Why Are Returns to Private Business Wealth So Dispersed? *

Corina Boar[†] Denis Gorea[‡] Virgiliu Midrigan[§]

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Abstract

We use firm-level data to document that private businesses experience large fluctuations in their profit shares. These are due to large, fat-tailed and transitory changes in output that are not fully accompanied by changes in their inputs. We interpret this evidence using a model of entrepreneurial dynamics. Because firms can limit their exposure to risk by operating at a smaller scale, our model predicts large macroeconomic losses from uninsurable business risk, much larger than those stemming from credit constraints. While self-financing allows entrepreneurs to quickly overcome credit constraints, even wealthy entrepreneurs remain considerably exposed to risk.

Keywords: entrepreneurship, risk, credit constraints, misallocation.

JEL classifications: E2, E44, G32

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[†]New York University and NBER, corina.boar@nyu.edu.

[‡]Bank for International Settlements, denis.gorea@bis.org.

[§]New York University and NBER, virgiliu.midrigan@nyu.edu.

1 Introduction

Private businesses account for a large share of macroeconomic activity in most economies. An important characteristic of private businesses is that they predominantly rely on internal savings and collateralized borrowing, as opposed to equity financing (Dyrda and Pugsley, 2018). A large literature in both macroeconomics and development has thus argued that constraints on credit prevent private businesses from operating at their optimal scale, and that improving access to credit would lead to large efficiency gains in the aggregate.[1]

An additional feature of private businesses is that their ownership is poorly diversified (Moskowitz and Vissing-Jørgensen, 2002), so firm owners are exposed to fluctuations in their business income. We argue in this paper that the macroeconomic costs of this lack of diversification are large–much larger than those caused by credit constraints. The crux of our argument is that entrepreneurs can reduce their exposure to risk by operating at a smaller scale. As a result, productive but poor entrepreneurs choose to operate at an inefficiently low scale and are unwilling to borrow to expand further. Firm size is therefore largely limited by risk, not by the availability of credit.

We develop this argument with a model of entrepreneurial dynamics, which we use to interpret microeconomic evidence on private businesses in Orbis. Though our main empirical analysis focuses on Spain, a country with particularly good coverage, all of our findings extend to other countries. We use the data to document that individual firms experience large fluctuations in the ratio of their profits to output–in short, their profit share.[2] For example, 5% of firms experience losses exceeding 20% of their output–a sizable amount, given that profit shares average only 13%.

We trace the source of these large fluctuations in profit shares to two factors. First, firms experience large, fat-tailed and transitory changes in their output. Second, these changes in output are not fully accompanied by changes in a firm's capital and, importantly, wage bill. In contrast to standard models of firm dynamics, in which payments to factors comove one-to-one with output, the wage bill of a firm only falls by approximately 0.5% for every 1% fall in output. This imperfect comovement is a high-frequency phenomenon: in the cross-section, the wage bill and capital comove nearly one-to-one with output. This suggests that the patterns we see are driven by frictions that prevent adjustments from one period to the next, rather than by other distortions, such as heterogeneous markups, that persist over time.

[1] See, for example, Buera et al. (2011), Midrigan and Xu (2014), Moll (2014), Bau and Matray (2023).

[2] As we explain below, the notion of output we use is value-added.

The model we study is a relatively standard model of entrepreneurial dynamics in which, as in Quadrini (2000) and Cagetti and De Nardi (2006), firms face two sources of financial frictions. First, motivated by the evidence that private businesses are poorly diversified, we assume that each firm is owned by a single entrepreneur. Second, we assume a collateral constraint that limits a firm's ability to borrow. The model consists of a large number of households who differ in their entrepreneurial ability and choose whether to be workers or entrepreneurs. Entrepreneurs produce a homogeneous good with a decreasing returns to scale technology that uses capital and labor hired in competitive markets. We make two additional assumptions motivated by the patterns in the data. First, both capital and labor are chosen before the firm observes its productivity. This allows the model to parsimoniously capture the imperfect comovement between output and inputs. Second, productivity is subject to both persistent and transitory shocks that are drawn from fat-tailed distributions. This allows the model to reproduce the distribution of output growth in the data.

