Trade Wedges, Inventories, and International Business Cycles

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Received Date; Received in Revised Form Date; Accepted Date

Abstract

The large, persistent fluctuations in international trade that can not be explained in standard models by changes in expenditures and relative prices are often attributed to trade wedges. We show that these trade wedges can reflect the decisions of importers to change their inventory holdings. We find that a two-country model of international business cycles with an inventory management decision can generate trade flows and wedges consistent with the data. Moreover, matching trade flows alters the international transmission of business cycles. Specifically, real net exports become countercyclical and consumption is less correlated across countries than in standard models. We also show that ignoring inventories as a source of trade wedges substantially overstates the role of trade wedges in business cycle fluctuations.

Keywords: Inventory, Net Exports, Trade Volatility, International Comovement.

JEL classification: E31, F12.

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†Nils Gornemann and David Richards provided excellent research assistance. We thank Sam Kortum, Jonathan Heathcote, Paolo Pesenti, Mario Crucini and seminar participants at Wharton, Yale, Wisconsin, Notre Dame, and Duke for their comments and suggestions.

‡The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia, the Federal Reserve Bank of Minneapolis, or the Federal Reserve System.

1. Introduction

The recent global collapse and rebound of international trade has renewed interest in understanding both the determinants of the cyclical fluctuations of international trade and the role of international trade in transmitting business cycles across integrated economies. Our understanding of international business cycles is limited, however, by the failure of standard models to account for the dynamics of international trade. As Levchenko, Lewis, and Tesar (2010) forcefully document, international trade tends to fluctuate much more than can be explained in standard models by the changes in expenditures on traded goods and relative prices. This is true even once one carefully controls for the different composition of the goods that are traded or consumed.[1] Since nearly all models of international business cycles fail to generate the magnitude of trade fluctuations observed in the data, these models lack a potentially important channel in the international propagation of business cycles.

In this paper, we consider a model of international trade and inventory management that can generate sizable fluctuations in international trade flows, similar to those observed in the data. We then use our model to re-examine the role of trade in propagating business cycles internationally. We find that the model generates countercyclical real net exports and relatively less comovement of consumption across countries. Hence, adding inventory frictions allows us to make progress on two dimensions along which standard models fair poorly: the cyclicality of real net exports and the consumption-output anomaly.

We focus on inventories in a business cycle setting because inventory management decisions have been shown to be an important feature in international trade. Since international trade takes time and is relatively costly, firms that engage in international trade tend to hold much larger stocks of inventories. Our previous work, Alessandria, Kaboski and Midrigan (2010a, 2010b, hereafter AKM), documents the role of inventories in international trade empirically. We document, using various sources of data, that importers hold much larger inventory stocks than non-importers do and order goods much less frequently. Moreover, we also show that inventories account for a sizable fraction of the import collapses following large devaluations or in the recent global recession. For example, AKM 2010b show that at the height of the trade collapse, US imports of automobiles fell more dramatically than final sales of imported autos in the US. Similarly, during the rebound of US trade, US imports of autos grew much faster than final sales of imported autos. US inventories of imported cars followed suit, falling during the collapse and being restocked during the trade recovery.

Motivated by these observations, we develop a model with domestic and foreign inventories that allows us to quantify the role of inventories in trade movements. In doing so, we introduce a dynamic component into the interpretation of trade wedges garnered from static (within-period) optimality conditions. We take a standard two-country real business cycle model and introduce a retail sector that has a stockout avoidance motive for holding inventories.[2] Our model is quite tractable as it introduces a small number of additional equilibrium conditions with analytical solutions. The analytical solutions shed light on the both the long-run and cyclical determinants of inventory investment. The additional parameters are disciplined by the salient facts on the inventory holdings of imported and domestically-produced goods in the data. Since we find that inventories are important, we emphasize that, given its simplicity, our approach can be easily applied to other work on international business cycles.

Our first goal is to see whether a plausibly calibrated model of inventory management and international trade can generate volatile and persistent fluctuations in international trade that are largely attributed to movements in a trade wedge of the type documented by Levchenko, Lewis, and Tesar (2010). We find that with the inventory mechanism we propose and international business cycles driven by productivity shocks, our model generates sizable fluctuations in inventories. These movements in inventories generate, in turn, large fluctuations in international trade and the trade wedge. Moreover, we find that the sources of these wedges matter a great deal. Our inventory-generated wedges imply fluctuations in consumption and output that are in line with the data. With wedges from “exogenous” taste or trade cost shocks, consumption becomes three times as volatile as in the data and these shocks account for an unreasonably high one-third of all aggregate fluctuations in output.

Our second goal is to explore whether a model with the appropriate fluctuations in international trade can generate international business cycles that match the data along other dimensions. Specifically, we consider two well-known failures of standard international business cycle models. First, as Raffo (2008) points out, standard models do not generate countercyclical real net exports, when the movements in investment in the model are constrained to match the data. With this constraint, exports expand more than imports and real net exports are procyclical. Second, Backus,

[1] Eaton, Kortum, Neiman and Romalis (2010) also study the recent trade collapse. They focus on the changes in the ratio of trade to GDP and attribute a large fraction of these movements to trade being relatively intensive in durables. Engel and Wang (2011) also focus on the role of durables in the volatility of trade. In our analysis, we focus on the movements of trade that cannot be accounted for by composition.

[2] Our earlier quantitative work on the recent recession (AKM, 2010b) applied a model without capital investment. It therefore lacked an important element for a quantitative analysis of international business cycle properties. The option of investing in capital and inventories, and their relationship with interest rates, has been shown to be important in analyzing the role of inventories in business cycles (Khan and Thomas, 2007a).

Kehoe and Kydland (1994, BKK hereafter) show that standard trade models predict consumption to be more correlated across countries than output, the opposite of what is observed in the data. This anomaly is now referred to as the consumption-output anomaly.[3]

We find that our model with inventories can make substantial progress regarding both of these failures of the standard model. Our model generates net exports that are countercyclical despite the fact that it accounts well for the variability of investment in the data. With inventories, following a good shock, imports expand more strongly and exports are dampened as domestic firms build their inventories of both goods. These dynamics reflect the different dynamics of net inventory investment and investment in equipment. In both the data and the model, net inventory investment movements are sharp but not very persistent, while investment in equipment has smaller and more persistent fluctuations.

In terms of the consumption-output anomaly, we find that inventories reduce the correlation of consumption across countries. The idea is simple. It is cheaper to consume from the stock of goods held locally than from goods that must be shipped internationally. Thus, consumption will depend on both the shocks and the stock of goods available. Since the stocks can move differently across countries, consumption becomes less correlated. For the same reasons, we also find that inventories tend to reduce the synchronization of production across countries, but the effect on consumption is much stronger.

Our paper is related to many papers that study trade dynamics and business cycles empirically and theoretically.[4] In terms of quantitative work, our paper is closely related to the work by Backus, Kehoe and Kydland (1992, BKK hereafter) and Stockman and Tesar (1995). BKK show that standard trade models imply a very tight link between relative quantities and relative prices and that, given this tight link, it is impossible for equilibrium business cycle models to generate relative prices and quantities that match the data. Stockman and Tesar show that shocks to tastes can break the link between relative quantities and prices and create a trade wedge. They consider the role of these shocks in the propagation of business cycles. Unlike their work, which takes the wedge as exogenous, we focus on understanding the source of the wedge. In our analysis, we show that the transmission of business cycles looks markedly different with endogenous wedges arising from inventories than with exogenous wedges arising from taste shocks or trade costs. Indeed, with only exogenous wedges, these taste shocks become an important driver of aggregate fluctuations. Lastly, this paper is related to our own work on inventories and trade. Similar to AKM (2010b), we also develop a general equilibrium model of international trade and inventory adjustment. That paper studies the fluctuations in trade in the global downturn in 2008-09 using a model that lacks capital and only considers transition dynamics following aggregate shocks. In contrast, here we work with a slightly simpler two-country general equilibrium model of inventory holdings and trade with capital accumulation. This model is linearizable, a feature which makes studying business cycle fluctuations very tractable.

The paper is organized as follows. In the next section, we discuss some evidence on the cyclical behavior of international trade. We also present some evidence about the relationship between the adjustment of inventories and the synchronization of production in the motor vehicle industry in the US, Europe and Japan. In Section 3 we build a model of international trade and inventory management. In Section 4 we calibrate the model. In Section 5 we discuss the main properties of the model and in Section 6 we consider alternative paramaterizations. Section 7 concludes.

2. Theory and Evidence

In this section, we provide evidence that inventory adjustment strongly influences the dynamics of imports, define the trade wedge, and summarize the key cyclical properties of the US trade flows. We also examine the role of inventories in accounting for the synchronization of global auto production in the last recession. In particular, we empirically document the effect of changes in the stock of Japanese-produced autos held overseas on production in Japan.

2.1. Evidence from Japanese Autos

We first study the dynamics of US imports of Japanese autos during the 2007-2011 period. This period includes two major events: the collapse and rebound of trade in the global recession from 2008 to 2010, as well as the tsunami in Japan in March 2011. An advantage of studying data on autos is that we can separately measure imports, sales, and inventories of imported Japanese autos. The dataset we study also allows us to distinguish between autos produced in Japan, as opposed to those produced by Japanese plants in the US.

The top panel of Figure 1 shows that imports fell substantially below sales in both episodes of large trade collapses. Consequently, both periods were characterized by sharp declines in the stock of inventories. Moreover, trade rebounds

[3] See Baxter and Crucini (1995) who propose one resolution to this puzzle, namely, incomplete markets and adding permanent productivity shocks.

[4] Husted and Kollintzas (1984) study import dynamics in the presence of inventory dynamics in a partial equilibrium model.

are associated with faster increases in imports than in sales and thus an increase in the stock of inventories. We next show that movements in imports are much larger than what can be explained by changes in relative prices or demand alone, and are therefore associated with large changes in *trade wedges*.

2.2. Trade Wedges and Cyclical Properties of Trade

We use the approach of Levchenko, Lewis, and Tesar (2010) to decompose changes in trade into those predicted by a standard gravity equation and those unexplained by theory – trade wedges. These wedges can arise due to changes in preferences (as in Stockman and Tesar, 1995), trade barriers, export participation by producers (Alessandria and Choi, 2007, and Ghironi and Melitz, 2005), or due to inventory changes (Alessandria, Kaboski, and Midrigan, 2011). We show next that trade wedges account for a substantial proportion of the movements in trade in the last 5 years and that inventories play an important role in determining the magnitude of the wedge.

To motivate our analysis, consider the following accounting identity:

$$M_t = C_{mt} + S_{mt} - S_{mt-1}, \tag{1}$$

where M_t is imports, C_{mt} is sales of imported goods, and S_{mt} is the inventory stock of imported goods at the end of period t so that $S_{mt} - S_{mt-1}$ is net inventory investment. We also assume a constant elasticity of demand for imported goods,

$$C_{mt} = T_t \left(P_{mt}/P_t\right)^{-\gamma} C_t, \tag{2}$$

where P_{mt} is the price of imported goods, P_t is the aggregate price index and C_t denotes total sales (or absorption). Equation (1) is an accounting identity, while (2) characterizes a large class of models of international trade in which preferences or technology is an Armington (CES) aggregator over imported and local goods.

