Debt Constraints and Employment: Not-for-Publication Appendix*

Patrick J. Kehoe† Virgiliu Midrigan‡ Elena Pastorino§

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†Stanford University, UCL, and Federal Reserve Bank of Minneapolis, pkehoe@stanford.edu.
‡New York University, virgiliu.midrigan@nyu.edu.
§Hoover Institution, Stanford University, and Federal Reserve Bank of Minneapolis, epastori@stanford.edu.

A Omitted Model Details

A.1 Proofs of Equivalence Results

We first consider the economy with housing and then the economy with liquid assets.

A.1.1 An Economy with Housing

In the economy with housing, families own houses and a family's borrowing is subject to collateral constraints based on the value of their houses. The preferences of the family are

$$\max_{c_t} \sum_{t=0} \beta^t [u(c_t) + \psi_t v(h_t)], \tag{1}$$

where c_t is the consumption of any of its members and h_t is the amount of housing consumed. The family faces a budget constraint,

$$c_t + qa_{t+1} + p_t h_{t+1} = y_t + d_t + a_t + p_t h_t. \tag{2}$$

At the beginning of period t, the household owns a house of size h_t with value $p_t h_t$ and decides whether to adjust the size of the house to h_{t+1}. The family faces a *collateral* constraint that limits the maximum amount it can borrow to a fraction $\bar{\chi}$ of the value of the family's home:

$$a_{t+1} \geqslant -\bar{\chi} p_t h_{t+1},$$

where $\bar{\chi}$ is the maximum loan-to-value ratio.

There is a fixed supply of houses, normalized to 1, and each unit of a house delivers one unit of housing services each period. The housing stock does not depreciate. The parameter ψ_t in the utility function governs the relative preference for housing. This parameter varies over time and is the source of changes in house prices and thus, through the collateral constraint, the amount of debt the family can borrow.

Now, given an exogenous sequence of taste parameters $\{\psi_t\}$ and a world bond price q, let $\{Q_t\}$ defined by $Q_t = \beta^t u'(c_t)/u'(c_0)$ be the resulting sequence of shadow prices of the family. Given these shadow prices, the labor market side of the model is identical to that above. That is, the value functions for consumers and firms, the matching and bargaining, and the free entry conditions are all identical. Essentially, the only ingredient needed from the family problem to solve the rest of the model is the sequence of shadow prices $\{Q_t\}$.

We claim that in terms of consumption, wages, and labor market outcomes, the debt constraint economy is equivalent to the economy with housing. We first show that given a debt constraint economy with a sequence of borrowing limits $\{\chi_t\}$, we can construct a sequence of taste parameters $\{\psi_t\}$ for the economy with housing such that, except for the price and quantity of housing, allocations and prices in the two equilibria coincide. We then show that given an economy with housing with taste parameters $\{\psi_t\}$, we can construct a sequence of borrowing limits $\{\chi_t\}$ such that, except for the price and quantity of housing, allocations and prices in the two equilibria coincide.

Let us start with a debt constraint economy with a budget constraint,

$$c_t + qa_{t+1} = y_t + d_t + a_t.$$

Note that the solution to the family problem can be summarized by the first-order conditions

$$qQ_t = Q_{t+1} + \theta_t \tag{3}$$

and

$$\theta_t \left(a_{t+1} + \chi_t\right) = 0, \tag{4}$$

the budget constraint and the debt constraint, where θ_t is the multiplier on the debt constraint. In the economy with housing, the solution to the family problem evaluated at the equilibrium value of $h_t = 1$ is summarized by the first-order conditions

$$qQ_t = Q_{t+1} + \theta_t, \tag{5}$$

$$\theta_t \left(a_{t+1} + \bar{\chi} p_t\right) = 0, \tag{6}$$

and

$$\beta \psi_{t+1} v'(1) + Q_{t+1} p_{t+1} + \theta_t \bar{\chi} p_t = Q_t p_t, \tag{7}$$

and the budget constraint (2). Note first that with $h_t = 1$, the budget constraints in the two economies coincide. Next, note that given the allocations and multipliers from the debt constraint economy, if the price of houses is equal to $p_t = \chi_t/\bar{\chi}$ and we set ψ_{t+1} according to

$$\psi_{t+1} = \frac{1}{\beta v'(1)}(Q_t p_t - Q_{t+1} p_{t+1} - \theta_t \bar{\chi} p_t),$$

then provided $\{c_t, a_{t+1}, \theta_t\}$ satisfy (3) and (4), these same variables along with p_t and ψ_{t+1} satisfy (5)–(7) and the collateral constraint. For the rest of the construction, since $\{c_t\}$ in the two economies coincide, so do the shadow prices $\{Q_t\}$. As we have noted, the labor market variables are uniquely pinned down by these shadow prices. Hence, consumption, output, and all labor market variables coincide in the two economies.

For the converse, we start with an economy with housing with taste parameters $\{\psi_t\}$ and construct a sequence of borrowing limits $\{\chi_t\}$ such that, except for the price and quantity of housing, the two equilibria coincide. Here we simply set $\chi_t = \bar{\chi} p_t$ and note that the budget constraints and first-order conditions for the two equilibria coincide. Thus, the shadow prices also coincide and, necessarily, so do consumption, output, and all labor market variables. We summarize this discussion with a proposition.

Proposition 1. *The debt constraint economy is equivalent to the economy with housing in terms of consumption, labor allocations, and wages.*

Note that in the economy with housing, we have time-varying taste parameters $\{\psi_t\}$ but a constant maximal loan-to-value ratio $\bar{\chi}$. It is immediate that a similar equivalence proposition holds if we consider a collateral constraint economy with a constant taste parameter $\psi_t = \psi$ and time-varying maximal loan-to-value ratios $\{\tilde{\chi}_t\}$.

A.1.2 An Economy with Illiquid Assets

Here we consider an economy with illiquid assets and liquid borrowing. Each family can save in assets that have a relatively high rate of return but are illiquid and can borrow at a relatively low rate. The budget constraint is

$$c_t + q_a a_{t+1} - q_b b_{t+1} = y_t + d_t + a_t - b_t - \phi(a_{t+1}, a_t), \tag{8}$$

where a_{t+1} denotes assets and b_{t+1} denotes debt. We assume that $q_a = 1/(1+r_a) < q_b = 1/(1+r_b)$, so the return on assets $1 + r_a$ is higher than the interest on debt. For simplicity, we assume that $\beta = q_a$, which can be interpreted as assuming that the family's patience is equal to that of the consumers in the rest of the world. We can think of the cost of borrowing as being $1 + r_b = 1 + r_a(1 - \tau)$, where τ is the income tax rate. Under this interpretation, the before-tax interest rate on saving equals the before-tax interest rate on borrowing, but the interest paid on borrowing is tax deductible. The function $\phi(a_{t+1}, a_t)$ represents the cost of adjusting assets from a_t to a_{t+1} and captures the idea that assets are illiquid. Borrowing is subject to the debt constraint

$$b_{t+1} \leq \tilde{\chi}_t, \tag{9}$$

where $\{\tilde{\chi}_t\}$ is a sequence of exogenous borrowing limits. The consumption problem of the family is to choose $\{c_t, a_{t+1}, b_{t+1}\}$ to solve

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

subject to the budget constraint (8) and the debt constraint (9). The first-order conditions for assets and debt imply

$$c_t + q_a a_{t+1} - q_b b_{t+1} = y_t + a_t - b_t - \phi(a_{t+1}, a_t) + d_t, \tag{10}$$

$$Q_t \left[q_a + \phi_1(a_{t+1}, a_t)\right] = Q_{t+1} \left[1 - \phi_2(a_{t+2}, a_{t+1})\right], \tag{11}$$

$$Q_t q_b = Q_{t+1} + \theta_t, \tag{12}$$

and

$$\theta_t(\tilde{\chi}_t - b_{t+1}) = 0, \tag{13}$$

where θ_t is the multiplier on the debt constraint.

Let $\{\hat{c}_t\}$ be the consumption allocations in the economy with debt constraints and $\{\hat{Q}_t\}$ be the associated shadow prices. We assume that the interest rate on borrowing is sufficiently low in the illiquid asset economy, so that the following inequality holds at the given allocations in the debt constraint economy:

$$q_b = \frac{1}{1 + r_b} > \frac{\hat{Q}_{t+1}}{\hat{Q}_t}. \tag{14}$$

We then have the following result.

Proposition 2. *Under (14), the debt constraint economy is equivalent to the economy with illiquid assets in terms of consumption, labor allocations, and wages.*

More precisely, given an allocation in the debt constraint economy that satisfies (14), there is a sequence of debt limits $\{\tilde{\chi}_t\}$ in the illiquid asset economy such that the consumption and marginal utilities in the illiquid asset economy coincide with the consumption and marginal utilities in the debt constraint economy. Hence, the labor market allocations and prices coincide as well.

The proof is constructive. We start with the allocations in the debt constraint economy, in particular, the consumption, wage income, and profit income denoted $\hat{c}_t$, $\hat{y}_t$, and $\hat{d}_t$, and use them to construct candidate allocations denoted by $\tilde{c}_t$, $\tilde{y}_t$, $\tilde{d}_t$, and so on for the illiquid asset economy. We next show that we can choose the debt limits $\{\tilde{\chi}_t\}$ in the illiquid asset economy so that the consumption and labor market outcomes in both economies coincide.