An entrepreneur's choices of labor and capital are distorted by two wedges that increase the expected marginal product of inputs relative to their costs. The first is a *risk wedge* that depends on the covariance between the entrepreneur's consumption and productivity. The second is a *credit wedge* that arises when the collateral constraint binds. The risk wedge distorts both labor and capital choices, whereas the credit wedge only distorts capital. As the entrepreneur accumulates wealth, the credit wedge quickly falls because wealth can be used to finance capital. Once the entrepreneur is sufficiently wealthy and overcomes the collateral constraint, the credit wedge vanishes, but the firm's scale remains inefficiently low because of risk. Intuitively, hiring a large amount of capital and labor exposes the entrepreneur to adverse shocks to productivity that can generate large declines in profit. More wealth insulates the consumption of the entrepreneur from such declines and reduces the risk wedge. The risk wedge, however, declines only gradually with wealth and, unlike the credit wedge, does not vanish even for wealthy entrepreneurs.

We use this framework to study the macroeconomic consequences of risk and credit constraints. These two frictions affect macroeconomic outcomes through two main channels. First, they distort the allocation of inputs across firms, generating aggregate productivity losses. Second, they reduce the overall demand for capital and labor, further depressing output and the equilibrium wage. We derive a mapping that traces how the risk and credit wedges shape all these aggregate outcomes. We do this by comparing the allocations in our economy to the efficient allocations–those chosen by a planner who seeks to maximize aggregate output and is subject to the same technology and timing restrictions as firms in our economy. This mapping allows us not only to quantify the overall impact of financial frictions, but also to assess the role of uninsurable risk and credit constraints in isolation.

We estimate the parameters of the model using the simulated method of moments. We target salient features of the data that summarize the prevalence and wealth of entrepreneurs, key aggregate ratios for private businesses–such as capital, labor and profit shares–and the dynamics of firm output. In addition to reproducing the targeted moments, the model is also consistent with the observed volatility of firm profits, the imperfect comovement between output and inputs, and the extent to which the consumption of business owners comoves with profits. The model thus provides a plausible account of the amount and sources of risk that entrepreneurs face, as well as the extent to which they can insure against this risk.

We use the estimated model to show that the aggregate losses from financial frictions are sizable–in their absence, output would be 15.8% larger. Importantly, these losses are largely accounted for by risk, not credit, wedges. Eliminating the risk wedge would increase output by 15.4%. In contrast, eliminating the credit wedge would only increase output by 0.4%. To understand this result, we show that aggregate outcomes are determined by the distribution of risk and credit wedges, weighted, as in Hopenhayn (2014), by the efficient firm size. In our economy, risk wedges are large precisely for high-ability entrepreneurs, who would be large under the efficient allocation. Although the majority of firms are credit-constrained, these firms are relatively unproductive and therefore have a small efficient size.

Our result on the importance of risk does not depend on the nature or the specific parameterization of the collateral constraint. To show this, we consider two extreme economies: one in which entrepreneurs cannot borrow at all, and one in which there are no limits on borrowing to finance capital. In both economies, the aggregate losses from financial frictions are similar to those in our baseline and are, once again, largely driven by risk. Thus, the extent to which firms can borrow to finance capital has little effect on aggregate outcomes. Intuitively, the presence of risk leads entrepreneurs to operate at a small scale and to accumulate wealth for precautionary reasons. Thus, they can self-finance most of their desired capital, which keeps credit wedges small, even in an economy without credit.

Risk emerges as the dominant friction that distorts macroeconomic aggregates because of three key ingredients that are necessary to match the data: fat-tailed shocks to productivity, transitory shocks to productivity, and labor inputs chosen before the realization of productivity shocks. To illustrate this point, we consider counterfactual economies in which we eliminate each ingredient in isolation. We recalibrate each economy and show that, in contrast to our baseline model, credit wedges emerge as the primary driver of macroeconomic losses. Thus, risk plays a critical role in our model precisely because firms are exposed to fat-tailed and transitory shocks to productivity, which translate into large movements in profits because of frictions that prevent the adjustment of labor. Though firms also face frictions that prevent the adjustment of capital, their effect is smaller because the capital share in production is smaller than that of labor.

Our conclusion that uninsurable business income risk has important macroeconomic consequences is not an artifact of assuming that agents are highly risk-averse. In our baseline model, we assume a coefficient of relative risk aversion of two. Reducing it to one-half, at the low end of the values typically used, does not alter our conclusions. We also show that our conclusions are robust to assuming that, in addition to private businesses, production is also carried out by corporate firms that are not subject to financial frictions.