To obtain a measure of the import wedge, researchers typically use imports, M_t, to proxy for sales of imported goods, C_{mt}, and construct a measure of the wedge as the gap between the actual data on (log) imports, m_t, and the prediction of the theory, $c_t + \gamma\left(p_{mt} - p_t\right)$:

$$\hat{\tau}_t = m_t - c_t + \gamma\left(p_{mt} - p_t\right). \tag{3}$$

We refer to this measure as the *naive wedge*. Imports and sales of imported goods are equal, however, only absent changes in inventories. More generally, the *actual wedge* is equal to

$$\tau_t = m_t - \frac{\overline{S}_m}{\bar{C}_m}\left(s_{mt} - s_{mt-1}\right) - c_t + \gamma\left(p_{mt} - p_t\right). \tag{4}$$

where $\bar{S}_m$ is the steady-state stock of imported inventories and $\bar{S}_m/\bar{C}_m$ is the inventory-to-sales ratio. To derive this expression, we assume that in the steady state sales of foreign goods equals imports, $\bar{C}_m = \bar{M}$, so that inventory investment is equal to zero.

To get a sense of the magnitude of these two wedges, we plot them using data for Japanese motor vehicles in the US in the bottom panel of Figure 1.[5] We note that the wedges are strongly correlated with fluctuations in trade. Importantly, the actual wedge, which adjust for inventory, is a lot smaller (its variance is 1/5th smaller) than the naive wedge that ignores inventories.

The top panel of Table 1 reports some properties of fluctuations in wedges, imports, sales, and relative prices of Japanese motor vehicles in the US from 2007m1 to 2011m12. Note that imports are twice as volatile as final sales of all autos, while sales of Japanese autos are only 40 percent more volatile than final sales. The naive wedge is almost 50 percent more volatile than sales and strongly correlated with imports (0.83), while the actual wedge is 40 percent less volatile and weakly correlated with imports (0.25).

We next measure the naive import wedge for all US imports of merchandise. For this calculation, we assume a conventional value of the Armington elasticity of $\gamma = 1.5$ and measure the relative price of imports, $\left(p_{mt} - p_t\right)$, as the ratio of the non-petroleum import price index relative to a price index on final expenditures of goods . Specifically, we measure the price of goods as

$$p_t = \alpha p_{gt} + (1-\alpha)\, p_{xt}.$$

where p_{gt} is the price of consumer goods and p_{xt} is the price of investment in equipment and software (from the BEA). We let $\alpha = 0.75$ to match the importance of the consumption of goods in goods expenditure. Our measure of aggregate

[5] We use $\gamma = 3$ here, our estimate of the elasticity of demand for Japanese cars over this period. When looking at more disaggregated data, it is common to find that imported goods tend to be more substitutable than they are with the aggregate data.

expenditure, C_t, is real domestic consumption of goods plus investment in equipment and software. We focus on the period 1995q1 to 2010q4.

Figure 2 plots the deviations from an HP-filtered trend (with a smoothing parameter of 1600) of US imports, the naive import wedge, and the actual import wedge. While imports are more volatile than the wedge, clearly, a substantial fraction of the fluctuations of imports is explained by the fluctuations in the naive wedge. The bottom panel of Table 1 summarizes the fluctuations in trade variables over the business cycle. Imports are about 1.4 times as volatile as US manufacturing industrial production (IP). Imports are strongly procyclical, with a correlation with IP of 0.92. The import wedge is slightly more volatile than IP and is also procyclical, with a correlation with IP of 0.86. Imports and the import wedge are persistent, with an autocorrelation of 0.86 and 0.78, respectively. The price of imports relative to final goods is about one-third as volatile as production and is not very correlated with either the import wedge or imports.

To measure the actual import wedge requires a measure of the inventory-to-sales ratio of imported goods as well as the changes in imported inventory. Unlike with autos, we lack direct measures of imported inventories and thus use the entire stock of US inventories as a proxy. Consistent with the micro evidence in AKM (2010a) that importers hold about double the inventory of non-importers, we set $\overline{S}_m/\bar{C}_m$ equal to 2.25, about twice the average inventory-to-sales ratio since 1997. We assume that fluctuations in imported inventories are perfectly correlated with fluctuations in aggregate inventories and then use equation (4) to calculate the actual import wedge.

Fluctuations in the actual import wedge, τ_t, are generally smaller than fluctuations in the naive wedge that ignores inventory adjustments, $\hat{\tau}_t$. Indeed, in the current recession, nearly one-third of the decline and all of the increase in the import wedge disappears and the size of the actual import wedge appears less unusual. Thus, inventory adjustments made a sizable contribution to recent trade fluctuations. In the last line of Table 1 we report the cyclical properties of the actual import wedge. With this adjustment, the actual wedge is 30 percent less volatile, 10 percentage points less persistent and 10 percentage points less correlated with imports than the import wedge.

This evidence clearly suggests that trade wedges are big and that the inventory management decisions of importers are an important source of these wedges. However, a key shortcoming of our empirical approach to estimating the role of inventory adjustment in fluctuations in trade at the aggregate level is that it requires a very strong assumption that imported inventories move one for one with total inventories. This is unlikely; it certainly is not the case for autos. Thus, we require a model of optimal inventory adjustment to accurately estimate the role of inventory adjustments in trade flows. That is what we do in Section 3.

2.3. ***Global Motor Vehicle Production and Sales***

To shed light on production and the global propagation of shocks, we turn again to the motor vehicle industry, a large and globally integrated industry. Figure 3 plots the quarterly production and sales of motor vehicles since 2007 in the US, Europe (the 27 countries in the EU), and Japan, the three largest markets prior to the Great Recession.

There are three key things to notice. First, production is quite synchronized, including the sharp, severe (42-70 log point) global collapse and rebound in production in 2008 and 2009 and the smaller reduction in production in the US and EU around the tsunami in 2011. Second, sales are less correlated across countries than production.[6] Third, relative to production, the drop in sales is smaller (30-60 percent) in each market, reflecting in part the drawing down of inventories. The relatively sharp decline in production relative to sales in Japan and Europe compared to the US could partly reflect a reliance on sales in the US market as well as a relatively large inventory adjustment of Japanese and EU cars in the US.

We can demonstrate the changes in foreign inventories account for a substantial share of production dynamics using the following accounting identity

$$\text{Production} = \text{Sales} + \Delta\text{Inventory} + \underbrace{\text{Sales}^* + \Delta\text{Inventory}^*}_{\text{Exports}},$$

where * denote foreign variables.

Table 2 reports the change in average exports, production, sales in Japan and outside of Japan. The first column reports the percent change in the average activity in the collapse period (November 2008 to August 2009) versus average activity prior to the collapse (May 2008 to October 2008). The second column reports the change in the average activity in the rebound (September 2009 to August 2010) against the collapse. Initially, production was essentially equal to domestic sales plus exports. Thus, the 42 percentage point drop in production relative to the 39 percentage point drop

[6] These very different sales dynamics in part reflect large differences in the size and timing of different national motor vehicle scrappage programs.

in domestic sales implies that 3 percentage points of the decline in production amounted to a reduction in domestic inventories. Now consider the role of foreign inventories. Again, US sales approximately equaled exports to the US initially, so the much larger percentage point decline in exports (65 vs. 26) indicates a reduction in US inventories. If we extrapolate the change in sales in the US market, which accounts for about 40 percent of exports, to exports overall, the total drop in global sales (foreign and domestic) would have been 20 percent.[7] Thus, a change in inventories would account for 22 of the original 42 percentage point drop, with the vast majority of this (19 percentage points) reflecting a decline in foreign inventories. An analogous calculation using the rebound implies that potentially 80 percent of the 25 percentage point increase in production in this latter period reflected inventory accumulation in foreign markets. These numbers demonstrate that inventories and trade play an important role in the international propagation of shocks.

The last thing we consider is the dynamics of Japanese net exports in this period. Here we scale real exports by total trade flows, so $nx = 2\left(ex - im\right)/\left(ex + im\right)$. Figure 4 plots the dynamics of real and nominal net exports as well as net exports of passenger cars (scaled by aggregate trade). Clearly, we see that in the 2008-09 period, real net exports dramatically moved from surplus to deficit and then back to surplus. The adjustment was large and sudden. Real net exports fell 30 percentage points from 2008Q3 to 2009Q1. The recovery was as large and almost as sudden, with net exports increasing 25 percentage points from 2009Q1 to 2009Q4. The movements in nominal net exports are more gradual and reflect in part the movements in energy prices. Net exports of autos show similar dynamics that are not as large, given autos' share in trade. Given our evidence from the auto industry, it is clear that the inventory adjustment overseas contributed to these net export dynamics.

In sum, the motor vehicle industry shows substantial synchronization of production in the recent recession. It also shows that production tends to fluctuate more than sales so that inventory stocks play an important role in the decline in production. Focusing on Japan, we see that the decline in exports drove the collapse and recovery in production and that overseas inventory dynamics strongly influenced the movements in exports. Indeed, based on the US, overseas inventory dynamics may have doubled the decline in production in Japan and lead to a rebound that was 5 times stronger.

3. Model

We now extend the two-country general equilibrium model of international trade of Backus, Kehoe, and Kydland (1994) to include a monopolistic retail sector that holds inventories of both domestic and imported intermediates. Inventories are introduced through the assumptions that orders must be placed before idiosyncratic demand is realized and that unsold goods can be stored with a cost. This gives retailers a stockout avoidance motive for holding inventories and allows for straightforward linearization. Specifically, in each country, a continuum of local retailers buy imported and domestic goods from a competitive intermediate goods sector in each country, and each retailer acts as a monopolist supplier in selling its particular variety of the good. This is very much like Walmart buying from many independent producers. Consumers purchase these varieties and then use an aggregation technology to transform home and foreign varieties into a final good used for consumption and investment.

3.1. Environment

Formally, consider an economy with two countries, Home and Foreign. In each period, t, the economy experiences one of finitely many states η_t. Let $\eta^t = (\eta_0, ..., \eta_t)$ be the history of events up to date t, with the initial state η_0 given. Denote the probability of any particular history η^t as $\pi(\eta^t)$.

The commodities in the economy are labor, a continuum of intermediate goods (indexed by $j \in [0,1]$) produced in Home, and a continuum of intermediate goods produced in Foreign. These intermediate goods are purchased and sold as retail goods to consumers. Finally, consumers combine intermediate goods to form final goods (consumption and capital), which are country-specific because of a bias for domestic intermediates. We denote goods produced in Home with a subscript H and goods produced in Foreign with a subscript F. (Foreign allocations and prices are denoted with an asterisk.) In addition, there are a full set of Arrow securities.

3.1.1. ***Consumers***

The consumer's preferences over final consumption $c\left(\eta^t\right)$ and leisure $l\left(\eta^t\right)$ are

$$\sum_{t=0}\sum_{\eta^t}\beta^t\pi\left(\eta^t\right)U\left[c\left(\eta^t\right) - hC\left(\eta^{t-1}\right), l\left(\eta^t\right)\right]. \tag{5}$$

[7] The overall drop in sales is the export-weighted drop in domestic and foreign (i.e., US) sales. Given an export share of 0.59, this calculation is 0.59(-0.26)+(1-0.59)(-0.12)=0.20

The consumer chooses his consumption but utility can also depend on past aggregate consumption $C(\eta^{t-1})$ for $h \neq 0$, which allows for habit formation. Habit formation is external in that the consumer does not take into account how its current consumption decision affects its future habit. Habit helps generate persistent fluctuations in consumption as in the data.