Formally, let $\tilde{c}_t = \hat{c}_t$. The first-order condition for assets (11), the associated shadow prices $\hat{Q}_t$ from the debt constraint economy, together with the transversality condition and an initial condition for assets a_0, imply a candidate path for assets $\{\tilde{a}_{t+1}\}$ for the illiquid asset economy. Given that the shadow prices in the two economies coincide, so do the wage income and net profits of firms, so that $\tilde{y}_t = \hat{y}_t$ and $\tilde{d}_t = \hat{d}_t$. To construct the debt limits $\{\tilde{\chi}_t\}$ in the illiquid asset economy, use that under (14) the debt constraints in the illiquid asset economy bind, so that $\tilde{b}_{t+1} = \chi_t$. Subtracting the budget constraints in the two economies gives

$$q_a\tilde{a}_{t+1} + \phi(\tilde{a}_{t+1}, \tilde{a}_t) + \tilde{\chi}_{t-1} - q_b\tilde{\chi}_t = \hat{\chi}_{t-1} - q\hat{\chi}_t \text{ for } t \geq 1 \tag{15}$$

and

$$q_a\tilde{a}_1 + \phi(\tilde{a}_1, \tilde{a}_0) + \tilde{b}_0 - \tilde{a}_0 - q_b\tilde{\chi}_0 = -\hat{a}_0 - q\hat{\chi}_0 \text{ for } t = 0. \tag{16}$$

Clearly, given some initial assets $\tilde{a}_0$ in the illiquid asset economy, the associated $\tilde{a}_1$ as well as the initial assets $\hat{a}_0$ and debt limit $\hat{\chi}_0$ from the debt constraint economy, we can choose initial debt $\tilde{b}_0$ and debt limit $\tilde{\chi}_0$ to satisfy (16). (Note there exists one degree of indeterminacy in $\tilde{b}_0$ and $\tilde{\chi}_0$.) Then for $t = 1$, we choose $\tilde{\chi}_1$ to satisfy (15) and proceed recursively, choosing $\tilde{\chi}_t$ to satisfy the budget constraint in each period t. Thus, the allocations in the two economies coincide. The proof of the converse is similar.

A.2 Computing the Benefit Flows from a Match

Here we describe how we decompose the surplus from a match, S_t, into period-by-period flows, s_t, that we then use to calculate the Macaulay duration.

The surplus from a match is equal to the present value of what a firm and a worker jointly earn,

$$E(z,h) = W(z,h) + J(z,h), \tag{17}$$

net of what the nonemployed earn, $U(z)$, where we focus on steady states and therefore drop the t subscript. The present value of what a firm and a worker jointly earn from forming a match is

$$E(z,h) = zh + \beta\phi\left[(1-\sigma)\int_{z'\geq\bar{z}(h')} E(z',h')\,dF_e(z'|z) + (1-\sigma)\int_{z'<\bar{z}(h')} U(z')\,dF_e(z'|z) + \sigma\int U(z')\,dF_e(z'|z)\right],$$

where $\bar{z}(h')$ is the exit cutoff. The present value of what the nonemployed earn is

$$U(z) = b(z) + \beta\phi\left\{\lambda^w(z)\int_{z'\geq\bar{z}(1)} W(z',1)\,dF_u(z'|z) + \lambda^w(z)\int_{z'<\bar{z}(1)} U(z')\,dF_u(z'|z) + [1-\lambda^w(z)]\int U(z')\,dF_u(z'|z)\right\}.$$

Notice that $U(z)$ depends on $W(z,h)$ and that Nash bargaining implies that $W(z,h)$ is a weighted average of $E(z,h)$ and $U(z)$,

$$W(z,h) = \gamma E(z,h) + (1-\gamma)U(z).$$

We can thus write the value of an unmatched consumer as

$$U(z) = b(z) + \beta\phi \left[\gamma\lambda^w(z) \int_{z' \geq \bar{z}(1)} E(z', 1)\, dF_u(z'|z) + (1-\gamma)\lambda^w(z) \int_{z' \geq \bar{z}(1)} U(z')\, dF_u(z'|z) \right]$$
$$+ \beta\phi \left\{ \lambda^w(z) \int_{z' < \bar{z}(1)} U(z')\, dF_u(z'|z) + [1 - \lambda^w(z)] \int U(z')\, dF_u(z'|z) \right\}.$$

We use these expressions to decompose the total surplus from a match into flows that accrue at different dates. Consider a consumer with general human capital z who is matched with a firm and thus starts producing with human capital $(z, 1)$ and another consumer who is not matched initially. As these expressions show, the surplus from the match is given by the expected value of what the employed consumer produces starting from today, relative to what the nonemployed consumer will produce, taking into account that each of these consumers may switch from employment to nonemployment and viceversa, may die, and experience changes in human capital according to transition probabilities that depend on the consumer's employment status. Importantly, the effective probability that a nonemployed consumer transits into employment is given by $\gamma\lambda^w$, not γ, which reflects the surplus sharing rule embedded in the Nash bargaining protocol.

We can think intuitively about what we do by considering simulating two large groups of consumers, all with some identical general human capital z_0, one group starting employed with human capital $(z_0, 1)$ and another group starting nonemployed with general human capital z_0. Then, we can simulate over time how these consumers' human capital evolves according to their respective laws of motion and how consumers switch employment status—with probability σ for employed and $\gamma\lambda^w$ for the nonemployed. The flows we calculate are the difference, at each date, of the total output produced by those in the first group and that produced by those in the second group.

A.3 Derivations for Proposition 3

We start with a model with only general human capital.

A.3.1 Setup

Let general human capital evolve according to the following law of motion: $z_{t+1} = (1+g)z_t$, when a consumer is employed, and $z_{t+1} = z_t$, when a consumer is nonemployed. We assume that home production and the cost of posting a vacancy are proportional to human capital in that $b(z) = bz$ and $\kappa(z) = \kappa z$. Recall that surplus is $S(z) = W(z) - U(z) + J(z)$.

From the free-entry condition and the formula for surplus, it is immediate that $\lambda_w(z) = \lambda_w$ and so it does not depend on z. Similarly, all value functions are proportional to z, so $S(z) = Sz$, $W(z) = Wz$, $U(z) = Uz$, and $J(z) = Jz$. Let $\beta = \tilde{\beta}\phi$, where $\tilde{\beta}$ is the primitive discount factor. Then, the value functions for an employed consumer, a nonemployed consumer, and a firm are given by

$$W(z) = w(z) + \beta \left[(1-\sigma) W((1+g)z) + \sigma U((1+g)z) \right],$$
$$U(z) = b(z) + \beta \left[\lambda_w W(z) + (1-\lambda_w) U(z) \right],$$
$$J(z) = z - w(z) + \beta(1-\sigma) J((1+g)z).$$

Substitute $W(z) = Wz$, $U(z) = Uz$, $w(z) = wz$, and $J(z) = Jz$, and then solve for S in $S(z) = Sz$ to obtain

$$W = w + \beta\left[(1-\sigma)(1+g)W + \sigma(1+g)U\right], \tag{18}$$
$$U = b + \beta\left[\lambda_w W + (1-\lambda_w)U\right], \tag{19}$$
$$J = 1 - w + \beta(1-\sigma)(1+g)J. \tag{20}$$

A.3.2 Surplus

To calculate match surplus, we use $S = W - U + J$ and equations (18)–(20) to obtain

$$S = 1 - b + \beta[(1+g)(1-\sigma) - \lambda_w\gamma]S + \beta g U. \tag{21}$$

We can also write the value of a nonemployed consumer as

$$U = b + \beta\gamma\lambda_w S + \beta U. \tag{22}$$

Using (22), we obtain

$$U = \frac{b + \beta\gamma\lambda_w S}{1-\beta}. \tag{23}$$

Substitute for U from (23) into (21) to get

$$S = 1 - b + \beta[(1+g)(1-\sigma) - \lambda_w\gamma]S + \frac{\beta(b + \beta\gamma\lambda_w S)}{1-\beta}g.$$

Thus, match surplus in the general human capital model is given by

$$S = \frac{1 - b + \frac{\beta g b}{1-\beta}}{1 - \beta\left[(1+g)(1-\sigma) - \lambda_w\gamma + \frac{\beta\gamma\lambda_w g}{1-\beta}\right]}. \tag{24}$$

Note that if the growth rate of general human capital is too high, then the present value of the surplus is infinity. Specifically, the denominator of the ratio in (24) is positive if, and only if,

$$g < \frac{\frac{1}{\beta} - (1-\sigma) + \lambda_w\gamma}{1 - \sigma + \frac{\beta\gamma\lambda_w}{1-\beta}}, \tag{25}$$

which can be thought of a joint restriction on g and the rest of the parameters.

A.3.3 Decompose Surplus Into Flows

We wish to decompose match surplus S into flows that accrue in each period, that is, to express it as $S = \sum_{k=0}^{\infty}\beta^k s_{k+1}$, starting from

$$S = 1 - b + \beta[(1+g)(1-\sigma) - \lambda_w\gamma]S + \beta U g$$

and $U = b + \beta(U + \gamma\lambda_w S)$. There are two natural ways to carry out this unique decomposition. The first way is to simply stack up the equations for the present value of surplus, S_t, and the present value of a nonemployed consumer, U_t, in recursive form and solve for the resulting surplus flows, s_k. The second way is to develop a recursive representation of the flows directly and solve the resulting difference equations in flows. These two methods are equivalent. The first way is a bit quicker but the second way is a bit more intuitive.

