta}\left(K-\bar{K}\right)^{1-\gamma-\beta}\right)^{\frac{1+\eta}{\eta}}$$

where $\bar{K}$ is the minimum stock of capital needed for operating (a fixed overhead stock). In this case the marginal product of capital is equal to

$$R'(K) = \frac{R}{K-\bar{K}}$$

and is thus different from the average product of capital

$$\frac{R}{K} = \frac{\left(\frac{\alpha}{u}A\right)^{\frac{1}{1-\alpha}}}{\left(\frac{\alpha}{u}A\right)^{\frac{1}{1-\alpha}} + \bar{K}}$$

where $u = \frac{r+\delta}{1+r}$ is again the user cost of capital and A is the revenue productivity. Clearly, as long as $\bar{K} > 0$, $\frac{R}{K}$ and K are both positively correlated with a plant's productivity. But this is in contrast to the negative relationship between $\frac{R}{K}$ and K we document in the data. Differences in productivity alone thus cannot generate the observed dispersion in the average product of capital under the assumption of (identical) fixed costs.

B. Measurement Error of Physical Capital
Initial Value

An important concern with our use of a perpetual inventory method to construct measures of capital stock is that the initial capital book value is measured with error. We use a counterfactual experiment for the group of plants that survive the entire sample to judge whether this type of measurement error can potentially account for the large dispersion in the average product of capital.

The counterfactual is as follows. Let us suppose that in the first year of the sample (1991) capital is efficiently allocated across all plants. We thus impute a measure of initial capital stock that is necessary to ensure that all plants have the same average product of capital. That is, we set $K^c_{i,91} = \frac{R_{i,91}}{mean\left(\frac{R_{i,91}}{K_{i,91}}\right)}$. We then construct measures of the stock of capital for all future years by using data on investment, retirements and depreciation, as we have done earlier, using $K^c_{i,91}$ as the initial condition. If the dispersion in average product of capital is indeed an artefact of mismeasured initial capital stock, we should see little increase in this measure in subsequent years. Figure A1 shows, however, the opposite. The dispersion in the average product of capital under this counterfactual grows rapidly over the subsequent years and converges to that in the data very quickly. Initial conditions alone do not contribute thus to our findings.

Variable capital utilization

To what extent do differences in the degree to which plants utilize their stock of capital drive some of our results? Can differences in the observed average product of capital among two plants simply reflect differences in the degree to which capital is utilized. We argue not.

First, recall that the bulk of the dispersion in the marginal product of capital is accounted for by persistent differences across plants, rather than time-series variation for a given plant. Although varying the work-week of capital may indeed be an optimal response to transitory shocks, it is presumably costly to make up shortages in the stock of capital by working it harder for a prolongued period of time (8 years in our data).

An alternative argument is available by making use of the electricy use data in our dataset. Assuming that electricity and capital services (the stock of capital times its work-week) enter the production function in a Leontieff fashion, we can use electricy consumption, rather than the actual stock of capital, to measures capital services, as the two are used in fixed proportions.

The correlation of electricity consumption with that of our measure of capital stock is equal to 0.77. This is reassuring given that data on electricy use is less likely to be contaminated by measurement error than the capital series. We then compute the marginal product of capital services, as measured by the ratio of a plant's revenue to its electricity use: $\ln\left(R_{it}/E_{it}\right)$. The variance of this object is only slightly lower than that of the average product of capital: 1.10 v.s. 1.23. We interpret these numbers as evidence that variable utilization of capital can account for up to 10% of the variation in the average product of capital in the data.

Finally, we also have information on the amount a plant spends on rental of structure and equipment: up to this point our focus was solely on the capital stock owned by plants. We augment now the plant's capital stock by adding the plant's expenditure on rent (divided by the user cost measure described earlier). Interestingly, this modification reduce the variance of the average product of capital to 0.85, thus 30% lower than our original calculation. This result is driven by the fact that rental capital is an important expenditure for plants in the data, especially for the small ones. For example the smallest 25% of the plants rent an average of 35% of their total capital stock, while the largest 25% of the plants rent only 11% of their capital stock. We thus conclude that ignoring rental capital contributes significantly to the measured misallocation in the data.