Using Home consumers as an example, the final consumption, $c(\eta^t)$, and investment, $x(\eta^t)$, good is produced by aggregating purchases of a continuum of domestic retail goods $y_H^d(j,\eta^t)$ and a continuum of imported retail goods $y_F^d(j,\eta^t)$ (where $j \in [0,1]$ indexes the good in the continuum).

$$D\left(\eta^t\right)^{\frac{\gamma-1}{\gamma}} = \left(\int_0^1 v_H\left(j,\eta^t\right)^{\frac{1}{\theta}} y_H^d(j,\eta^t)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta(\gamma-1)}{(\theta-1)\gamma}} + \tau^{\frac{1}{\gamma}}\left(\int_0^1 v_F\left(j,\eta^t\right)^{\frac{1}{\theta}} y_F^d(j,\eta^t)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta(\gamma-1)}{(\theta-1)\gamma}} \tag{6}$$

The weights $v_H(j,\eta^t)$ and $v_F(j,\eta^t)$ are subject to idiosyncratic shocks that are iid across j and t. We assume that these shocks are distributed Pareto, with domestic taste shocks drawn from $G^D(v) = 1 - v^{-a_{DOM}}$ and the taste shocks on imported goods from $G^I(v) = 1 - v^{-aIMP}$. These stochastic idiosyncratic demand shocks lead to the precautionary stockout avoidance motive for holding inventories. Allowing a_{DOM} and a_{IMP} to differ allows incentives to carry inventory to differ across imported and domestic goods.[8]

The parameter $\tau \in [0,1]$ captures the lower weight on Foreign goods (i.e., a Home bias). For simplicity, we make the innocuous assumption that the shocks to retail varieties are identical across consumption and investment.[9] The Foreign consumer uses analogous technologies except that the lower weights τ multiply the Home goods.

The final composite good is used for consumption, investment, and investment adjustment costs so that

$$c\left(\eta^t\right) + x\left(\eta^t\right)\left(1 + \frac{\xi}{2}\left(\frac{x\left(\eta^t\right)}{x\left(\eta^{t-1}\right)} - 1\right)^2\right) = D\left(\eta^t\right).$$

The left-hand side of the resource constraint shows that investment is subject to quadratic adjustment costs on the change in investment, parameterized by ξ. This type of adjustment cost is useful to get investment to be hump-shaped as in the data.

The law of motion for country-specific capital, which depreciates at rate δ, is:

$$k(\eta^{t+1}) = (1-\delta)\,k(\eta^t) + x(\eta^t). \tag{7}$$

The consumer purchases domestic and imported retail goods at prices $p_H(j,\eta^t)$ and $p_F(j,\eta^t)$, respectively, supplies labor at a wage $W(\eta^t)$, and earns capital income at the rental rate $R(\eta^t)$ and profits $\Pi(\eta^t)$ (from retailers). In addition, it trades Arrow securities $B\left(\eta^{t+1}\right)$ that are purchased at time t and pay off one unit next period in state η^{t+1}. We denote the price of the security in state η^t that pays one unit in state η^{t+1} as $Q\left(\eta^{t+1}|\eta^t\right)$. Suppressing the dependence of all variables on η^t for brevity, the consumer's period t budget constraint is therefore expressed:[10]

$$\sum_{i=\{H,F\}} \int_0^1 p_i(j)\, y_i(j)\, dj + \sum_{\eta^{t+1}} Q(\eta^{t+1}) B\left(\eta^{t+1}\right) = Wl + Rk + \Pi + B \tag{8}$$

The foreign consumers is analogous except that prices and profits are those in the Foreign country. The prices of Arrow securities $Q(\eta^{t+1}|\eta^t)$ are the same in both countries, since they can be traded internationally at no cost. The consumer takes prices and profits as given and maximizes (5) by choosing a series for labor supply, retail purchases, investment, and Arrow securities subject to (6), (7), and (8).

[8] In AKM (2010a) the different inventory holdings of retailers of domestic and imported goods arise from differences in the transaction costs (fixed order costs and delivery lags) between international and domestic orders. We find that the dynamic properties of our stockout-avoidance inventory model are quite similar to those of the micro-founded transaction cost model in a variation where both models lack capital accumulation.

[9] It is straightforward to introduce different inventory holdings for investment and consumption goods as well as different levels of tradability in consumption and investment.

[10] We also need to set a borrowing limit in order to rule out Ponzi schemes, $B(\eta^t) > \underline{B}$, but this borrowing limit can be set arbitrarily large, i.e., $\underline{B} << 0$.

3.1.2. **Producers**

For each country, we model a single representative producer that supplies both the Home and Foreign markets. Intermediate goods in Home are produced by competitive firms using the following technology:

$$M\left(\eta^t\right) = A\left(\eta^t\right) K\left(\eta^t\right)^{\alpha} L\left(\eta^t\right)^{1-\alpha} \tag{9}$$

where $M\left(\eta^t\right)$ is output of intermediates, $K\left(\eta^t\right)$ is aggregate capital and $L\left(\eta^t\right)$ is aggregate labor used for production of intermediates.

Aggregate productivity in Home evolves according to

$$\log A\left(\eta^t\right) = \rho \log A\left(\eta^{t-1}\right) + \varepsilon\left(\eta^t\right)$$

Finally, we assume an analogous production function for Foreign-produced intermediates with a country-specific aggregate productivity shock. Producers are competitive, maximizing static profit-taking prices as given.

3.1.3. **Retailers**

In Home there is a unit mass of retailers selling goods that were produced in Home, and another unit mass of retailers selling goods that were produced in Foreign. Retailers purchase intermediates from producers and sell them to consumers. For a Home retailer of good j produced in Home, retail sales are denoted $y_H\left(j, \eta^t\right)$, while purchases from intermediate goods producers are denoted $z_H\left(j, \eta^t\right)$. We focus on Home retailers operating in Home. Retailers operating in Foreign face an identical problem, as do Foreign retailers operating in Home. (The subscript F continues to distinguish goods *produced* in Foreign, while an asterisk continues to denote the corresponding arguments for the *retailers* in the Foreign market.)

The key friction motivating the holding of inventories is that a retailer must choose the amount of goods to have in its store at time t *before* learning $v_H\left(j, \eta^t\right)$. We denote this stock on hand as $z_H\left(j, \tilde{\eta}^t\right)$, where $\tilde{\eta}^t$ signifies the history up to date t excluding the retailer's demand realization at t. However, the retailer chooses its price $p_H\left(j, \eta^t\right)$ *after* learning $v_H(j, \eta^t)$. We also allow the retailer to return the unsold stock, but only at $t+1$ so the retailer will be able to sell it at next period's price $\omega\left(\eta^{t+1}\right)$ after incurring the inventory carrying costs of depreciation. Allowing the resale of unsold goods in the following period at the market price means we do not need to keep track of the distribution of inventory holdings.

The profit maximization problem of a Home retailer selling home goods is:

$$\max_{z_H(j,\tilde{\eta}^t),\ p_H(j,\eta^t)} \sum_{t=0}^{\infty} \sum_{\eta^t} Q\left(\eta^t\right) \left[p_H\left(j, \eta^t\right) y_H\left(j, \eta^t\right) - \omega\left(\eta^t\right) \left[z_H\left(j, \tilde{\eta}^t\right) - s_H\left(j, \eta^{t-1}\right)\right]\right]$$
$$s.t.\ \ y_H\left(j, \eta^t\right) \leq z_H\left(j, \tilde{\eta}^t\right)$$
$$s_H\left(j, \eta^t\right) = \left(1 - \delta_s\left(\eta^t\right)\right) \left[z_H\left(j, \tilde{\eta}^t\right) - y_H\left(j, \eta^t\right)\right] \tag{10}$$

where $Q\left(\eta^t\right) = Q\left(\eta^t | \eta^{t-1}\right) Q\left(\eta^{t-1} | \eta^{t-2}\right) ... Q\left(\eta^1 | \eta^0\right)$ is the date 0 Arrow-Debreu price of 1 unit of the numeraire to be delivered in state η^t, and $y^H\left(j, \eta^t\right)$ is the demand the retailer faces at price $p_j^H\left(\eta^t\right)$. Unsold inventory $z_H\left(j, \tilde{\eta}^t\right) - y_H\left(j, \eta^t\right)$ can be carried forward, but this entails a cost from physical depreciation, captured by $\delta_s\left(\eta^t\right)$. The end-of-$t$ stock of inventories of undepreciated inventories is denoted $s_H\left(j, \eta^t\right)$. Thus the retailer will optimally choose inventories to trade off being able to satisfy demand when demand is high with the costs of carrying unsold inventories into the next period when demand is low.

The Home retailer that sells Foreign goods faces a similar problem, except its wholesale cost is $\omega^*\left(\eta^t\right)$. Foreign retailers also face analogous problems.

3.2. *Equilibrium*

We first define and then show some preliminary characterization of the equilibrium, which will be solved numerically.

3.2.1. *Definition*

In this economy, an equilibrium is defined as (i) an allocation of aggregate and individual quantities $\{C\left(\eta^t\right), c(\eta^t), L\left(\eta^t\right), l\left(\eta^t\right), K(\eta^t), k(\eta^t), M\left(\eta^t\right), y(\eta^t), B\left(\eta^t\right), \Pi\left(\eta^t\right)\}_{t=0}^{\infty}$, and disaggregate goods $\{y_i\left(j, \eta^t\right), s_i\left(j, \eta^t\right), z_i\left(j, \tilde{\eta}^t\right)_{i=H,F}\}_{t=0}^{\infty}$

for both Home and Foreign, and (ii) prices of goods $\left\{ \{p_i(j,\eta^t)\}_{i=H,F},\ \omega(\eta^t),\right\}$ and factors in $\{W(\eta^t), R(\eta^t)\}_{t=0}^{\infty}$ for both Home and Foreign, and (iii) Arrow security prices $\left\{Q\left(\eta^{t+1}|\eta^t\right)\right\}_{t=0}^{\infty}$, such that:

- Given prices, the allocations satisfy the consumers' problems, the intermediate producers' problems, and retailers' problems in Home and Foreign;
- Individual consumption $c(\eta^t)$ equals aggregate consumption, $C(\eta^t)$; and
- The retail goods, labor, and capital markets clear in each country, and the intermediate goods markets and Arrow security markets clear for the world economy.

We briefly describe the market clearing conditions. First, Arrow securities are in zero net supply, so the bond market clearing condition requires $B(\eta^t) + B^*(\eta^t) = 0$. Second, all capital and labor is used in intermediate goods production.

$$\begin{aligned} L\left(\eta^t\right) &= l\left(\eta^t\right) \\ K\left(\eta^t\right) &= k(\eta^t) \end{aligned}$$

The resource constraint for intermediate goods requires that production equal orders:

$$M\left(\eta^t\right) = \int_0^1 \left[z_H\left(j,\tilde{\eta}^t\right) - s_H\left(j,\eta^{t-1}\right)\right] dj + \int_0^1 \left[z_H^*\left(j,\tilde{\eta}^t\right) - s_H^*\left(j,\eta^{t-1}\right)\right] dj \tag{11}$$

Notice that intermediate goods produced in Home, $M(\eta^t)$, have two uses: they go to domestic retailers of Home goods, $z_H(j)$, and to foreign retailers of exported Home goods, $z_H^*(j)$. The resource constraint for individual retail goods $y_H(j,\eta^t)$ involves those sold as consumption goods $c_H(j,\eta^t)$ and investment goods $x_H(j,\eta^t)$:

$$y_H\left(j,\eta^t\right) = c_H\left(j,\eta^t\right) + x_H\left(j,\eta^t\right)$$

A parallel set of market clearing constraints holds for foreign goods.