Our results have implications for two other prominent strands of research. One studies how income is distributed between labor, capital and profits and attributes the latter to economic rents, unmeasured inputs or financial frictions. Our model predicts that profits largely reflect compensation for risk. The other strand studies the determinants of dispersion in entrepreneurial returns to wealth, an important driver of wealth inequality. Our model generates a large dispersion in returns to wealth, which mostly reflects compensation for risk.

Related Work. Our paper contributes to the literature on misallocation and production distortions (Hsieh and Klenow, 2009; Buera et al., 2011; Moll, 2014; Midrigan and Xu, 2014; Gopinath et al., 2017). Our emphasis on risk relates our paper to Tan (2018), Robinson (2021) and David et al. (2022a), who study how risk distorts investment. In contrast to these papers, we study how risk distorts labor choices. This relates our paper to Arellano et al. (2019), who study the decline in aggregate employment during the Great Recession, and David et al. (2022b) who study the role of aggregate, rather than idiosyncratic risk, in distorting the labor share. Partly motivated by our empirical findings, Di Tella et al. (2024) study optimal policy in an environment similar to ours, in which uninsurable risk distorts firm production choices, and show that optimal policy is the opposite of what would be optimal when misallocation is caused by markups resulting from imperfect competition.

2 Data and Motivating Facts

In this section, we describe the datasets we use and document several facts that motivate our modeling choices and inform our quantitative analysis. Specifically, we document that private businesses experience substantial volatility in their profits from one year to the next, largely because changes in their output are not accompanied by changes in their capital and, importantly, their wage bill. We also document that private businesses are poorly diversified.

2.1 Orbis Data

We use the historical product of Orbis, compiled by Moody's Bureau van Dijk. The data covers the period 1995-2019 and has harmonized information on annual balance sheets and income statements of privately and publicly traded firms (see Gopinath et al., 2017 and Kalemli-Ozcan et al., 2015 for a detailed description). We focus our analysis on Spain, a country with excellent coverage across the entire firm size distribution, as shown by Gopinath et al. (2017). However, as we show in Appendix A, all of our results hold for other countries.

Variable definition. We first define the variables we use in the analysis. Our measure of output, y_{it}, for firm i in year t is value added, which we define as the difference between production (revenues + change in inventories) and all costs other than labor and depreciation. Our measure of wage payments, wl_{it}, is the firm's wage bill, including pension costs. The capital stock, k_{it}, is the book value of property, plant, equipment and intangibles. We define profits, π_{it}, as output net of the wage bill and the user cost of capital. We compute the user cost of capital using the reported information on depreciation and assuming an interest rate of 2%. Lastly, we define firm equity as the difference between total assets and total liabilities. Using this measure, we calculate debt as the difference between the firm's physical capital and its equity, whenever this difference is positive. Our notion of debt is therefore a net one, corresponding to the difference between financial liabilities and financial assets. We deflate nominal variables using the CPI deflator and set 2015 as the base year. Appendix A provides further detail on variable construction.

Sample selection. Given our interest in private businesses, we restrict attention to partnerships and private limited companies. We exclude firms that operate in finance, insurance and real estate, public administration, defense and education. Additionally, we exclude firms with negative output, capital, wage bill and depreciation, as well as firms with missing output, capital, wage bill, equity, profits and depreciation. To minimize the concern that variables are measured with error, we exclude observations in the top and bottom 0.1% of the distri-

Table 1: Summary Statistics

	mean	p10	p25	p50	p75	p90
output	486	32	67	151	350	792
wage bill	351	24	52	118	268	594
capital	594	6	21	83	294	844
profit	63	-24	-1	9	38	117
employment	12	1	2	5	10	21

Notes: The numbers in the first four rows are expressed in thousands of 2015 EUR and are based on 6,298,358 firm-year observations. The employment counts in the last row are based on 5,734,570 firm-year observations.

bution of growth rates of value added, capital and wage bill, as well as the distribution of the capital-output ratio, the labor share, the profit share, and the debt-to-capital ratio. Lastly, because we compute growth rates over horizons of up to three years, we restrict attention to firms that have at least four years of data.[3] Our final sample consists of 622,883 firms, observed for an average of 10 years. Table 1 summarizes the distribution of key variables. The average (median) firm has 12 (5) workers, a value added equal to 486 (151) thousand euros, a wage bill of 351 (118) thousand euros and profit of 63 (9) thousand euros.