Method 1: Directly Solve Vector Difference Equation for Values. Here we simply write the value of match surplus and of a nonemployed consumer as a vector system, and solve it. To this purpose, we write (21) and (19) as

$$\begin{vmatrix} S_t \\ U_t \end{vmatrix} = \begin{vmatrix} 1-b \\ b \end{vmatrix} + \beta \begin{vmatrix} \alpha & g \\ \gamma\lambda_w & 1 \end{vmatrix} \begin{vmatrix} S_{t+1} \\ U_{t+1} \end{vmatrix}, \tag{26}$$

where $\alpha = (1+g)(1-\sigma) - \gamma\lambda_w$. Letting $X_t = (S_t, U_t)'$ and $x = (1-b, b)'$, we have

$$X_t = x + \beta A X_{t+1} = x + \beta(Ax) + \beta^2(A^2x) + \beta^3(A^3x) + \beta^4(A^4x) + \dots. \tag{27}$$

The solution is then of the form

$$\begin{vmatrix} S_t \\ U_t \end{vmatrix} = \begin{vmatrix} \sum_{k=0}^{\infty} \beta^k s_{t+k} \\ \sum_{k=0}^{\infty} \beta^k u_{t+k} \end{vmatrix}, \tag{28}$$

where s_{t+k} is the (expected) surplus flow in the k-th period of a match that starts in period t and u_{t+k} is the (expected) payoff flow to a consumer k periods after having *started* nonemployed in period t. It is critical to understand that the flows $\{u_{t+k}\}$ are the expected flows of output accruing in period $t+k$ to a consumer who was nonemployed in t but may be either employed or not at any $t+k$. That is, the expected flow at $t+k$ is the sum of the probabilities of the different paths of possible realizations of employment and nonemployment between t and $t+k$ multiplied by the output conditional on that path. (In particular, u_{t+k} is not just the output produced when the consumer is currently nonemployed at $t+k$.) Comparing (27) and (28), we see that the surplus flow in the k-th period of a match is the first row of

$$A^k x = A^k \begin{vmatrix} 1-b \\ b \end{vmatrix}.$$

Now let δ_s and δ_ℓ denote the eigenvalues of

$$A = \begin{vmatrix} \alpha & g \\ \gamma\lambda_w & 1 \end{vmatrix},$$

which are given by

$$\delta_s = \frac{1+\alpha}{2} - \left[\frac{(1+\alpha)^2}{4} - (\alpha - \gamma\lambda_w g)\right]^{1/2} \text{ and } \delta_\ell = \frac{1+\alpha}{2} + \left[\frac{(1+\alpha)^2}{4} - (\alpha - \gamma\lambda_w g)\right]^{1/2}. \tag{29}$$

Then, write the eigendecomposition of A as VDV^{-1}, where V are eigenvectors and D is a diagonal matrix with eigenvalues on the diagonal of the form

$$D = \begin{vmatrix} \delta_s & 0 \\ 0 & \delta_\ell \end{vmatrix}.$$

Hence, $A^k = VD^kV$. Since s_k is the first row of $A^k x = VD^kVx$, (26) and (27) imply that s_k has the form $s_k = c_s\delta_s^k + c_\ell\delta_\ell^k$.

One way to determine the constants is to compute the first row of VD^kV by hand. We follow a simpler approach, which delivers the same answer, namely, we use two initial conditions for s_0 and s_1. To do so, we note that from the form of (27) and (28) it follows that

$$\begin{vmatrix} s_0 \\ u_0 \end{vmatrix} = \begin{vmatrix} 1-b \\ b \end{vmatrix} \text{ and } \begin{vmatrix} s_1 \\ u_1 \end{vmatrix} = \begin{vmatrix} \alpha & g \\ \gamma\lambda_w & 1 \end{vmatrix} \begin{vmatrix} 1-b \\ b \end{vmatrix},$$

so that

$$s_0 = 1 - b \text{ and } s_1 = \alpha(1-b) + gb. \tag{30}$$

Then, combining these formulas for s_0 and s_1 with the form of $s_k = c_s \delta_s^k + c_\ell \delta_\ell^k$ at $k = 0, 1$ gives two equations in two unknowns, c_s and c_ℓ, namely

$$\begin{cases} c_s + c_\ell = 1 - b \\ c_s \delta_s + c_\ell \delta_\ell = \alpha(1-b) + gb \end{cases}.$$

Solving these two equations gives the constants in Proposition 3.

Method 2: Develop Difference Equations in Flows. We wish to decompose match surplus S into flows that accrue in each period, that is, to express it as

$$S = \sum_{t=0}^{\infty} \beta^t s_t. \tag{31}$$

We can do so using equations (21) and (22). As an intermediate step, we also need to decompose the value of a nonemployed consumer, U, into flows accruing at each date, so write

$$U = \sum_{t=0}^{\infty} \beta^t u_t. \tag{32}$$

To find an expression for s_t, simply plug in equations (31) and (32) into (21) and (22). We obtain

$$\begin{aligned} s_0 + \beta s_1 + \beta^2 s_2 + \ldots = 1 - b + \beta\left[(1-\sigma)(1+g) - \gamma\lambda_w\right]\left(s_0 + \beta s_1 + \beta^2 s_2 + \ldots\right) \\ + \beta g \left(u_0 + \beta u_1 + \beta^2 u_2 + \ldots\right) \end{aligned}$$

and

$$u_0 + \beta u_1 + \beta^2 u_2 + \ldots = b + \beta\gamma\lambda_w\left(s_0 + \beta s_1 + \beta^2 s_2 + \ldots\right) + \beta\left(u_0 + \beta u_1 + \beta^2 u_2 + \ldots\right).$$

Matching up the terms for each β^t gives $s_0 = 1 - b$, $u_0 = b$, and for $t \geq 1$

$$s_t = [(1-\sigma)(1+g) - \gamma\lambda_w] s_{t-1} + g u_{t-1} \tag{33}$$

and

$$u_t = \gamma\lambda_w s_{t-1} + u_{t-1}. \tag{34}$$

This gives a system of first-order difference equations. To solve this system, first-difference (33) and rewrite (34) as $u_t - u_{t-1} = \gamma\lambda_w s_{t-1}$ to obtain

$$s_t - s_{t-1} = \left[(1-\sigma)(1+g) - \gamma\lambda_w\right](s_{t-1} - s_{t-2}) + g\gamma\lambda_w s_{t-2}$$

or

$$s_t - [1 + (1-\sigma)(1+g) - \gamma\lambda_w] s_{t-1} + (1 - \sigma - \gamma\lambda_w)(1+g)\, s_{t-2} = 0.$$

This is a second-order difference equation with solution

$$s_t = c_s \delta_s^t + c_\ell \delta_\ell^t,$$

where the roots are given by

$$\delta_s = \frac{1+\alpha}{2} - \left[\frac{(1+\alpha)^2}{4} - (\alpha - \gamma\lambda_w g)\right]^{1/2} \text{ and } \delta_\ell = \frac{1+\alpha}{2} + \left[\frac{(1+\alpha)^2}{4} - (\alpha - \gamma\lambda_w g)\right]^{1/2}, \quad (35)$$

with $\alpha = (1+g)(1-\sigma) - \gamma\lambda_w$. To find the coefficients, we use $s_0 = c_s + c_\ell = 1 - b$ and

$$s_1 = c_s\delta_s + c_\ell\delta_\ell = \alpha(1-b) + gb,$$

which give the expressions in Proposition 3 in the paper.

Durations. Given the solution $s_t = c_s\delta_s^t + c_\ell\delta_\ell^t$ derived from either method, the rest of the proposition follows immediately. Equation (40) in Proposition 3 follows by using

$$\sum_{t=0}^{\infty}\beta^t s_t = \sum_{t=0}^{\infty}\beta^t(c_s\delta_s^t + c_\ell\delta_\ell^t) = \frac{c_s}{1-\beta\delta_s} + \frac{c_\ell}{1-\beta\delta_\ell}.$$

To establish equation (38) in Proposition 3, note that the Macaulay duration can be computed as

$$\sum_{t=0}^{\infty} t\frac{\beta^t s_t}{S} = \frac{1}{S}\sum_{t=0}^{\infty} t\beta^t(c_s\delta_s^t + c_\ell\delta_\ell^t) = \frac{1}{S}\left[\frac{c_s\beta\delta_s}{(1-\beta\delta_s)^2} + \frac{c_\ell\beta\delta_\ell}{(1-\beta\delta_\ell)^2}\right]. \quad (36)$$

To establish equation (39) in Proposition 3, note that the Modified Macaulay duration can be written as

$$\begin{aligned}\sum_{t=0}^{\infty}\frac{1-\varrho^t}{1-\varrho}\frac{\beta^t s_t}{S} &= \frac{1}{1-\varrho}\sum_{t=0}^{\infty}\frac{\beta^t s_t}{S} - \frac{1}{1-\varrho}\sum_{t=0}^{\infty}\varrho^t\frac{\beta^t(c_s\delta_s^t + c_\ell\delta_\ell^t)}{S}\\ &= \frac{1}{1-\varrho} - \frac{1}{S}\frac{1}{1-\varrho}\left[\frac{c_s}{1-\beta\varrho\delta_s} + \frac{c_\ell}{1-\beta\varrho\delta_\ell}\right],\end{aligned}$$

since $\sum_{t=0}^{\infty}\beta^t s_t/S = 1$.