3.2.2. Preliminary Characterization

We briefly offer a preliminary characterization of the features of the equilibrium. Notationally, expressions are simplified by dropping the η^t dependence where it does not cause confusion (e.g., static conditions). Perfectly competitive producers simply pay factors their marginal products and price at marginal cost, $\omega = r^\alpha w^{1-\alpha}/A$.

The consumer's maximization can be solved step-wise, with the consumer choosing an allocation of retail purchases $y_H^d(j)$ and $y_F^d(j)$ to minimize the expenditure necessary to deliver D units of the final composite good. With respect to aggregates, the consumer's optimization conditions are standard. The zero net supply condition on Arrow securities leads to the following pricing $Q(\eta^t) = \beta^t \pi(\eta^t) \frac{U_c(\eta^t)/P(\eta^t)}{U_c(\eta^0)/P(\eta^0)}$.

The cost-minimizing first-order conditions define the demand for the retail varieties:

$$\begin{aligned} y_H^d(j) &= v_H(j)\left(\frac{p_H(j)}{P_H}\right)^{-\theta}\left(\frac{P_H}{P}\right)^{-\gamma} D \\ y_F^d(j) &= v_F(j)\,\tau\left(\frac{p_F(j)}{P_F}\right)^{-\theta}\left(\frac{P_F}{P}\right)^{-\gamma} D \end{aligned} \tag{12}$$

where we have defined the following aggregate price indexes for Home-produced output, Foreign-produced output, and output overall:

$$P_H = \left(\int_0^1 v_H(j)\,p_H(j)^{1-\theta}\,dj\right)^{\frac{1}{1-\theta}} \tag{13}$$

$$P_F = \left(\int_0^1 v_F(j)\,p_F(j)^{1-\theta}\,dj\right)^{\frac{1}{1-\theta}} \tag{14}$$

$$P = \left[P_H^{1-\gamma} + \tau P_F^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{15}$$

We characterize the optimal decisions for retailers of Home goods in Home, but the other retailers are analogous. Given the ex ante symmetry of the problem, all j retailers have the same desired stock-on-hand, $z_H(j) = z_H$. The retailers' pricing decision rule depends on its idiosyncratic demand shock relative to a threshold value, $\hat{v}_H$, and is:

$$p_H(j) = \begin{cases} \hat{p}_H = \frac{\theta}{\theta-1} \sum_{\eta^{t+1}} \left(1 - \delta_s\left(\eta^t\right)\right) \frac{Q\left(\eta^{t+1}\right)}{Q\left(\eta^t\right)} \omega\left(\eta^{t+1}\right) \text{ if } v_H(j) \leq \hat{v}_H \\ \left(\frac{z_H}{v_H(j)\left(\frac{P_H}{P}\right)^{-\gamma}\left(\frac{1}{P_H}\right)^{-\theta} D}\right)^{-\frac{1}{\theta}} \text{ if } v_H(j) > \hat{v}_H \end{cases} \tag{16}$$

For a low demand shock, it sets the price, $\hat{p}_H$, as a $\theta/(\theta-1)$ markup over its marginal shadow cost, the expected discounted value of carrying the inventories forward. For a sufficiently high demand shock, the retailer sells at the price to just sell its entire inventory.

Given this pricing policy, our assumption of Pareto-distributed taste shocks leads to an analytical solution for optimal stock-on-hand, z_H. Equating the two branches of the pricing function yields the stockout threshold of demand, $\hat{v}_H$:

$$\hat{v}_H = \frac{z_H}{\left(\frac{P_H}{P}\right)^{-\gamma}\left(\frac{\hat{p}_H}{P_H}\right)^{-\theta} D}$$

Notice that the price of firms that stock out is now equal to $p_H(j) = \hat{p}_H\left[v_H(j)/\hat{v}_H\right]^{\frac{1}{\theta}}$, and so the price index for home goods sold in Home equals:

$$P_H = \hat{p}_H\left[\left(\hat{v}_H\right)^{1-a_D}\left(\frac{a_{DOM}}{1-a_{DOM}} - \frac{a_{DOM}}{\frac{1}{\theta} - a_{DOM}}\right) - \frac{a_{DOM}}{1-a_{DOM}}\right]^{\frac{1}{1-\theta}}$$

The prices are all functions of the threshold values and can be substituted into the expressions for optimal stock on hand. Continuity of the prices at $\hat{v}_H$ (i.e., again equating both branches of (16)) yields the threshold value:

$$\hat{v}_H = \left(\frac{\frac{1}{\theta}}{a_{DOM} - \frac{1}{\theta}}\right)^{1/a_{DOM}}\left(\frac{\omega_t}{E_t\left(1-\delta_s\right)\frac{Q_{t+1}}{Q_t}\omega_{t+1}} - 1\right)^{-1/a_{DOM}}$$

We can substitute this to solve for the z_H, which is the product of three terms:

$$\begin{aligned} z_H &= \left(\frac{P_H}{P}\right)^{-\gamma}\left(\frac{\hat{p}_H}{P_H}\right)^{-\theta} D \\ &\times\left(\frac{1/\theta}{a_{DOM} - 1/\theta}\right)^{1/a_{DOM}} \\ &\times\left(\frac{\omega\left(\eta^t\right)}{\sum_{\eta^{t+1}}\left(1-\delta_s\left(\eta^t\right)\right)\frac{Q\left(\eta^{t+1}\right)}{Q\left(\eta^t\right)}\omega\left(\eta^{t+1}\right)} - 1\right)^{-1/a_{DOM}} \end{aligned} \tag{17}$$

Desired stock-on-hand is increasing in the average level of demand (first term) and the stockout avoidance motive (second term) but decreasing in the price today relative to the expected price tomorrow (third term). This last term gives insight into inventory investment motives in response to transitory productivity shocks; there is an incentive to invest when goods are cheap, interest rates are low, and inventory costs are low.

The aggregate stock of inventories held in Home is given by

$$S = S_H + S_F = \int_0^1 s_H(j)\,dj + \int_0^1 s_F(j)\,dj \tag{18}$$

Additionally, inventory depreciation is assumed to depend on the stock of local inventories:

$$\delta_s = \delta_{s,0} + \delta_{s,1} e^{\left(S/\bar{S}-1\right)}$$

where $\bar{S}$ is the steady-state level of inventories. If $\delta_{s,1} < 0$, there are economies of scale to holding inventories, while with $\delta_{s,1} > 0$, there are congestion costs.

We stress here that the equilibrium equations are a minimum departure from the standard international business cycle literature. The retailers' problem and the taste shocks introduced into the consumer's demand problem result in only two changes in equilibrium conditions. The pricing formula in (16) replaces the simple markup over marginal cost. Second, we have the additional equations that track inventories: the optimal stock-on-hand equation (17), and the definitions of retailer and aggregate end-of-period inventories (equations (10) and (18), respectively.) Finally, since the model has no fixed costs or occasionally-binding constraints complicating the decision rules or laws of motion of aggregates, the aggregate equilibrium is linearizable.

4. Calibration

We now describe the functional forms and parameter values considered for our benchmark economy. The parameter values used in the simulation exercises are reported in Table 3. Similar to Raffo (2008) we use a GHH instantaneous utility function. Unlike Raffo, we allow for habit persistence in consumption.

$$U(c,l) = \log\left((c - hC_{-1}) - \frac{\psi}{1+\eta} l^{1+\eta}\right).$$

With external habit the household takes the path of C_{-1} as given.

For several parameters, we assign typical values that are relatively standard in the international real business cycle literature. These parameters include the preference parameters $\{\beta, \gamma, \psi, \eta\}$ and technology parameters $\{\delta, \alpha\}$. Our period is a quarter so $\beta = 0.99$. We set the depreciation rate of capital to $\delta = 0.025$ and the capital share to $\alpha = 0.33$. We choose ψ, the relative weight on leisure in the utility function in order to match a labor supply of one-third. We set η so that the Frisch elasticity is 2. We assign the elasticity of substitution between domestic and imported goods $\gamma = 1.5$, a standard value.

The remaining parameters $\{\theta, \delta_{s,0}, \delta_{s,1}, a_{DOM}, a_{IMP}, \tau_F\}$ are particular to our inventory/retailing set-up. We start by assigning $\theta = 3$, which implies that the ratio of manufacturing to total sales is 40 percent, as in the US. Although all four moments are jointly determined by all parameters, the parameter τ is the main determinant of trade flows, while a_{DOM}, a_{IMP}, δ_s primarily determine the trade share, stock of inventories, and premium of imported inventories relative to domestic inventories. We target three moments for the US from 1997 to 2010. First, imports are 26.5 percent of manufacturing sales. Second, inventory holdings are equal to 1.5 times final quarterly expenditures on consumption plus investment. The third target is that importing firms hold twice the inventory (relative to sales) as firms that source domestically. This ratio is consistent with inventory-to-sales ratios for importers vs. domestic firms that we observe for Chilean plants and for US manufacturing industries (AKM 2010b). We set depreciation to be $\delta_{s,0} = 0.016$. This implies inventory holding costs, including interest costs, of about 2.6 percent per quarter. This is quite low relative to estimates in the literature. Of course, our model misses out on some key channels that lead to inventory holdings. We undertake sensitivity to δ_s, however.

The technology shock process follows much of the literature. The persistence of national productivity shocks is 0.95 and the correlation of innovations across countries is 0.25. The size of the shocks is set to match the volatility of industrial production.

The investment adjustment costs and cyclicality of inventory holding costs are chosen to target the volatility of investment in equipment and overall investment. Matching the cyclicality of inventory investment requires $\delta_{s,1} = -0.00445$, which implies that, in booms, the costs of managing inventories fall and this encourages additional investment in inventories.[11] Finally, we set our habit parameter to match the autocorrelation of consumption in our benchmark model. This requires a habit parameter h of 0.30.

To clarify the role of inventories, we also consider the properties of a model with no inventories. This is a version of the BKK model with retailers charging a constant markup over marginal cost. In the model with no inventories, we set the investment adjustment cost so that total investment, which includes net inventory investment, is 2.89 times as volatile as production, as in the data.

5. Results

We now show that our model with inventories generates highly volatile trade flows and trade wedges as in the data and that matching these trade flows alters the transmission of business cycles in that real net exports are countercyclical and the consumption-output anomaly is weakened. The improved fit of the model arises because economic expansions

[11] An alternative approach to affect the cyclicality of net inventory investment is to introduce a physical cost of managing inventories and let this cost vary over the cycle.

are now associated with strong inventory investment for both domestic and imported goods. This inventory investment strengthens import demand and weakens both export supply and terms of trade movements in response to a productivity shock.

To clarify the role of inventories, we compare the properties of our benchmark model economy to the model without inventories. Tables 4 and 5 report some key cyclical properties of the model. Figures 5 plots the impulse response to a positive (one standard deviation) productivity shock of key variables in the benchmark inventory model and the no inventory model, respectively.