C. Measurement Error in Revenue

Can measurement error in revenue (or alternatively productivity shocks that the plant does not observe), account for some of the dispersion in the measured average product of capital? We answer this question by filtering out the 'noise' in the revenue variable using a technique developed by Olley and Pakes (1996) and Levinsohn and Petrin (2003). The idea is to recognize that a plant's use of (freely) variable factors of production (we use materials) is monotonically increasing in its productivity x and the plant's capital stock K. We can thus separate observed productivity, x, from measurement error, by identifying the latter as sources of variation in a plant's revenue uncorrelated with the plant's stock of capital and materials. We use a non-parametric regression of measured revenue on capital stock K and material use M to obtain a "purified" measure of revenue R^p. The variance of $\ln \frac{R^p}{K}$ is 1.04, thus an additional 1/6th lower than the original measure we report in Section 2.

We conclude thus that measurement issues can indeed account for a substantional fraction of the observed dispersion in the marginal product of capital. We have ignored in this analysis, due to data limitations, the possibility of heterogeneity across plants in the production technology, markups, differences in the overhead requirement etc. Understanding the role heterogeneity in production technology plays is an important question that remains to be addressed.

Additional estimates of α

Let our production function:

$$Y = exp(x)K^{1-\beta-\gamma}(L^{\gamma}M^{\beta})$$

Firms are monopolistically competitive and face a demand

$$P = (\frac{Y}{exp(z)})^{\frac{1}{\eta}}$$

Since revenue $R \equiv PY = Y^{1+\frac{1}{\eta}}exp(z)^{-\frac{1}{\eta}}$, we can write the log revenue function as:

$$r = (1+\frac{1}{\eta})y - \frac{1}{\eta}z = (1+\frac{1}{\eta})[(1-\beta-\gamma)k + \gamma l + \beta m] + a$$

where the lower case represent revenue and inputs in logs and $a \equiv x^* + z^* = (1+\frac{1}{\eta})x - \frac{1}{\eta}z$.

Now consider the short-run problem of profit maximization wrt to L and M:

$$(1+\frac{1}{\eta})\gamma = \frac{wL}{R} \equiv s_L$$
$$(1+\frac{1}{\eta})\beta = \frac{p_m M}{R} \equiv s_M$$

Given this set of equations, we can move to the estimation of $\alpha = \frac{1+\frac{1}{\eta}-(1+\frac{1}{\eta})(\beta+\gamma)}{1-(1+\frac{1}{\eta})(\beta+\gamma)}$. As immediately seen, if we could obtain consistent estimates for s_L and s_M, the only piece missing is $1 + \frac{1}{\eta}$, which we recover from our revenue equation.

We consider the case where individual plant data revenue contain measurement or optimization error ϵ such that $s_L = \frac{wL}{R_d} exp(\epsilon)$ and $s_M = \frac{p_m M}{R_d} exp(\epsilon)$. In addition, there might be differences in terms of technology across different sectors, so we estimate $(1 \hat{+} \frac{1}{\eta})\gamma$ and $(1 \hat{+} \frac{1}{\eta})\beta$ with mean (log) factor shares $\bar{s}_L$ and $\bar{s}_M$ within each 5-digit industry.

Make use of these estimates in the revenue equation, we have:

$$\tilde{r} = r_d - \bar{s}_L(l-k) - \bar{s}_M(m-k) = (1 + \frac{1}{\eta})k + a + \epsilon$$

Depending on the specification of dynamic process for a, we employ different strategies to estimate $(1 \hat{+} \frac{1}{\eta})$.