A.3.4 An Extension of Proposition 3

We consider a generalization of the setup in the paper when workers only acquire general human capital, in which we now allow for differential accumulation rates for consumers when employed and not employed.

Setup. Assume now

$$z_{t+1} = \begin{cases} (1+g)z_t \text{ if employed} \\ (1+d)z_t \text{ if nonemployed} \end{cases}.$$

Now, if human capital decays during nonemployment, then $d < 0$. The analogues of (18)–(20) are

$$\begin{aligned}&W = w + \beta\left[(1-\sigma)(1+g)W + \sigma(1+g)U\right],\\ &U = b + \beta(1+d)\left[\lambda_w W + (1-\lambda_w)U\right],\\ &J = 1 - w + \beta(1-\sigma)(1+g)J,\end{aligned}$$

so that $U = b + \beta(1+d)(U + \gamma\lambda_w S)$.

Surplus. Calculate the surplus as

$$S = 1 - b + \beta[(1+g)(1-\sigma) - (1+d)\lambda_w\gamma]S + \beta U(g-d). \tag{37}$$

From $U[1-\beta(1+d)] = b + \beta(1+d)\lambda_w\gamma S$, we obtain

$$U = \frac{b + \beta\gamma\lambda_w(1+d)S}{1-\beta(1+d)}. \tag{38}$$

Substitute for U from (38) into (37) to get

$$S = 1 - b + \beta[(1+g)(1-\sigma) - (1+d)\lambda_w\gamma]S + \beta\frac{b + \beta\gamma\lambda_w(1+d)S}{1-\beta(1+d)}(g-d). \tag{39}$$

Hence, match surplus in the general human capital model is

$$S = \frac{1 - b + \frac{\beta(g-d)b}{1-\beta(1+d)}}{1 - \beta\left[(1+g)(1-\sigma) - (1+d)\lambda_w\gamma + \frac{\beta\gamma\lambda_w(1+d)(g-d)}{1-\beta(1+d)}\right]}. \tag{40}$$

Of course, to ensure that we can solve (39) to give (40), we require that the growth rates g and d are such that the present value in (39) converge.

Decompose Surplus Into Flows. Following the same steps as for the surplus flow decomposition in the general human capital model, we can show that $s_k = c_s\delta_s^k + c_\ell\delta_\ell^k$, where

$$\delta_s = \frac{1+d+\alpha}{2} - \left\{\frac{(1+d+\alpha)^2}{4} - (1+d)\left[\alpha - \gamma\lambda_w(g-d)\right]\right\}^{1/2},$$

$$\delta_\ell = \frac{1+d+\alpha}{2} + \left\{\frac{(1+d+\alpha)^2}{4} - (1+d)\left[\alpha - \gamma\lambda_w(g-d)\right]\right\}^{1/2},$$

$$c_\ell = \frac{\alpha - \delta_s + b\left[\sigma(1+g) - (1-\gamma\lambda_w)(1+d) + \delta_s\right]}{\delta_\ell - \delta_s},$$

and

$$c_s = 1 - b - c_\ell.$$

A.4 Derivations for Proposition 4

We next study the model with firm-specific human capital.

A.4.1 Setup

The value functions for an employed consumer, a nonemployed consumer, and a firm are given by

$$\begin{aligned} &W(h) = \omega(h) + (1-\sigma)\beta W((1+g)h) + \sigma\beta U, \\ &U = b + \lambda_w\beta W(1) + (1-\lambda_w)\beta U, \\ &J(h) = h - \omega(h) + (1-\sigma)\beta J((1+g)h). \end{aligned}$$

Using the Nash Bargaining surplus sharing rule, the Bellman equation for surplus is

$$S(h) = h - b + \beta(1-\sigma)S((1+g)h) - \beta\gamma\lambda_w S(1). \tag{41}$$

A.4.2 Surplus

We solve for $S(h)$ using the method of undetermined coefficients. Specifically, we guess that $S(h)$ is linear in h with $S\left(h\right)=a_{0}+a_{1}\left(h-1\right)$ so that $S\left(1\right)=a_{0}$. Then, we write (41) as

$$a_{0}+a_{1}\left(h-1\right)=h-b+\left(1-\sigma\right)\beta a_{0}+\left(1-\sigma\right)\beta a_{1}\left(\left(1+g\right)h-1\right)-\gamma\lambda_{w}\beta a_{0}, \tag{42}$$

which holds for all h. Evaluating (42) at $h=0$ gives

$$a_{0}-a_{1}=-b+\left(1-\sigma\right)\beta a_{0}-\left(1-\sigma\right)\beta a_{1}-\gamma\lambda_{w}\beta a_{0}, \tag{43}$$

whereas evaluating (42) at $h=1$ gives

$$a_{0}=1-b+\left(1-\sigma\right)\beta a_{0}+\left(1-\sigma\right)\beta a_{1}\left(1+g-1\right)-\gamma\lambda_{w}\beta a_{0}. \tag{44}$$

Subtracting (43) from (44) gives $a_{1}=1+\left(1-\sigma\right)\beta a_{1}\left(1+g\right)$ or

$$a_{1}=\frac{1}{1-\beta\left(1-\sigma\right)\left(1+g\right)}.$$

By (43), a_0 satisfies

$$a_{0}=\frac{a_{1}[1-\beta\left(1-\sigma\right)]-b}{1-\beta\left(1-\sigma\right)+\beta\gamma\lambda_{w}}.$$

We thus have

$$S\left(1\right)=\frac{\frac{1-\beta(1-\sigma)}{1-\beta(1-\sigma)(1+g)}-b}{1-\beta\left(1-\sigma\right)+\beta\gamma\lambda_{w}}. \tag{45}$$

A.4.3 Decompose Surplus into Flows

Since the free-entry condition is $\kappa=\gamma\lambda_{f}\beta S\left(1\right)$, the response of λ_f to a change in discount rates depend on how $S\left(1\right)$ responds to a change in discount rates. So let us decompose the surplus when the match is formed, $S(1)$, into flows accruing at each date.

To keep the algebra cleaner, let us use the date since the match was formed, t, as the argument of the surplus function, instead of $h=\left(1+g\right)^{t}$. So $S_{t}=S\left(\left(1+g\right)^{t}\right)$ is the surplus received t periods after the match was formed. Thus, we have

$$S_{t}=\left(1+g\right)^{t}-b+\beta\left(1-\sigma\right)S_{t+1}-\beta\gamma\lambda_{w}S_{0}. \tag{46}$$

We would like to decompose $S\left(1\right)=S_{0}$ into flows, so write

$$S_{0}=\sum_{t=0}^{\infty}\beta^{t}s_{t}.$$

Method 1: Directly Solve Vector Difference Equation for Values. Here we follow a shorter version of what we did for the general human capital case. Inspection of the vector difference equations for the firm specific human capital case, makes it clear that a solution for the surplus flows has the same form as in the general human capital case, namely

$$s_{t}=c_{s}\delta_{s}^{t}+c_{\ell}\delta_{\ell}^{t}. \tag{47}$$

We need only solve for the constants, c_s and c_ℓ, and the roots, δ_s and δ_ℓ. We do so by developing four equations in these four unknowns and by directly solving them. To do so, we directly calculate from recursive substitution into the stock equations. The flows are defined by

$$S_0 = s_0 + \beta s_1 + \beta^2 s_2 + \dots, \tag{48}$$

that is, the flow s_t corresponds to the term multiplied by β^t. To find s_0, use the recursion

$$S_0 = 1 - b + \beta\left[(1-\sigma)S_1 - \gamma\lambda_w S_0\right], \tag{49}$$

and note that the term s_0 is the constant

$$s_0 = 1 - b, \tag{50}$$

because all the other terms are multiplied by at least β. To calculate s_1, we substitute to find all the terms multiplied by β. To this purpose, we use

$$S_1 = 1 + g - b + \beta\left[(1-\sigma)S_2 - \gamma\lambda_w S_0\right] \tag{51}$$

to substitute for S_1 in the right side of (49). We also substitute for the term S_0 on the right side of (49) using (49) itself. By doing so and regrouping terms, we can then rewrite (49) as

$$\begin{aligned} S_0 = {} & 1 - b + \beta\left[(1-\sigma)(1+g-b) - \gamma\lambda_w(1-b)\right] \\ & + \beta^2\left\{(1-\sigma)\left[(1-\sigma)S_2 - \gamma\lambda_w S_0\right] - \gamma\lambda_w\left[(1-\sigma)S_1 - \gamma\lambda_w S_0\right]\right\}. \end{aligned} \tag{52}$$

Matching up the terms in β between (48) and (52) gives

$$s_1 = (1-\sigma)(1+g-b) - \gamma\lambda_w(1-b). \tag{53}$$

To calculate s_2, we substitute into the right side of (52) for S_0, S_1, and S_2 using (49), (51), and

$$S_2 = (1+g)^2 - b + \beta\left[(1-\sigma)S_3 - \gamma\lambda_w S_0\right]. \tag{54}$$