Specifically, in Table 4, we find that imports and exports are now about 7 percent more volatile than production (compared to 40-49 percent in the data and 11 percent less volatile than production with no inventories). These fluctuations in trade generate a sizable import wedge, with relative volatility of 0.79 (compared to 1.08 in the data). The large increase in wedge volatility despite the moderate increase in trade volatility arises because the inventory model generates smaller relative price fluctuations. For instance, the relative volatility of the terms of trade is 0.4 in the benchmark model against 0.57 in the no inventory model (and 0.27 in the data).

With inventories, imports are substantially more procyclical in the benchmark model (0.85) than in the model without inventories (0.69) as shown in Table 5. Neither is as procyclical as in the data, however, where the correlation with production is 0.92. The wedge is procyclical in the benchmark model (0.68) but less so than in the data (0.86). However, the model matches the correlation of the wedge with imports (0.86, model, 0.88, data).

In terms of real net exports, the inventory model generates somewhat larger fluctuations in net exports compared to the no inventory model (0.33 vs 0.21 in Table 4), but both are similar to the data (0.28). With inventories, however, net exports (normalized by sales) are countercyclical (-0.25) as in the data (-0.42). This is in strong contrast to the model without inventories, where they are procyclical (0.33). Net exports are now countercyclical because inventories make exports considerably less procyclical. The correlation of exports with production is 0.63, as compared to 0.90 without inventories.

In both models there is a consumption-output anomaly, in that consumption is more correlated across countries than output. However, we find that the anomaly, measured by the difference between the consumption and output cross-correlation, is smaller in the inventory model (0.21) than in the no inventory model (0.33). In terms of the comovement of business cycles, whose correlations are presented in the bottom panel of Table 5, we find that there is actually less synchronization of business cycles in the inventory model than in the no inventory model. For instance, the cross-correlation of production is 0.35 in the inventory model and 0.43 in the no inventory model. Similarly, the cross-correlation of consumption in the inventory model is 0.56 and 0.71 in the no inventory model. One reason for the weaker comovement is that inventories provide another way to smooth production (and consumption). We explore this in greater detail in our sensitivity analysis.

A key problem with both models, however, is that the fluctuations in trade they generate are not persistent enough. For instance, the autocorrelation of imports in Table 6 is 0.67 with and without inventories and 0.86 in the data. The model with inventories does generate wedges, but these are also not persistent enough, with an autocorrelation of 0.57 that is lower than the 0.78 in the data. Nevertheless, movements in net exports are relatively persistent, with an autocorrelation of 0.71, similar to the data. Without inventories, net exports are too transitory.

The source of these transitory fluctuations is clear from Figure 5. Following a productivity shock at home, the need to build up inventory in the more productive location leads to an initial jump in imports but a much weaker export response in the model with inventories relative to the model without inventories. Consequently, initially net exports go into a deficit and that deficit is reversed in later periods when imports fall sharply and exports expand.

6. Sensitivity

In this section, we perform further analysis to examine the role of inventories for the cyclicality of trade and the propagation of business cycles. First, we consider how the cyclicality of real net exports affects the transmission of business cycles in these models. Second, we evaluate the model's response to a global productivity shock that hits both countries symmetrically. Third, we introduce exogenous taste shocks to imports that yield exogenous trade wedges and evaluate the relationship between these exogenous wedges and the endogenous wedges driven by inventories. Finally, we change the correlation of shocks in the model of inventories in order to more closely examine the role of inventories in alleviating the comovement puzzle.

6.1. Balanced Real Net Exports

We consider how the cyclicality of real net exports influences the propagation of shocks by constraining real trade flows to be balanced each period. In order to better understand propagation, we do not recalibrate the adjustment costs on investment or inventories. The results are reported in the columns *Balanced Real Trade* for the inventory and no inventory models.

The first thing to notice is that, with balanced real trade, consumption is more correlated across countries than production in the inventory model (0.47 vs. 0.40) and less correlated in the no inventory model (0.39 vs. 0.40). The higher comovement of consumption in the inventory model reflects the use of inventories to smooth out consumption. By comparing the balanced trade to our benchmark models, we can more easily see the role of the cyclical movements in net exports. In the inventory model, where net exports are countercyclical, the consumption-output anomaly only increases 14 percentage points, while it increases 34 percentage points in the no inventory model, where net exports are procyclical. Thus, the procyclical net exports in the inventory model clearly generate substantially less synchronization in consumption across countries. Viewed differently: given a particular consumption correlation across models, we would expect output to be more correlated in the inventory model than in the no inventory model.

6.2. Global Shocks

Figure 6 shows the impulse response to a global positive productivity shock. That is, we shock both countries with a symmetric and synchronized positive productivity shock. The model with inventories leads to a large increase in trade, with imports and exports, of course, increasing symmetrically. This boom in trade exceeds the boom in production, and the increase in production is partially used for inventory investment. Hence, the increase in production exceeds the increase in absorption (consumption plus capital investment). The wedge is consequently sizable, but these inventory-driven dynamics are short-lived. In contrast, the model without inventories yields no wedge, and the increases in trade, production, and absorption are equally sized and follow the identical pattern.

Although the figures evaluate the response to a *positive* productivity shock and global boom, similar, but opposite, dynamics would arise in a global recession like that described in Section 2. They also demonstrate how correlated shocks can lead to greater volatility in trade than in production observed in the data (Table 5).

6.3. Exogenous Wedge Shocks

To show that the source of the trade wedge matters, we introduce exogenous shocks that lead to trade wedges. Specifically, we allow the home bias parameter τ to be subject to a stochastic shock $\hat{\tau}_t$ as follows

$$\tau(\eta^t) = e^{\hat{\tau}(\eta^t)}\overline{\tau}$$

These wedges are similar to the taste shocks introduced by Stockman and Tesar (1995), but here the shocks are only on foreign goods, which more closely approximates a shock to trade costs, where decreased trade costs resemble import-specific productivity shocks. Indeed the results we discuss below from this simple formulation are essentially the same as those from a more involved model with explicit shocks to iceberg trade costs.[12]

We assume the shock $\hat{\tau}\left(\eta^t\right)$ follows a first-order autoregressive process

$$\begin{aligned}
\hat{\tau}\left(\eta^t\right) &= \rho\hat{\tau}\left(\eta^{t-1}\right) + \sigma_w \varepsilon\left(\eta^t\right) \\
\hat{\tau}^*\left(\eta^t\right) &= \rho\hat{\tau}^*\left(\eta^{t-1}\right) + \sigma_w \varepsilon^*\left(\eta^t\right)
\end{aligned}$$

where σ_w governs the variance of the wedge shocks relative to the shock to productivity $\varepsilon\left(\eta^t\right)$.

We calibrate the shock to match the volatility of the trade wedge in the inventory and no inventory models. For the inventory model, $\sigma_w = 0.28$, while the no inventory model requires $\sigma_w = 2.7$, nearly ten times the size of shocks needed in the model with inventories. The results of this exercise are in the columns denoted "Trade Shocks" in Tables 4 and 5.

In the inventory model, these trade shocks bring trade more in line with the data without doing much harm to the business cycle properties of the model.[13] Specifically, output is only slightly more volatile (3.53 vs. 3.33), while trade is substantially more volatile (1.30 vs. 1.07) and quite close to the data (1.30 vs. 1.4). The real exchange rate and terms of trade become more volatile as well. Net exports become slightly less countercyclical (-0.23 vs. -0.25), while output comovement increases slightly (0.35 to 0.37), and consumption comovement falls slightly (0.56 to 0.53).

In the no inventory model, these trade shocks bring trade more in line with the data but mess up the business cycle properties of the model. Specifically, output is much more volatile (5.12 vs. 3.40), trade is substantially more volatile (1.63 vs. 0.89), and the terms of trade are substantially more volatile (1.03 vs. 0.57 and 0.27 in the data). Consumption becomes 1.34 times more volatile than IP compared to 0.46 in the data. Net exports become slightly less procyclical

[12] This assumes the trade costs are not included in the export price. Details of this model and the results are available upon request.

[13] Impulse response functions in the model verify this. Other than the movements in trade wedges and trade, they are nearly identical to the model without wedges (see our earlier working paper, Alessandria, Kaboski, and Midrigan, 2012).

(0.29 vs. 0.33), while output and consumption comovement both fall 8 percentage points. Given the movements in output and consumption, these trade shocks become a major driver of international business cycles. In short, we find that the trade dynamics from endogenous wedges reflecting inventory adjustment lead to a substantially better fit than those reflecting exogenous shocks to trade.

6.4. Equalized Comovement

The final columns in Tables 4 and 5 present the results for the model with no inventories, where the correlation of productivity shocks across the two countries has been set in order to match the comovement in production across countries in the model with inventories. This facilitates an easier comparison of the impact of inventories on the relative comovement of production and consumption. Clearly, even after the comovement in production is equalized, the model with inventories is yielding much less comovement in consumption. Again, this is because the presence of inventories impacts the cost of consumption. Thus, consumption will depend on both productivity shocks and the stock of available inventories. While productivity shocks hit the costs of goods symmetrically, inventory stocks move differently across countries, especially since inventory motives differ across imported and domestic goods. This allows consumption to be less correlated across countries.[14]

6.5. Additional Sensitivity

We consider a variety of alternative calibrations. Specifically, we consider how the imported inventory premium, depreciation rate of inventories, and asymmetries in inventory holdings affect our results.[15] The results of these alternative calibrations are also reported in Tables 4 and 5.

We first consider the impact of eliminating the inventory premium on imported goods. The results are reported in the column *No Import Premium*. As one should expect, eliminating the import premium reduces the volatility of trade flows from 1.07 to 0.92. The lower volatility of trade arises primarily from a smaller wedge as this is reduced from 0.79 to 0.61. Real net exports are now more countercyclical than in the benchmark (-0.31 vs. -0.25) as exports become slightly less procyclical and imports slightly more procyclical.

We next consider how our choice of inventory depreciation affects our results. In the column titled *Low Inventory Depreciation* we consider the case with $\delta_{0s} = 0.008$. With this lower depreciation rate, we must reduce the idiosyncratic uncertainty to hit the same inventory targets as before. The low depreciation rate lowers the volatility of the inventory stock from 0.54 to 0.49. The volatility of trade flows falls slightly from 1.07 to 1.04. The wedge also becomes slightly less volatile. Net exports become slightly less countercyclical.

Lastly, we consider the properties of the model when there are asymmetric inventory holdings across countries. Specifically, we consider the case where the Foreign country produces and exports a good that requires more inventory holdings than the Home country. An example of this might be Japan intensively producing and exporting autos while the US intensively produces and exports less inventory-intensive services. Specifically, we calibrate the model so that the retailers of Home goods in the foreign country face the same idiosyncratic uncertainty as retailers of locally produced goods in the Foreign country. The last two columns of the table report the statistics from the *Asymmetric Countries* (High denotes the country with high final good inventories and Low denotes the country with low final good inventories). With the asymmetric inventory holdings, import and export volatility are no longer equal as the incentive to adjust inventories differs across destinations. Indeed, Home imports are more volatile than Home exports and the wedge is larger in the Home country than in the Foreign country. Business cycle correlations do not change dramatically, although the High inventory country now has real net exports that are more countercyclical, while the Low inventory country has slightly less countercyclical net exports.