D. a as a first-order markov process

If we assume that a is a first-order markov process:

$$a_t = a_0 + \rho a_{t-1} + \xi_t$$

The revenue equation then can be written as:

$$\tilde{r}_t = a_0 + \rho \tilde{r}_{t-1} + (1 + \frac{1}{\eta})(k_t - \rho k_{t-1}) + \epsilon_t^*$$

where $\epsilon_t^* = \epsilon_t - \rho\epsilon_{t-1} + \xi_t$ If we assume that ξ_t is i.i.d overtime, then a simple NLS will generate a consistent estimate of $(1 \hat{+} \frac{1}{\eta})$. Our NLS procedure gives an estimate $(1 \hat{+} \frac{1}{\eta}) = .783$. This in turn implies a mean $\alpha = 0.44$ across different 5-digit industries.

E. x^* as a first-order markov process

If there is a transitory shock component such that z^*, the previous estimate is not consistent because

$$\tilde{r}_t = x_0 + \rho \tilde{r}_{t-1} + (1 + \frac{1}{\eta})(k_t - \rho k_{t-1}) + (z_t^* - \rho z_{t-1}^*) + \epsilon_t^*$$

There are two ways to pursue from here. One is to instrument endogenous variable $\tilde{r}_{t-1}$ with other lagged variable that is not correlated with z^*_{t-1}. A natural candidate is lagged investment i_{t-1}. It will responds to any permanant shocks summarized by x^*, but doesn't depend on transitory shocks. Obvsiouly we can also use any twice-lagged variables like k_{t-2}, but this brings a trade-off between endogeneity vs sample selection if the exit decisions of plants in our sample is conditional on size. We exploit the moment conditions that

$$E[i_{t-1}(z^*_t - \rho z^*_{t-1} + \epsilon^*_t)] = 0$$

The GMM estimator gives an estimate $(1 \hat{+} \frac{1}{\eta}) = .784$, very similar to the NLS estimator we had. But the persistence estimator ρ increases substantially from .77 to .85.

F. Olley and Pakes Method

As a robustness check, we also use an estimator proposed by Olley and Pakes (1996). It shares the similar theoretical underpinning that investment $i(k_t, x^*_t)$ depends only on persistent productivity shock x^*_t and current capital stock k_t. In addition, if investment is monotonically increasing in productivity x^*_t, then it can be expressed as an inversion $x^*_t = i_{-1}(k_t, i_t)$. Thus the revenue equation can be written as:

$$\tilde{r}_t = (1 + \frac{1}{\eta})k_t + a_t + \epsilon_t = \phi(k_t, i_t) + z^*_t + \epsilon_t$$

where $\phi(k_t, i_t) = (1 + \frac{1}{\eta})k_t + i_{-1}(k_t, i_t)$, a non-parametric function of k_t and i_t. Given a consistent estimate of $\hat{\phi}_t$ of $\phi(k_t, i_t)$, we can furuther make use of the assumption of x^* as a first order markov process such that

$$\hat{\phi}_t = (1 + \frac{1}{\eta})k_t + \rho(\hat{\phi}_{t-1} - (1 + \frac{1}{\eta})k_{t-1}) + \xi_t$$

The Olley and Pakes estimator gives an estimate of $(1 \hat{+} \frac{1}{\eta}) = .731$, which is close to our benchmark case. It will imply a value of mean $\alpha = .34$.

G. OLS

Finally, simply using ordinary least squares and ignoring the correlation between k_{it} and a_{it} in:

$$r_{it} = \alpha k_{it} + a_{it}$$

we obtain an estimate of α equal to 0.65.

H. Aggregate Productivity Losses

The table below reports calculations similar to those we have conducted in the main text in Table 3 (correlation between k_{it} and a_{it} and aggregate productivity losses from misallocation) for the different α coefficients we estimate above (two of the estimates above lie very close to those reported in text, $\alpha = .44$, so we do not report those here).

Aggregate productivity losses are largest for the OLS estimate and lowest for the Olley-Pakes estimate, but are nevertheless large in all cases.

	$corr\,(a_{it}, k_{it})$	$\Delta \log A$
$\alpha = .34$	0.48	-0.24
$\alpha = .45$	0.33	-0.40
$\alpha = 0.65$	0.00	-0.98

Figure A1: Counterfactual initial stock of capital