Now consider the β^2 term in (52), that is, the term

$$\beta^2\{(1-\sigma)\left[(1-\sigma)S_2 - \gamma\lambda_w S_0\right] - \gamma\lambda_w\left[(1-\sigma)S_1 - \gamma\lambda_w S_0\right]\},$$

which can be rewritten as

$$\beta^2\left[(1-\sigma)^2 S_2 - \gamma\lambda_w(1-\sigma)S_1 + (1-\sigma-\gamma\lambda_w)\gamma\lambda_w S_0\right]. \tag{55}$$

Each of the values S_0, S_1, and S_2 are of the form of a constant term, respectively, $1-b$, $1+g-b$, and $(1+g)^2-b$, plus β times the continuation value. Now substitute only the constant terms from S_2, S_1, and S_0 into (55) to get

$$\beta^2\left\{(1-\sigma)^2\left[(1+g)^2 - b\right] - \gamma\lambda_w(1-\sigma)\left[(1+g) - b\right] + (1-\sigma-\gamma\lambda_w)\gamma\lambda_w(1-b)\right\}, \tag{56}$$

and note that all other omitted terms are multiplied by a higher power of β than β^2. Clearly, comparing the β^2 terms in (48) and (56) gives

$$s_2 = (1-\sigma)^2\left[(1+g)^2 - b\right] - \gamma\lambda_w(1-\sigma)\left[(1+g) - b\right] + (1-\sigma-\gamma\lambda_w)\gamma\lambda_w(1-b). \tag{57}$$

Now, we can use the form of (47) to get four equations in four unknowns, namely,

$$\begin{cases} c_s + c_\ell = s_0 \\ c_s\delta_s + c_\ell\delta_\ell = s_1 \\ c_s\delta_s^2 + c_\ell\delta_\ell^2 = s_2 \\ \dfrac{c_s}{1-\beta\delta_s} + \dfrac{c_\ell}{1-\beta\delta_\ell} = S(1) \end{cases},$$

where s_0, s_1, s_2, and $S(1)$ are given by (50),(53),57), and (45). It is immediate to verify that the constants c_s and c_ℓ and the roots δ_s and δ_ℓ given in Proposition 4 satisfy these equations. (We also checked $c_s\delta_s^3 + c_\ell\delta_\ell^3 = s_3$ holds.) The formulas for the Macaulay duration and the modified Macaulay duration follow immediately from $s_t = c_s\delta_s^t + c_\ell\delta_\ell^t$ and are of the form derived for the durations in Proposition 3. This proves Proposition 4 in the main paper.

Method 2: Develop Difference Equations in Flows. Here we discuss a more intuitive method by developing a difference equation directly in the flows. We cannot do this directly by matching terms as in the general human capital model, because the Bellman equation for S_0 involves $S_1 \neq S_0$ in that

$$S_0 = 1 - b + \beta\left(1-\sigma\right)S_1 - \beta\gamma\lambda_w S_0. \tag{58}$$

Thus, we need to relate S_1 to S_0. It turns out to be easier to work with $S_1 - S_0$, so rewrite (58) as

$$S_0 = 1 - b + \beta\left(1-\sigma\right)\left(S_1 - S_0\right) + \beta\left(1-\sigma-\gamma\lambda_w\right)S_0. \tag{59}$$

We wish to figure out how the change in the surplus function, $S_1 - S_0$, evolves over time. To do so, first-difference (46) to obtain

$$S_t - S_{t-1} = \left(1+g\right)^{t-1} g + \beta\left(1-\sigma\right)\left(S_{t+1} - S_t\right). \tag{60}$$

Next, divide both sides of (60) by $\left(1+g\right)^t$ and define

$$X_t = \frac{S_t - S_{t-1}}{\left(1+g\right)^t}.$$

We can then rewrite (60) as

$$X_t = \frac{g}{1+g} + \beta\left(1-\sigma\right)\left(1+g\right)X_{t+1}.$$

Clearly, X_t does not depend on t so we have $X_{t+1} = X$ and

$$X = \frac{g}{1+g} + \beta\left(1-\sigma\right)\left(1+g\right)X. \tag{61}$$

We can now write the Bellman equation for date-0 surplus as

$$S_0 = \left(1-b\right) + \beta\left(1-\sigma\right)\left(1+g\right)X + \beta\left(1-\sigma-\gamma\lambda_w\right)S_0. \tag{62}$$

The system of equations (61) and (62) is now in a form that does not depend on t (or h) so we can apply the same logic as in the general human capital model. Let

$$X = \sum_{t=0}^{\infty} \beta^t x_t,$$

and plug this into (61) to obtain

$$x_0 + \beta x_1 + \beta^2 x_2 + \beta^3 x_3 + \ldots = \frac{g}{1+g} + \beta\left(1-\sigma\right)\left(1+g\right)\left(x_0 + \beta x_1 + \beta^2 x_2 + \beta^3 x_3 + \ldots\right).$$

Matching up the terms raised to the some power of β gives $x_0 = g/(1+g)$,

$$x_1 = \left(1-\sigma\right)\left(1+g\right)x_0 = \left(1-\sigma\right)\left(1+g\right)\frac{g}{1+g},$$

and

$$x_t = \left(1-\sigma\right)\left(1+g\right)x_{t-1} = \left(1-\sigma\right)^t\left(1+g\right)^t\frac{g}{1+g}.$$

Similarly, equation (62) reads

$$\begin{aligned}&s_0 + \beta s_1 + \beta^2 s_2 + \beta^3 s_3 + \ldots = 1 - b + \beta\left(1-\sigma\right)\left(1+g\right)\left(x_0 + \beta x_1 + \beta^2 x_2 + \beta^3 x_3 + \ldots\right)\\ &\quad + \beta\left(1-\sigma-\gamma\lambda_w\right)\left(s_0 + \beta s_1 + \beta^2 s_2 + \beta^3 s_3 + \ldots\right),\end{aligned}$$

or, using the above expressions for x_t,

$$\begin{aligned}&s_0 + \beta s_1 + \beta^2 s_2 + \beta^3 s_3 + \ldots\\ &= 1 - b + \beta\left(1-\sigma\right)\left(1+g\right)\left[\frac{g}{1+g} + \beta\left(1-\sigma\right)\left(1+g\right)\frac{g}{1+g} + \beta^2\left(1-\sigma\right)^2\left(1+g\right)^2\frac{g}{1+g} + \ldots\right]\\ &\quad + \beta\left(1-\sigma-\gamma\lambda_w\right)\left(s_0 + \beta s_1 + \beta^2 s_2 + \beta^3 s_3 + \ldots\right).\end{aligned}$$

We next match up the terms of various β powers to obtain $s_0 = 1 - b$,

$$s_1 = \left(1-\sigma\right)\left(1+g\right)\frac{g}{1+g} + \left(1-\sigma-\gamma\lambda_w\right)s_0,$$

and

$$s_2 = \left(1-\sigma\right)^2\left(1+g\right)^2\frac{g}{1+g} + \left(1-\sigma-\gamma\lambda_w\right)s_1.$$

More generally,

$$s_{t-1} = \left(1-\sigma\right)^{t-1}\left(1+g\right)^{t-1}\frac{g}{1+g} - \left(1-\sigma-\gamma\lambda_w\right)s_{t-2}$$

and

$$s_t = \left(1-\sigma\right)^t\left(1+g\right)^t\frac{g}{1+g} + \left(1-\sigma-\gamma\lambda_w\right)s_{t-1}.$$

We can express this more compactly as

$$s_t - A\left(1-\sigma\right)^t\left(1+g\right)^t = \left(1-\sigma-\gamma\lambda_w\right)\left[s_{t-1} - A\left(1-\sigma\right)^{t-1}\left(1+g\right)^{t-1}\right],$$

where A is a coefficient we need to determine, which satisfies

$$A\left(1-\sigma\right)^t\left(1+g\right)^t - \left(1-\sigma-\gamma\lambda_w\right)A\left(1-\sigma\right)^{t-1}\left(1+g\right)^{t-1} = \left(1-\sigma\right)^t\left(1+g\right)^t\frac{g}{1+g}.$$

Write the left side of this last expression as

$$A\left(1-\sigma\right)^t\left(1+g\right)^t\left[1 - \frac{1-\sigma-\gamma\lambda_w}{\left(1-\sigma\right)\left(1+g\right)}\right] = \left(1-\sigma\right)^t\left(1+g\right)^t\frac{g}{1+g},$$

so we have

$$A = \frac{(1-\sigma)(1+g)}{(1-\sigma)(1+g) - (1-\sigma-\gamma\lambda_w)} \frac{g}{1+g} = \frac{(1-\sigma) g}{(1-\sigma) g + \gamma\lambda_w}.$$

Let $\hat{s}_t = [s_t - A(1-\sigma)^t (1+g)^t]$. We then have $\hat{s}_t = (1-\sigma-\gamma\lambda_w)\hat{s}_{t-1}$ or

$$\hat{s}_t = (1-\sigma-\gamma\lambda_w)^t \hat{s}_0.$$

Since $\hat{s}_0 = s_0 - A = 1 - b - \frac{(1-\sigma)g}{(1-\sigma)g+\gamma\lambda_w}$, we obtain

$$s_t = \frac{(1-\sigma) g}{(1-\sigma) g + \gamma\lambda_w} (1-\sigma)^t (1+g)^t + (1-\sigma-\gamma\lambda_w)^t \left[1 - b - \frac{(1-\sigma) g}{(1-\sigma) g + \gamma\lambda_w}\right].$$

This completes the argument.