7. Conclusions

Over the business cycle, fluctuations in international trade involve substantial, persistent departures from theory in that the movements in trade generally cannot be fully explained by movements in final expenditures and relative prices. We show empirically and theoretically that an important reason for the failure of standard models to explain these trade flows is that they ignore the inventory management decisions of importers. We show a two-country GE model with an

[14] With the presence of retailers and inventories there are issues with measuring the shocks hitting the economy that make the measured correlation of shocks in the inventory model lower than the actual shocks. In the no inventory model the bias goes the other way. Thus, in this experiment the correlation of measured Solow residuals in the two models is about the same.

[15] We also consider the role of the elasticity of substitution and assets trade for propagation in our working paper. These generally have a moderate impact in our framework.

inventory management decision and business cycles driven by productivity shocks can generate some of the explained and unexplained movements in international trade over the business cycle.

In terms of the propagation of business cycles, we find that bringing trade flows more in line with the data generates international business cycles more like the data. Specifically, with inventories, real net exports are countercyclical as in the data. Following a positive productivity shock in the home country, inventory investment motives give the home country a stronger desire to import and a weaker desire to export than in a standard model without inventories. Moreover, with countercyclical net exports, inventories lead consumption to become less correlated across countries for a given amount of comovement in production. This occurs because the stock of inventories is local and influences the consumption decision. Lastly, we find that introducing shocks to preferences for foreign goods into our benchmark inventory model, a natural stand-in for changes in trade costs and an alternative source of the trade wedge, can generate all of the movements in the trade wedge without dramatically altering international business cycles. Introducing these same shocks in a model without inventories requires much larger shocks to trade costs and generates business cycles that don't look much like the data.

The importance of inventories in the international transmission of business cycles suggests several avenues for further investigation. Our model of inventories has an explicit supply chain, but it would be interesting to introduce a more involved input-output structure, where manufacturing production involves intermediates. The differing importance of inventories across sectors may also have implications for how shocks filter through the input-output structure. Such an analysis would require disaggregate data on industry-level holdings of imported and domestic inventories that are more broadly representative than our automobile case study. Assembly and analysis of informative disaggregate data on sales and inventory of imported goods would be helpful.

Finally, our analysis only considers business cycles arising from supply shocks. In practice, monetary, government, and financial shocks are also likely to matter. To the extent that these shocks affect inventory investment, they will generate trade wedges. The framework we have developed is tractable enough to consider these types of shocks.

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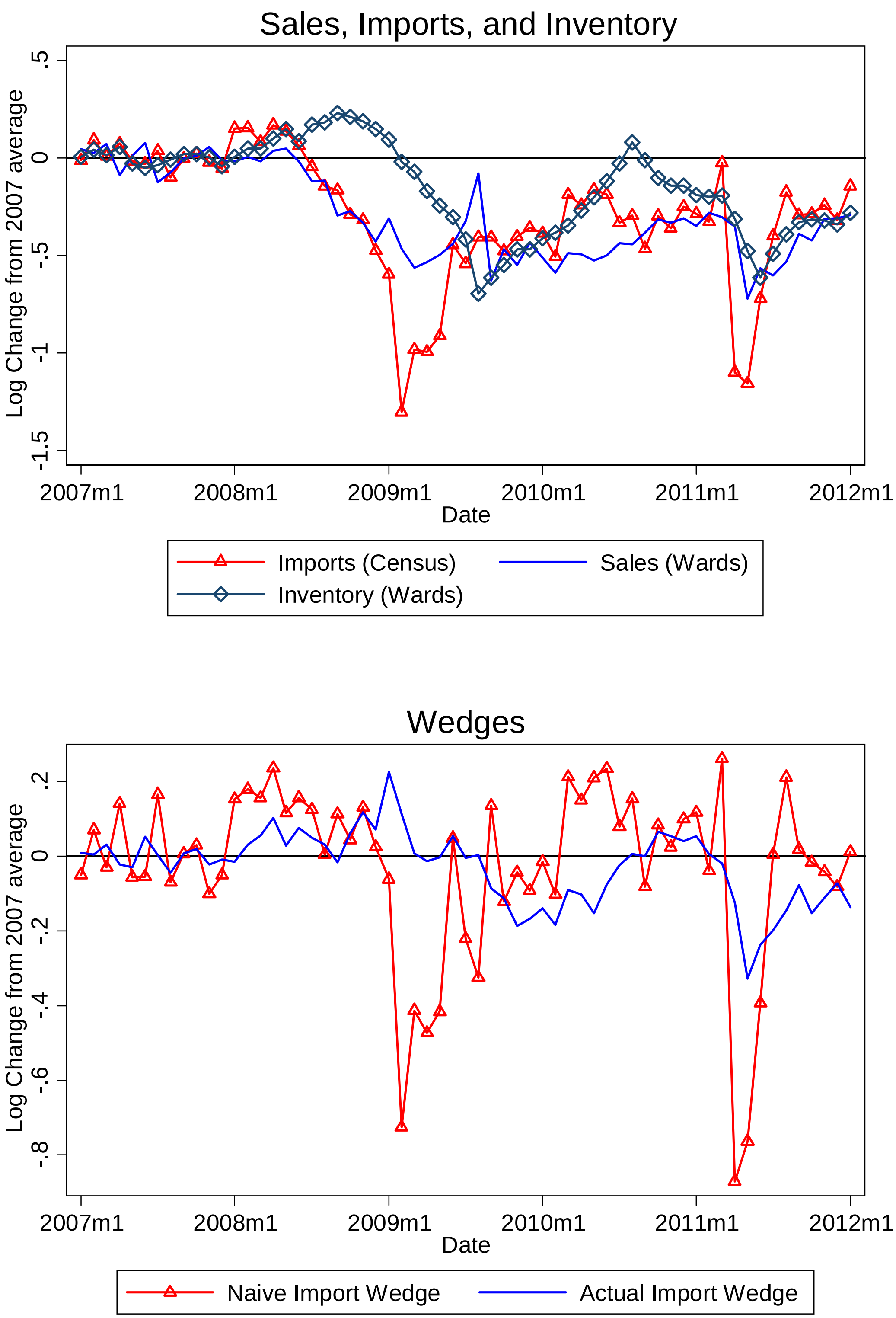

Figure 1: US Car Sales, Imports, and Inventory of Japanese cars

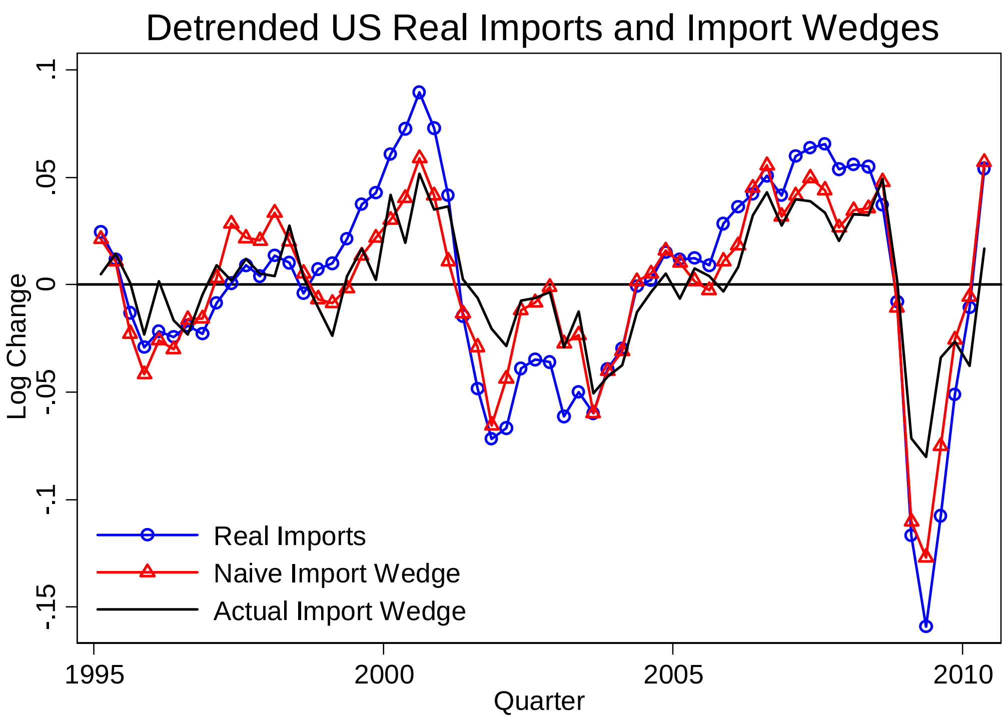

Figure 2: Deviations from Trend of US Imports, Wedge, and Import Price

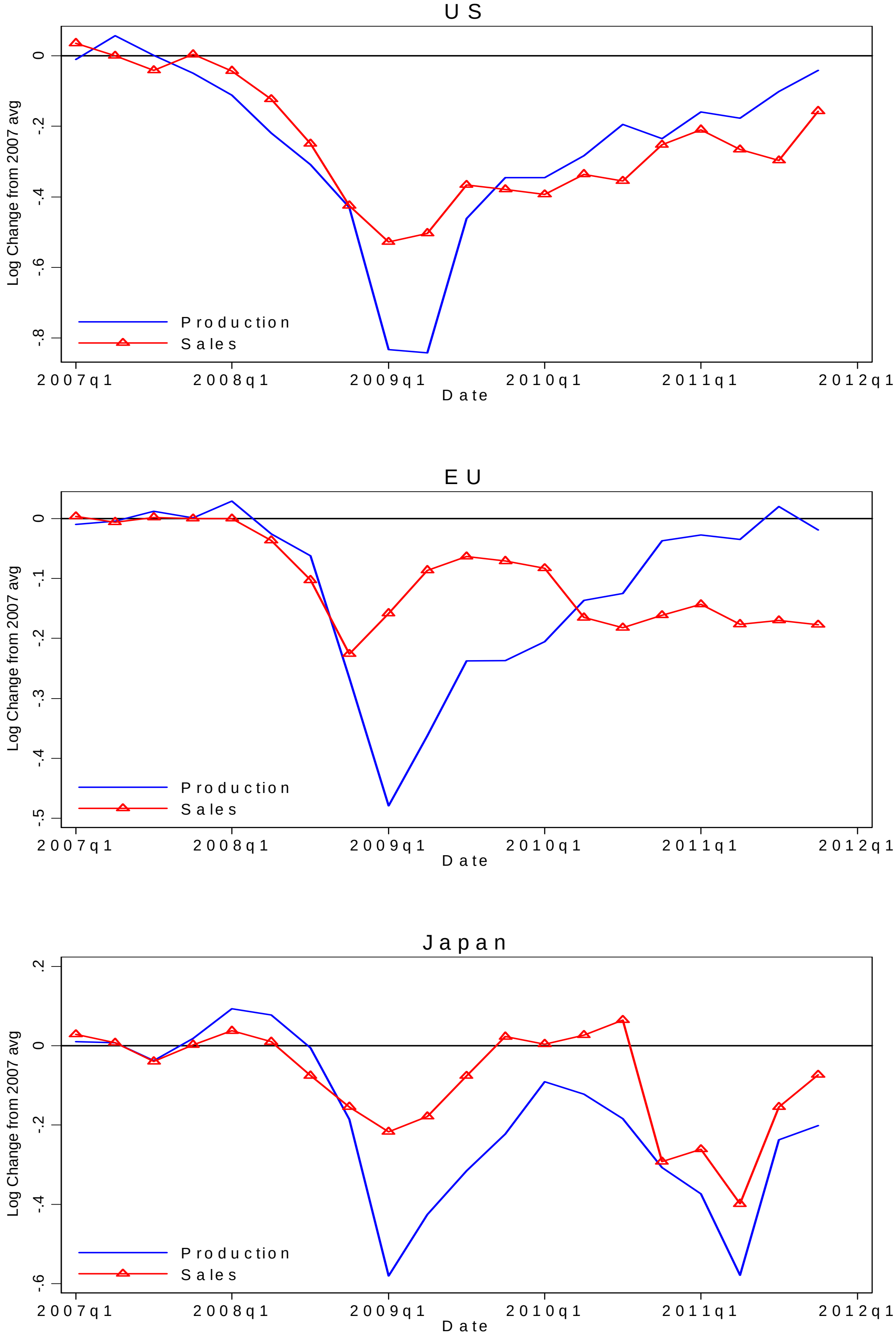

Figure 3: Production and Sales by Market

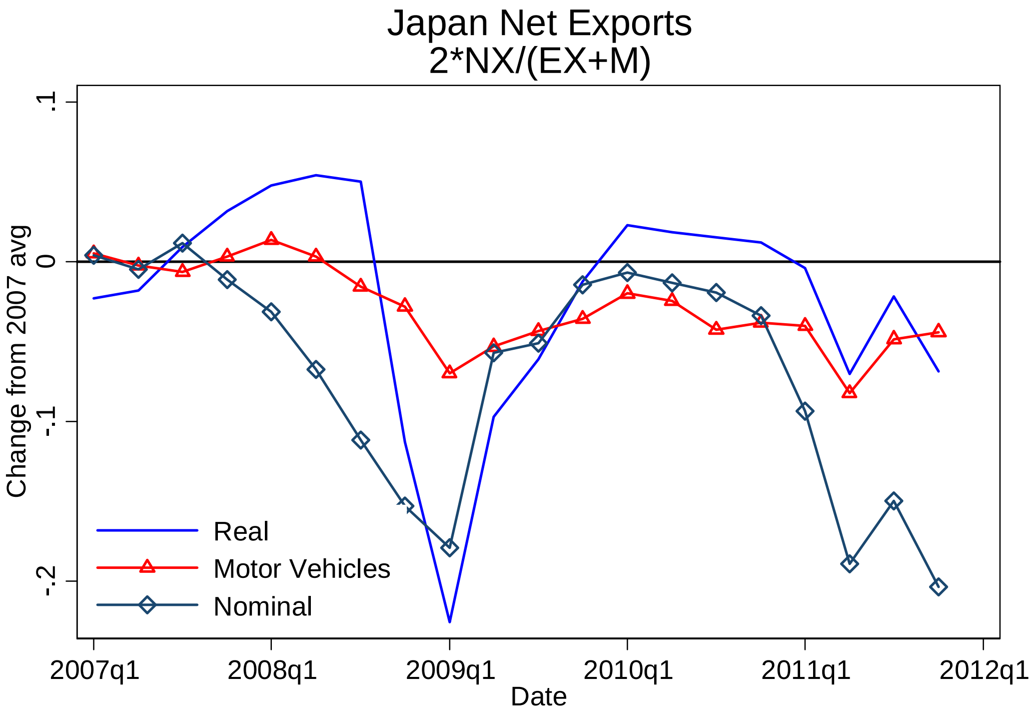

Figure 4: Real Net Exports in Japan

1

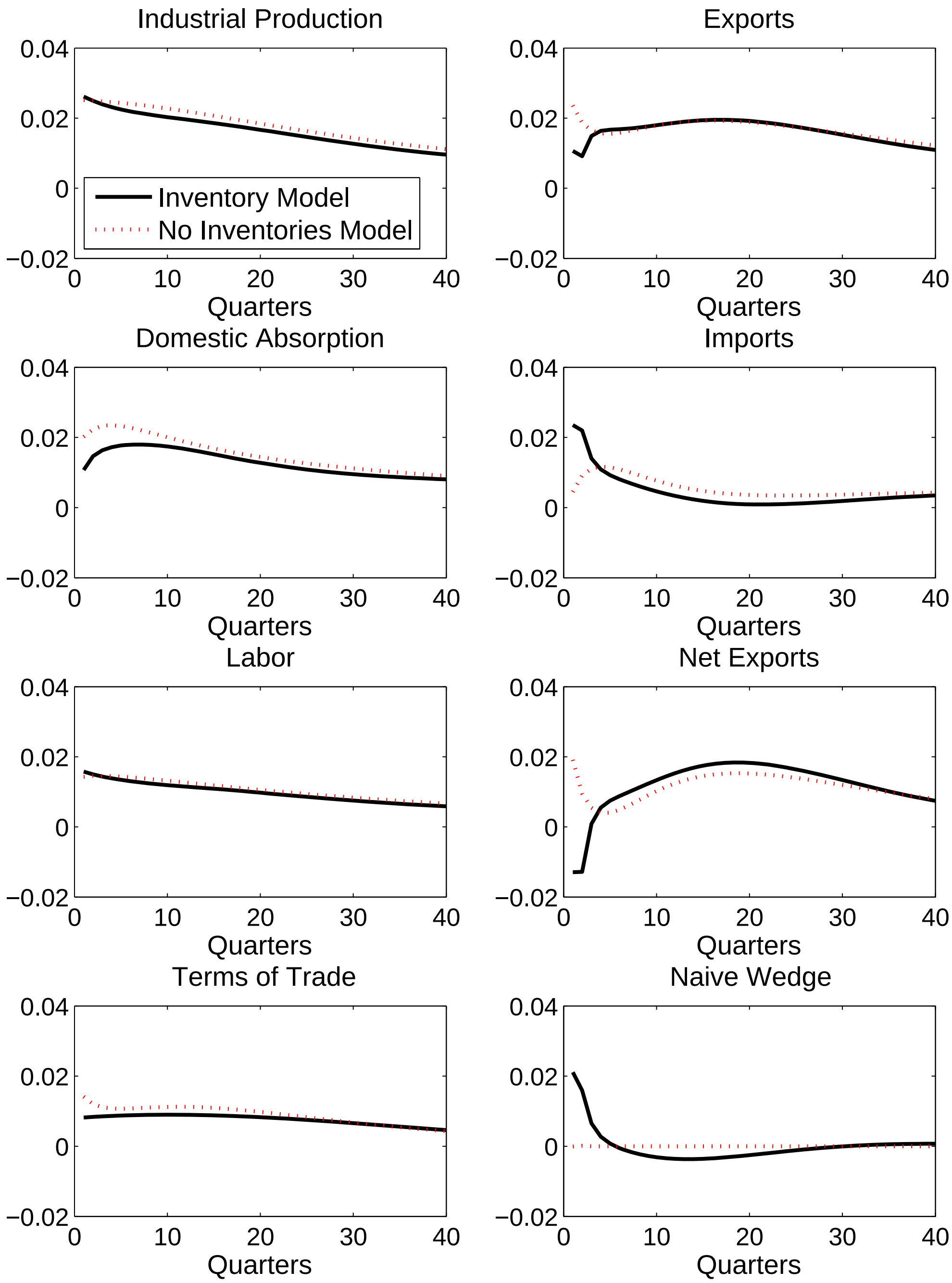

Figure 5: Impulse Response of Positive Home Productivity Shock

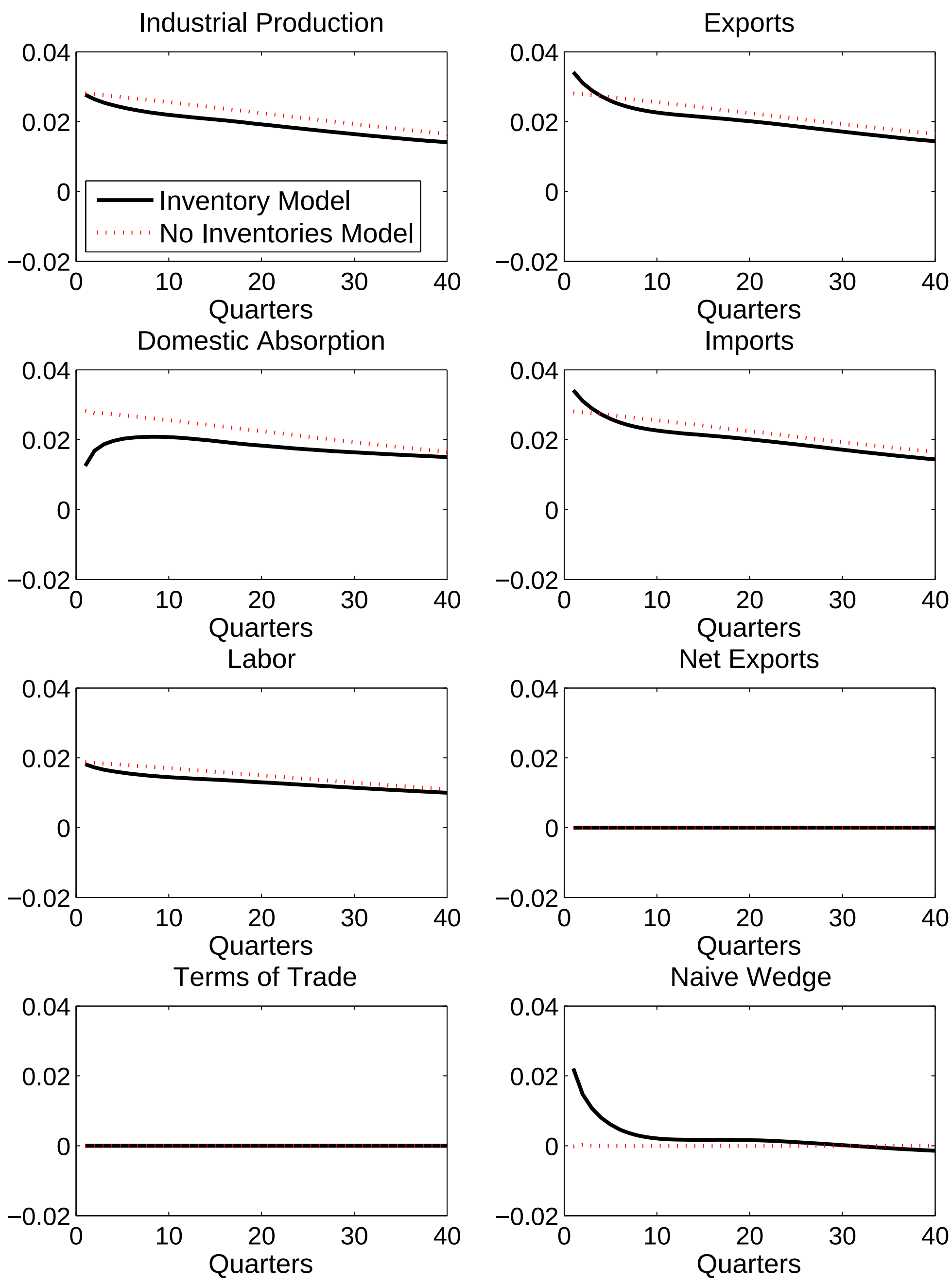

Figure 6: Impulse Response to Positive Global Productivity Shock

Table 1: US Business Cycle Statistics on Imports

US Imports and Sales of Japanese Motor Vehicles (2007m1 to 2011m12)

	Volatility rel. to Sales	Autocorrel.	Correlation with Sales	Correlation with Imports
Sales (all cars)*	16.4	0.81	1	
Imports (Census)	2.06	0.78	0.71	1
US Sales Japan cars'	1.38	0.85	0.90	0.72
Import Price	0.09	0.94	-0.54	-0.47
Naïve Import Wedge	1.47	0.49	0.22	0.83
Actual Import Wedge	0.60	0.79	0.15	0.25

US Aggregate Imports (1995Q1 to 2010q4)

	Volatility rel. to IP	Autocorrel.	Correlation with IPMFR	Correlation with Imports
Industrial Production (IP)*	3.44	0.91	1.00	
Imports Goods	1.40	0.86	0.92	1.00
Import Price	0.36	0.83	0.08	0.21
Naïve Import Wedge	1.08	0.78	0.86	0.94
Import Ratio	0.84	0.73	0.78	0.93
Actual Import Wedge	0.80	0.67	0.81	0.85

* Sales and IP volatility are absolute, not relative. Import Price measured relative to price of final basket.