B Data

B.1 State-Level Data

We use state-level information on consumption, house prices, employment, wages, total population, and working-age population between 2007 and 2009 on all continental U.S. states, that is, all states of the United States except for Alaska and Hawaii.

Consumption. We measure consumption of nondurable goods (food and beverages purchased for off-premises consumption, clothing and footwear, gasoline and other energy goods, and other nondurable goods) and services (transportation services, recreation services, food services and accommodations, financial services and insurance, and other services) using state-level information on personal consumption expenditures from the BEA Regional Economic Accounts Data. Specifically, we use data from the BEA archive "Personal Consumption Expenditures (PCE) by State" in the file `PCEbyState.xlsx`, available at `http://www.bea.gov/regional/downloadzip.cfm`. The BEA provides annual information on state-level consumption for the 50 states of the United States and the District of Columbia from 1997 to 2014 in millions of U.S. dollars of the corresponding year. We use data for the period 1997-2012, but similar results can be obtained by using the more recent data. We obtain a measure of consumption per capita using census information on the size of the state population, described below. We deflate this measure by the national CPI.

House Prices. We construct a measure of state-level house prices using the state-level Zillow Home Value Index (ZHVI) for "All Homes." (See Zillow's website for a description of the methodology used by Zillow to compute this index.) In particular, we use ZHVI in the version "All Homes (SFR, Condo/Co-op) Time Series ($)." This monthly house price index is available for 48 U.S. states between January 2006 and December 2012. However, three states—Kansas, Maine, and Texas—are missing throughout the period, and only 41 U.S. states are consistently sampled over this period. In particular, 7 states—New York, Iowa, West Virginia, Montana, North Dakota, Vermont, and Wyoming—are missing in 2006 at the onset of the recession. We obtain an annual measure of house prices by averaging this monthly house price index for each of the 41 states that are consistently in the sample over the 12 months of each year of interest. We deflate this measure by the national CPI.

Employment. We use information on state-level total nonfarm employment, excluding government and construction, from the "Local Area Personal Income Accounts" archive of the BEA Regional Economic Accounts Data on the total number of individuals employed full-time and part-time by state and year, made available by the BEA as "CA25N: Total Full-Time and Part-Time Employment by NAICS Industry." We classify nontradable and tradable employment as follows. We consider tradable employment as employment in:

i) forestry, fishing, and related activities, namely forestry and logging (1131-1133), fishing, hunting, and trapping (114), and support activities for agriculture and forestry (115). We exclude from this category crop production (111) and animal production (112);

ii) mining (21);

iii) manufacturing (31-33).

We consider nontradable employment as employment in the remaining industries in total nonfarm employment excluding government and construction, namely:

i) utilities (22);

ii) wholesale trade (42);

iii) retail trade (44-45);

iv) transportation and warehousing (48-49);

v) information (51);

vi) finance and insurance (52);

vii) real estate and rental and leasing (53);

viii) professional, scientific, and technical services (54);

ix) management of companies and enterprises (55);

x) administrative and support and waste management and remediation services (56);

xi) educational services (61);

xii) health care and social assistance (62);

xiii) arts, entertainment, and recreation (71);

xiv) accommodation and food services (72);

xv) other services, except public administration (81).

Wages. We compute nominal hourly wages following Beraja, Hurst, and Ospina (2016) based on information on total labor income, hours, and weeks worked available from the census Integrated Public Use Microdata Series (IPUMS). Like Beraja, Hurst, and Ospina (2016), we focus on working-age males from non-group-quarter households who are employed full-time, work at least 30 hours per week and 48 weeks during the previous 12 months, and whose labor income is above \$5,000. We calculate hourly wages by dividing total labor income during the year—the sum of nominal wages, salary earnings, and business non-farm earnings during the prior 12 months—by hours worked. We obtain a measure of hours worked in a year by multiplying hours usually worked per week by the number of weeks worked in the year.

Following Beraja, Hurst, and Ospina (2016), we adjust this measure of nominal hourly wages for the differential demographical composition of different U.S. states. To derive such a measure of adjusted nominal hourly wages, we first run the following OLS regression in each year:

$$\log w_{itk} = \gamma_t + \Gamma_t X_{itk} + \eta_{itk},$$

where w_{itk} is the hourly wage for household/individual i in period t residing in state k; X_{itk} is a vector of controls for demographic variables, which include dummy variables for usual hours worked, age, education, citizenship, and race. We next compute adjusted nominal wages within a state as the average of $e^{\gamma_t+\eta_{itk}}$ across all individuals in state k. Following this procedure, we replicate the adjusted nominal hourly wages constructed by Beraja, Hurst, and Ospina (2016).[1] We deflate this measure by the national CPI to obtain a measure of adjusted real hourly wages.

Total and Working-Age Population. Estimates of total and working-age population are available at `http://www.census.gov/popest/data/intercensal/state/state2010.html` for each U.S. state. We use estimates of state-level total and working-age population from the "Intercensal Estimates of the Resident Population by Five-Year Age Groups, Sex, Race and Hispanic Origin for States and the United States: April 1, 2000 to July 1, 2010" from the data file `ST-EST00INT-ALLDATA.csv`. In measuring working-age population, we restrict attention to individuals between 15 and 64 years of age (included), who correspond to individuals between age group 4, that is, individuals aged 15 to 19 years, and age group 13, that is, individuals aged 60 to 64 years. (See the documentation contained in `ST-EST00INT-ALLDATA.pdf`, available from the census.)

B.2 Main Variables and Statistics

B.2.1 Measures of Annual Consumption Changes

One of our model's main predictions concerns the relationship between changes in state-level consumption and employment between 2007 and 2009. We isolate the component of consumption changes due to credit constraints by projecting consumption changes on house price changes following the approach of Beraja, Hurst, and Ospina (2016). We then analyze the economy's response to the associated changes in consumption.

Formally, we compute annual changes in consumption in each U.S. state relative to 2006 based on the growth rate of consumption between 2007 and 2009 predicted by changes in house prices in those years. To do so, we first regress log consumption growth on log house price growth between 2006 and 2007 to obtain the growth rate of predicted consumption between 2006 and 2007, g_{67}. We repeat this exercise for the periods between 2007 and 2008 and between 2008 and 2009 to obtain, respectively, the growth rate of predicted consumption between 2007 and 2008, g_{78}, and the growth rate of predicted consumption between 2008 and 2009, g_{89}. We then normalize consumption in 2006 to be equal to zero for all states and calculate annual consumption in each subsequent year for each state as

$$c_{2007,s} = c_{2006,s} + g_{67},\ c_{2008,s} = c_{2007,s} + g_{78},\ c_{2009,s} = c_{2008,s} + g_{89},$$

with 2006 as baseline year. We then linearly interpolate these data to construct a quarterly series of consumption for each state.

B.2.2 Cross-Sectional Wage Growth

In our main quantification exercise, we use moments of the cross-sectional distribution of earnings from U.S. census data for 1970, 1980, and 1990 for high school graduates (individuals with 12 years of education) following Elsby and Shapiro (2012). We also use an alternative measure of life-cycle wage growth for this group of workers based on the findings of Rubinstein and Weiss (2006).

[1] We thank these authors, in particular Erik Hurst, for generously helping us to replicate their construction.

Elsby and Shapiro (2012). We closely follow Elsby and Shapiro (2012), ES hereafter, and measure earnings as the annual wage and salary income of census respondents, and experience as potential experience, that is, an individual's age minus years of education minus six. We focus on full-time, full-year workers, defined as individuals who work 35 hours or more per week and are employed for 50 or more weeks per year. Like ES, we apply these selection rules so as to minimize the risk of sampling respondents who have not left full-time education, to make it more likely that observed profiles reflect variation in wages rather than in hours or weeks worked, and to minimize the risk of discrepancies between actual and potential experience.

ES calculate experience-earnings profiles in any given year using normalized log earnings, which are computed as the difference between the average log nominal wage of individuals in the given year with a given level of experience and the average log nominal wage of individuals in the same year with zero years of experience. We target the difference in average log wages between workers with 30 years of experience and workers just entering the labor market, averaged across the three years 1970, 1980, and 1990, which is equal to 1.21 or, equivalently, 4.1% per year of experience.

As we discuss in the paper, estimates from Buchinsky et al. (2010) from the PSID imply an average annual growth rate of wages over an employment spell of 7% for workers who have between 1 and 40 years of experience. To see that the different growth rates of wages implied by cross-sectional and longitudinal experience-earnings profiles are broadly consistent with each other, we also calculate experience-earnings profiles using (weighted) PSID data from 1975, 1981, and 1991 applying the same sample selection rules of ES. This choice of years reflects the fact that no individual with 12 years of education is observed with zero or one year of experience between 1970 and 1974, and that the PSID is a retrospective panel, so years 1981 and 1991, respectively, refer to labor income in 1980 and 1990. As Figure 1 reveals, the experience-earnings profiles computed based on PSID data are remarkably similar to those computed based on census data.