Table 2: Change in Japan Passenger Car Production, Sales, and Exports

Change from	Nov. 08 to Aug. 09 vs May 08 to Oct. 08	Sep. 09 to Aug 10 vs Nov. 08 to Aug. 09
Export share of production in previous period	0.59	0.48
Production	-0.42	0.25
Domestic Sales	-0.12	0.21
Exports	-0.63	0.27
Exports plus Domestic sales	-0.39	0.23
Global Sales*	-0.20	0.05
US Sales	-0.26	-0.11
US Exports	-0.65	0.28

* Global Sales measures the change in Domestic Sales + Foreign Sales where US Sales is a proxy for sales outside of Japan

Table 3: Parameter Values

		Benchmark	No Habit	No Inventory
	Assigned Parameters			
β	discount factor	0.99	0.99	0.99
γ	Armington elasticity of H vs. F	1.5	1.5	1.5
θ	elasticity across varieties in H & F	3	3	3
$\delta 0s$	inventory depreciation	0.016	0.016	
$\delta 1s$	Elasticity of inventory depreciation	-0.0044	-0.0045	
μ	Elasticity of inventory costs	0	0	
η	Frisch Elasticity	0.5	0.5	0.5
h	Habit	0.30	0	0.3
δ	Capital Depreciation	0.025	0.025	0.025
α	Capital Share	0.33	0.33	0.33
	Calibrated Parameters			
ad	home taste shocks	1.3	1.3	1.3
af	foreign taste shocks	1.0001	1.0001	1.3
τ	foreign weight	0.335	0.335	0.36

Table 4: Business cycle statistics model and data

		Inventory Model - Endogenous Costs							No Inventory			
Standard Deviations:	*Data*	*Benchmark*	*Balanced RNX*	*Trade shocks*	*No Import Premium*	*Low Depreciation*	*Asymmetry - High*	*Asymmetry - Low*	*Benchmark*	*Balanced RNX*	*Trade shocks*	*Comove fixed*
Production	3.44	3.33	3.27	3.53	3.34	3.33	3.33	3.35	3.4	3.45	5.12	3.39
NX, NX/(EX+M)	2.67	3.08	0	2.98	2.79	2.51	3.01	3.01	1.96	0	6.45	2.44
NX/sales	0.28	0.33	0	0.32	0.29	0.26	0.32	0.29	0.21	0	0.68	0.26
NII/sales	0.45	0.82	0.62	0.88	0.80	0.77	0.86	0.72				
Standard Deviations (rel. to IP):												
Consumption, C	0.46	0.53	0.56	0.61	0.53	0.54	0.55	0.52	0.63	0.69	1.34	0.62
Employment, L	0.82	0.62	0.61	0.68	0.62	0.62	0.62	0.62	0.62	0.62	0.96	0.61
Total investment, X + Delta S	2.89	2.89	2.3	2.89	2.89	2.88	2.89	2.89	2.9	2.69	2.88	2.89
Investment, X	1.62	1.62	1.31	1.62	1.62	1.62	1.62	1.62	2.9	2.69	2.87	2.89
Inventory Stock	0.63	0.54	0.44	0.55	0.51	0.49	0.56	0.47				
Exports	1.49	1.07	0.99	1.3	0.92	1.04	0.95	1.08	0.89	0.84	1.63	0.9
Imports	1.4	1.07	0.99	1.3	0.92	1.04	1.09	0.94	0.89	0.84	1.63	0.9
RER	0.89	0.2	0.24	0.32	0.18	0.19	0.19	0.19	0.27	0.23	1.18	0.29
TOT	0.27	0.4	0.54	0.45	0.37	0.40	0.38	0.38	0.57	0.49	1.03	0.61
Inventory Sales Ratio	0.82	0.52	0.52	0.53	0.53	0.53	0.48	0.56				
Sales (incl Mfr)	0.72	0.78	0.76	0.81	0.78	0.78	0.78	0.77	0.97	1	1.29	0.97
Wedge	1.08	0.79	0.66	1.09	0.61	0.74	0.85	0.58			1.07	
AutoCorrelations:												
Production, IP	0.91	0.7	0.69	0.69	0.70	0.69	0.69	0.70	0.72	0.72	0.7	0.73
NX, NX/(EX+M)	0.78	0.71		0.56	0.70	0.71	0.61	0.61	0.4		0.47	0.32
NX, NX/sales	0.76	0.71		0.56	0.70	0.71	0.61	0.61	0.4		0.47	0.32
NII/salesM	0.61	0.55	0.52	0.58	0.53	0.54	0.49	0.60				
Consumption, C	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.63	0.72	0.64	0.74
Employment, L	0.91	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.73	0.72	0.71	0.74
Total investment, X + Delta S	0.79	0.64	0.63	0.65	0.64	0.63	0.58	0.71	0.86	0.73	0.92	0.81
Investment, X	0.9	0.95	0.95	0.95	0.95	0.95	0.95	0.95				
Inventory Stock	0.92	0.92	0.93	0.93	0.92	0.92	0.91	0.93				
Exports	0.85	0.67	0.66	0.51	0.69	0.66	0.76	0.55	0.68	0.72	0.67	0.66
Imports	0.86	0.67	0.66	0.51	0.69	0.66	0.55	0.76	0.68	0.72	0.67	0.66
RER	0.76	0.78	0.75	0.75	0.78	0.76	0.77	0.77	0.65	0.72	0.67	0.61
TOT	0.71	0.74	0.74	0.74	0.75	0.74	0.74	0.74	0.65	0.72	0.62	0.61
Inventory Sales Ratio	0.78	0.73	0.71	0.71	0.73	0.72	0.70	0.76				
Sales (incl Mfr)	0.91	0.79	0.78	0.79	0.79	0.79	0.79	0.80	0.74	0.72	0.72	0.75
Wedge	0.78	0.57	0.56	0.43	0.55	0.56	0.48	0.59			0.72	

Balanced RNX denotes a case where real exports = real imports. Trade shocks denotes a shock to trade weight that matches the volatility of the trade wedge in the inventory and no inventory models. Asymmetry High and Low denote countries with high and low retail inventory levels. Comove fixed means choosing the international correlation of productivity shocks to achieve the same cross-correlation of output as in our benchmark inventory model.

Table 5: Business cycle statistics model and data: Cross Correlations

		Inventory Model - Endogenous Costs							No Inventory			
Correlation with IP:	*Data*	*Benchmark*	*Balanced RNX*	*Trade shocks*	*No Import Premium*	*Low Depreciation*	*Asymmetry - High*	*Asymmetry - Low*	*Benchmark*	*Balanced RNX*	*Trade shocks*	*Comove fixed*
NX, NX/(EX+M)	-0.04	-0.25		-0.23	-0.31	-0.18	-0.32	-0.19	0.33		0.25	0.29
NX/sales	-0.42	-0.25		-0.23	-0.31	-0.18	-0.32	-0.19	0.33		0.25	0.29
NII/sales	0.56	0.71	0.82	0.73	0.73	0.71	0.69	0.74				
Consumption, C	0.8	0.96	0.96	0.96	0.96	0.96	0.96	0.95	0.97	1	0.97	0.97
Employment, L	0.91	1	0.99	1	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99
Total investment, X + NII	0.86	0.94	0.99	0.95	0.95	0.95	0.94	0.95	0.95	0.99	0.81	0.95
Investment, X	0.92	0.67	0.6	0.62	0.70	0.66	0.66	0.70	0.95	0.99	0.82	0.95
Inventory Stock	0.81	0.71	0.64	0.69	0.70	0.74	0.77	0.64		-0.31		
Exports	0.85	0.63	0.83	0.68	0.59	0.70	0.67	0.57	0.9	0.84	0.91	0.87
Imports	0.92	0.85	0.83	0.83	0.87	0.83	0.85	0.83	0.69	0.84	0.72	0.64
RER	-0.38	0.55	0.54	0.56	0.54	0.56	0.61	0.49	0.53	0.55	0.45	0.55
TOT	0.69	0.56	0.55	0.55	0.56	0.56	0.60	0.52	0.53	0.55	0.44	0.55
Inventory-Sales Ratio (IS)	-0.03	-0.96	-0.98	-0.96	-0.95	-0.97	-0.95	-0.96		-0.33		
Sales (incl Mfr)	0.97	0.97	0.97	0.97	0.98	0.97	0.97	0.97	1	1	0.99	0.99
Wedge	0.86	0.68	0.83	0.74	0.65	0.67	0.65	0.66		-0.01	0.97	
Correlations:												
IP and IPs*	0.6	0.35	0.4	0.37	0.34	0.35	0.35	0.35	0.42	0.4	0.57	0.35
L and Ls*	0.39	0.49	0.61	0.51	0.47	0.50	0.35	0.35	0.65	0.61	0.72	0.6
C and Cs*	0.38	0.56	0.47	0.53	0.55	0.58	0.48	0.48	0.75	0.39	0.71	0.67
X and Xs*	0.33	0.09	0.73	0.16	0.20	0.26	0.23	0.23	0.22	0.42	0.25	0.17
IS and Sales	-0.13	-0.91	-0.93	-0.9	-0.90	-0.92	-0.89	-0.92				
Total Investment and NII	0.87	0.56	0.58	0.54	0.60	0.56	0.52	0.65				
Exports and Imports	0.85	0.63	1	0.79	0.59	0.74	0.61	0.61	0.79	1	0.7	0.68
TOT and RER	-0.16	1	1	1	1.00	1.00	1.00	1.00	1	1	0.99	1
NIIY AND X	0.47	0.04	0.05	0.02	0.10	0.01	-0.01	0.15				
Wedge and TOT	0.09	0.24	0.41	0.27	0.29	0.20	0.29	0.25			0.6	
Wedge and Imports	0.88	0.86	0.78	0.91	0.79	0.84	0.87	0.77			0.57	

*Taken from Chari, Kehoe, and McGrattan (2002) based on the US and Europe