Rubinstein and Weiss (2006). In our robustness exercises in the paper, we show that our model's implications for the labor market responses to a credit tightening are robust to using alternative evidence from Rubinstein and Weiss (2006) on how wages grow over the life cycle. Figure 3a in Rubinstein and Weiss (2006), on which, as discussed in the paper, our alternative calibration is based, plots cohort and cross-sectional wage profiles for white male high school graduates over the period 1964-2002 from the March supplements of the CPS. The figure shows the experience-wage profiles for the cohort of high school graduates born between 1951 and 1955, who entered the labor market between 1971 and 1975. The graph also reports the evolution of cross-sectional experience-wage profiles from 1971 to 2000 in five-year intervals, where each such cross-sectional profile depicts the mean wages of workers with the indicated experience in a given time interval.

As Rubinstein and Weiss remark, it is apparent from the figure that cohort-based wage profiles are affected by changes in market conditions that shift the cross-sectional profiles over time. For instance, high school graduates of all experience levels earned lower wages during the period between 1970 and 2000, which is why the mean wage profile of the cohort of high school graduates born between 1951 and 1955 exhibits almost no wage growth after 10 years in the labor market.

B.2.3 On-the-Job Wage Growth

To pin down the parameters of the processes for general and firm-specific human capital, we target statistics on the longitudinal growth of individual wages implied by three distinct sets of estimates from well-known studies of returns to tenure and experience: Altonji and Shakotko (1987), Topel (1991), and Buchinsky et al. (2010). The relative size of these returns has long been debated in the labor economics literature. We take estimates from these three influential studies as representative of the spectrum of returns documented in the literature. Here we describe how the estimates of returns to tenure and experience from these papers informed our calibration.

As we explain in the paper, for a given parametrization of our model, we simulate paths for wages, experience, and tenure for a panel of individuals. Given the simulated experience and tenure profiles from our model, we compute the annualized wage growth implied by the estimated wage equations in Altonji and Shakotko (1987), Topel (1991), and Buchinsky et al. (2010). We then choose the parameters of the human capital processes in our model to ensure that our model's implied wage growth at various levels of experience is consistent with the wage growth implied by the wage equations in these three studies. We next provide details about the wage equations in these studies and the estimates from them that we used.

Altonji and Shakotko (1987). We use the estimates of the effects of education, experience, and tenure on log wages by Altonji and Shakotko (1987) from a PSID sample of white males observed between 1968 and 1981. These authors estimate the log wage equation given by

$$W_{ijt} = b_0 Z_{ijt} + b_1 T_{ijt} + b_2 T_{ijt}^2 + b_3 OLDJOB_{ijt} + \varepsilon_{ijt},$$

by instrumental variables (IV), where W_{ijt} is the log real wage of individual i in job j at time t, computed as the log of labor earnings during the year divided by the product of annual hours and the GNP implicit price deflator for consumption, $Z_{ijt} = (Edu, Edu^2, Time, X_{ijt}, X_{ijt}^2/10, X_{ijt}^3/100,\text{-} Edu \times X_{ijt})$ is a vector of characteristics of the individual such as education (Edu) and of the job including labor market experience (X_{ijt}), T_{ijt} and T_{ijt}^2 are the number and the square of the number of years individual i has held job j as of time t, and $OLDJOB_{ijt}$ is equal to one if $T_{ijt} > 1$ and to zero otherwise. This latter variable allows the impact of the first year of tenure on wages to be more flexible than implied by the quadratic specification of the impact of tenure on wages. The variable ε_{ijt} is the error term, which consists of the sum of a fixed individual effect, ε_i, a fixed job match effect, ε_{ij}, and a transitory component, η_{ijt}. The principal instrumental variables the authors use to address the likely correlation between the tenure variables and the error term are deviations of the tenure variables around their means for the sample observations on a given job spell.

The authors' main finding is that the effect of seniority on wages is small once the effects of general labor market experience and secular wage growth, captured by a time trend, are taken into account. Their preferred estimates imply that 10 years of tenure lead to a wage increase of 6.6%, with much of this growth occurring in the first year on the job. The estimates of Altonji and Shakotko (1987) of the effects of education, experience, and tenure on wages are reported in Table 1, which replicates column IV_1 of Table 1 in Altonji and Shakotko (1987) for the estimates of the effects of Edu and Edu^2 and the second row of the panel titled "'Corrected' IV_1 Estimates" of Table IV in Altonji and Shakotko (1987) for the remaining estimates.

Topel (1991). We use the estimates of the effects of tenure and experience on annual within-job wage growth obtained by Topel (1991) from a PSID sample of white males observed between 1968 and 1983. These estimates are the outcome of the first step of a two-step procedure followed by Topel to separately estimate returns to tenure and experience. In the first step, Topel estimates

Table 1: Estimates from Altonji and Shakotko (1987)

Edu	Edu^2	X	$X^2/10$	$X^3/100$	$Edu \times X$	T	T^2	$OLDJOB$
0.0155	0.0019	0.0339	-0.0111	0.00096	0.00045	0.0005	0.00012	0.0467

parameters of within-job wage growth, whereas in the second step, the author provides an estimate of the effects of experience and, residually, of tenure obtaining a lower bound on the average return to tenure. The main finding of the paper is that 10 years of tenure raise the wage of the typical male worker in the United States by over 25%.

Formally, let y_{ijt} denote the log wage of individual i in job j at time t, computed as the log average hourly earnings deflated by a wage index for white males calculated from the annual demographic (March) files of the CPS (see Murphy and Welch (1992)). Let X_{ijt} denote total labor market experience, T_{ijt} current job tenure, and ε_{ijt} a mean-zero error.[2] Topel (1991) estimates wage growth within a job as

$$\begin{aligned}\Delta Y_{ijt} =& \beta_{11}\Delta X_{ijt} + \beta_{21}\Delta T_{ijt} + \beta_{12}\Delta X^2_{ijt} \times 10^2 + \beta_{22}\Delta T^2_{ijt} \times 10^2 \\ &+ \beta_{13}\Delta X^3_{ijt} \times 10^3 + \beta_{23}\Delta T^3_{ijt} \times 10^3 + \beta_{14}\Delta X^4_{ijt} \times 10^4 + \beta_{24}\Delta T^4_{ijt} \times 10^4 + \Delta\varepsilon_{ijt}, \end{aligned} \tag{63}$$

where $\Delta Y_{ijt} = y_{ijt} - y_{ijt-1}$, the parameters β_{11} and β_{21} correspond to parameters β_1 and β_2, respectively, in Topel (1991), ε_{ijt} is the sum of a worker-firm effect, ϕ_{ijt}, an individual-specific effect, μ_i, and a transitory component, ν_{ijt}, and $Z_{ijt} \in \{X_{ijt}, T_{ijt}\}$, with

$$\begin{aligned}\Delta Z_{ijt} &= Z_{ijt} - Z_{ijt-1} = 1, \\ \Delta Z^2_{ijt} &= Z^2_{ijt} - Z^2_{ijt-1} = 2Z_{ijt} - 1, \\ \Delta Z^3_{ijt} &= Z^3_{ijt} - Z^3_{ijt-1} = 3Z_{ijt}(Z_{ijt} - 1) + 1, \\ \Delta Z^4_{ijt} &= Z^4_{ijt} - Z^4_{ijt-1} = (2Z_{ijt} - 1)[2Z_{ijt}(Z_{ijt} - 1) + 1].\end{aligned}$$

These expressions are obtained by repeatedly using the fact that $Z_{ijt-1} = Z_{ijt} - 1$. Topel's estimates of the effects of experience and tenure are reported in Table 2, which replicates column 3 of Table 2 in Topel (1991).

Table 2: Estimates from Topel (1991)

$\Delta X + \Delta T$	$\Delta X^2(\times 10^2)$	$\Delta X^3(\times 10^3)$	$\Delta X^4(\times 10^4)$	$\Delta T^2(\times 10^2)$	$\Delta T^3(\times 10^3)$	$\Delta T^4(\times 10^4)$
0.1258	-0.4067	0.0989	0.0089	-0.4592	0.1846	-0.0245

Buchinsky et al. (2010). Much of the literature on returns to tenure and experience has focused on the possible endogeneity of job changes and its effect on the estimation of returns to tenure. Buchinsky et al. (2010), BFKT hereafter, contribute to this debate by also considering the possible endogeneity of labor market experience and its impact on the estimation of both returns

[2]This index is meant to filter out both real aggregate wage growth and changes in any aggregate price level so that wage data from different time periods of the panel are expressed in comparable units. The GNP price deflator for consumption was used to construct it.

to tenure and experience. To address this issue, they explicitly model individuals' decisions about employment as well as interfirm mobility. They use data from the PSID from 1975 to 1992 and restrict attention to all heads of households between the ages of 18 and 65 interviewed for at least three years during the sample period.

Using a Bayesian approach, these authors estimate their model for three separate education groups: high school dropouts, high school graduates, and college graduates. Although these authors' estimated returns to experience are somewhat higher than those previously found in the literature, they are of similar magnitude. In contrast, these authors' estimates of the returns to tenure are much higher than those previously found in the literature, including those obtained by Topel (1991). Consequently, BFKT's estimates of total within-job wage growth are significantly higher than Topel's estimates. These results hold true for all three of the education groups they analyze.

Their study sheds light on several important factors leading to the apparent differences between their estimates and those from previous studies. First, their study highlights the importance of explicitly modeling employment as well as mobility decisions, which directly affect experience and tenure. Second, their study underscores the importance of accounting for unobserved heterogeneity in participation and mobility decisions, in addition to accounting for unobserved heterogeneity in the wage function. Third, their study demonstrates the importance of explicitly controlling for job-specific components in the wage function. The authors control for these job-specific components by introducing into their wage equation a function that provides a summary statistic of an individual's *career path*, as summarized by a worker's tenure and experience in previous jobs. BFKT find that the magnitude of the estimated returns changes markedly when they account for this factor but that the qualitative results are similar.

Formally, BFKT specify the observed log wage equation for individual i in job j at time t as

$$w_{ijt} = w^*_{ijt} I(y_{it} = 1), \tag{64}$$

where, by definition, $w^*_{ijt} = x'_{wijt}\delta_0 + \varepsilon_{ijt}$ and is measured in the data by the log of the (deflated) wage in year t, measured as total (real) labor income from the PSID. (When an individual does not work for the entire year, the authors compute earnings per week, multiply this wage figure by 52, and impute the resulting earnings as the individual's earnings for the year.) Here x'_{wijt} is a vector of observed characteristics, including education, labor market experience, and firm tenure, of an individual in the current job j, and $I(\cdot)$ is an indicator function that equals one if, and only if, $y_{it} = 1$, that is, if, and only if, the i-th individual participates in the labor market at time t. Note that w^*_{ijt} represents the individual's offered wage, which is observed only if the individual chooses to work.

BFKT decompose the error term ε_{ijt} into three components,

$$\varepsilon_{ijt} = J^W_{ijt} + \alpha_{wi} + \xi_{ijt},$$

where α_{wi} is a person-specific correlated random effect, analogous to μ_i in Topel (1991), and ξ_{ijt} is an idiosyncratic error term. The term J^W_{ijt} is analogous to the term ϕ_{ijt} in Topel (1991), with the important difference that in BFKT, it explicitly provides a summary statistic for the individual's work history and career: J^W_{ijt} captures the timing and magnitude of all discontinuous jumps in the individual's wage profiles that resulted from all job changes experienced by the individual until date t. BFKT specify J^W_{ijt} as a piecewise linear function of experience and tenure at the time of a job change and given by

$$J_{ijt}^{W} = (\phi_0^s + \phi_0^e e_{i0}) d_{i1} + \sum_{l=1}^{M_{it}} \left[\sum_{k=1}^{4} (\phi_{k0} + \phi_k^s s_{it_l - 1} + \phi_k^e e_{it_l - 1}) d_{kit_l} \right],$$

where $d_{1it_l} = 1$ if the l-th job of the i-th individual lasted less than a year and equals 0 otherwise, $d_{2it_l} = 1$ if the l-th job of the i-th individual lasted between 2 and 5 years and equals 0 otherwise, $d_{3it_l} = 1$ if the l-th job of the i-th individual lasted between 6 and 10 years and equals 0 otherwise, and $d_{4it_l} = 1$ if the l-th job of the i-th individual lasted more than 10 years and equals 0 otherwise; M_{it} denotes the number of job changes experienced by the i-th individual at time t (not including the individual's first sample year). If an individual changed jobs in the first sample year, then $d_{i1} = 1$, otherwise $d_{i1} = 0$. The variables $s_{it_l - 1}$ and $e_{it_l - 1}$ denote tenure and experience in year t_l, respectively, when individual i leaves job l. Note that while the ϕ's are fixed parameters, the size of the jumps within each of the four brackets of tenure may differ depending on the level of tenure and labor market experience at the time of a job change.

In Table 3 we report the estimates of BFKT of the effects of experience and tenure on wages for high school graduates, which replicates column 4 of Table 4 in their paper.

Table 3: Estimates from Buchinsky et al. (2010)

Variable	Mean
X	0.076
$X^2/100$	-0.467
$X^3/1000$	0.123
$X^4/10,000$	-0.012
T	0.069
$T^2/100$	-0.285
$T^3/1000$	0.077
$T^4/10,000$	-0.007
Job switches after:	
Up to 1 year (ϕ_{10})	0.109
2 to 5 years (ϕ_{20})	0.107
6 to 10 years (ϕ_{30})	0.190
Over 10 years (ϕ_{40})	0.373
Tenure at job that lasted:	
2 to 5 years (ϕ_2^s)	0.028
6 to 10 years (ϕ_3^s)	0.018
Over 10 years (ϕ_4^s)	0.035
Experience at job that lasted:	
Up to 1 year (ϕ_1^e)	-0.002
2 to 5 years (ϕ_2^e)	0.0001
6 to 10 years (ϕ_3^e)	-0.003
Over 10 years (ϕ_4^e)	-0.017

B.2.4 Wage Losses from Separation and Nonemployment Durations

We use monthly PSID data to compute the mean log wage difference between the first wage after a nonemployment spell and the last wage before a nonemployment spell as well as nonemployment spell durations. Following Krolikowski (2017), we use the PSID waves covering the years between 1988 and 1997.[3] The reason why we complement the sample of BFKT with these additional data is two-fold. First, in the survey years prior to 1988, the PSID did not collect monthly information on employment status at different employers so it is not possible to calculate either monthly transition rates across labor market states or the duration of either unemployment or out-of-the-labor-force spells at monthly frequency for those years. (For consistency with the model, we treat unemployment and out-of-the-labor-force as corresponding to the same state of nonemployment.) Given the relatively high job-finding rates commonly measured in various data sources, the ability to measure transitions across employment states at monthly, rather than annual, frequency is critical to accurately measuring worker flows and thus the distribution of durations of spells of nonemployment.

Second, in addition to information on employment status, these monthly data include information on wages. For an employed individual, these data provide the starting wage at the present employer as well as the ending wage at the former employer. Similarly, for a nonemployed individual, these data contain information on the starting wage at the former employer and the ending wage at the employer before the former one. This information, coupled with information on labor force status, allows us to measure wage changes associated with employment-to-nonemployment-to-employment transitions, commonly referred to as *ENE* transitions. Using this information, we can then assess the ability of our model to replicate this salient dimension of the data.

For consistency with BFKT and Krolikowski (2017), we restrict attention to individuals who are heads of households between the ages of 18 and 65. Like Krolikowski (2017), we focus on males, exclude observations on individuals who report being self-employed, and only consider waves up to the 1997 survey to avoid the complication of biennial interviews.[4] We use individual weights to properly account for the PSID's poverty over-sample and nonrandom attrition. Specifically, like Krolikowski (2017), we include the supplementary low-income sample and the 1997 immigrant sample in the analysis but exclude the Latino sample introduced in 1990.

Based on these data, the mean log wage difference experienced by a worker after a nonemployment spell is −4.4%, as reported in the paper. Excluding workers who report quitting a job, the sample reduces to workers who separate due to "plant closing/employer moved" or because they were "laid off/fired." Since this information is available only at yearly frequency, for this calculation we restrict attention to the last transition of an individual from employment to nonemployment in a particular year in order to identify the relevant nonemployment spell. For such workers, the mean log wage difference after nonemployment is −5.5%.

For a comparison, we also computed the wage loss of workers with fewer than 35 years of labor market experience who experience up to one year of nonemployment in the yearly PSID sample of BFKT. The resulting wage loss equals 5.7%, which is remarkably similar to the loss we measure in the monthly sample reported above. (Wage losses for workers who experience longer spells of nonemployment tend to be larger in this sample but are less relevant for our analysis—transitions

[3] We are indebted to Pawel Krolikowski for assisting us in the replication of his results.

[4] BFKT use all individuals, including the self-employed, but exclude the poverty sub-sample. They perform sensitivity analyses by excluding the self-employed and women, and report that they do not detect any visible differences in their results.

rates into employment are fairly high, as noted—and quite noisy since very few workers experience them. For instance, only 105 observation-years experience nonemployment spells of two years and less than 45 observation-years experience nonemployment spells of three years or more.)

B.2.5 Figure 11 in the Paper

In Figure 11 in the paper, panels A through C, we plot changes in total employment, employment in the nontradable sector, and employment in the tradable sector against changes in consumption, as predicted by changes in house prices, across U.S. states between 2007 and 2009. As we explain in the paper, total employment in each state is computed as the weighted average of employment in the nontradable and tradable goods sectors. The relationship between changes in wages and changes in employment across U.S. states between 2007 and 2009 is depicted in panel D of Figure 11. In all panels, changes are normalized by those of Louisiana, which is the state that experienced the median change in consumption across states between 2007 and 2009.

The elasticities reported in panels A through C are computed following an IV approach in which in the first stage we regress changes in consumption to changes in house prices, weighted by the working-age population in each state. In the second stage, we regress changes in the relevant measure of employment to predicted consumption changes from the first stage, weighted by the working-age population in each state. The elasticity reported in panel D is computed by regressing changes in wages to changes in employment, weighted by the working-age population in each state.

References

[1] Murphy, K.M., and F. Welch (1992): "The Structure of Wages," *Quarterly Journal of Economics*, 107 (1), 285-326.

Figure A.1: Cross-Sectional Experience-Earnings Profiles from Census and PSID Data