2.2 Spanish Survey of Household Finances

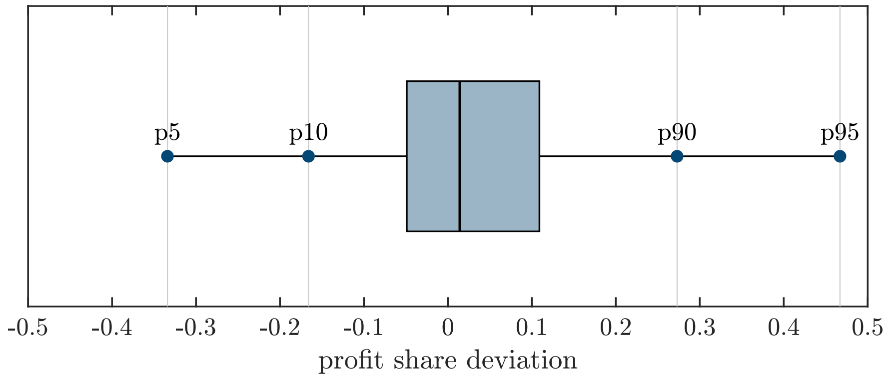
We also use data from the Spanish Survey of Household Finances (EFF), a representative survey of approximately $6,000$ households conducted every three years. A subset of households is surveyed across multiple waves, creating a panel component. We use data for the 2008-2020 survey waves and restrict the sample to households in which the reference household member is between 22 and 79 years old. We define an entrepreneur as a household who owns a business, is actively involved in running or managing that business, and reports positive business wealth. Appendix A provides further detail.

2.3 Motivating Facts

We use these two datasets to document facts that suggest that private businesses experience considerable risk and are poorly diversified. We first use Orbis to show that they experience

[3] Table A.1 reports the impact these sample restrictions have on the sample size.

Figure 1: Distribution of Profit Share Deviations



Notes: The figure is a boxplot of the distribution of profit share deviations $\pi_{it}/y_{it} - \overline{\pi_{it}/y_{it}}$, weighted by average firm output. The 1^{st} and 99^{th} percentiles, not shown for visual clarity, are -1.66 and 1.27.

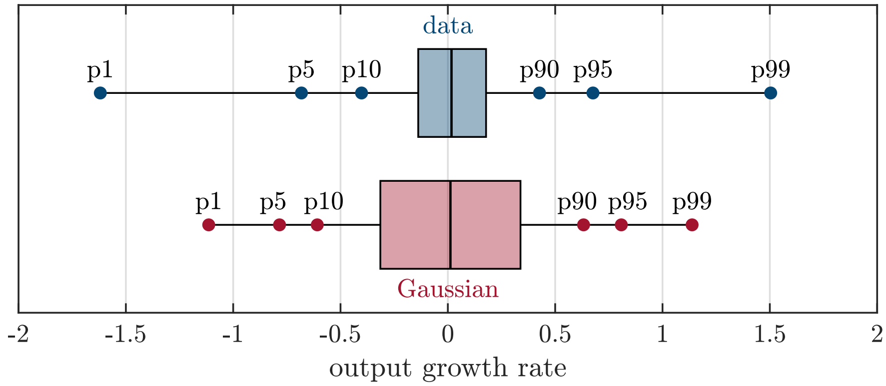
sizable fluctuations in their profits. We then show that these fluctuations are primarily due to large changes in output that are not fully accompanied by changes in capital and the wage bill. We finally use the EFF to show that, as in the U.S. (see Moskowitz and Vissing-Jørgensen, 2002), private businesses in Spain are poorly diversified, in that they are typically owned by a single household that owns a substantial fraction of the business.

Profit Shares Fluctuate Considerably. We first document that firms experience large fluctuations in their profits, echoing the findings of DeBacker et al. (2023) for the U.S. To interpret the magnitudes, we scale profits by output and thus report statistics for profit shares. We calculate, for each firm, the deviation of its profit share, π_{it}/y_{it}, from its time-series mean, $\overline{\pi_{it}/y_{it}}$. Figure 1 visualizes the distribution of profit share deviations using a boxplot that, in addition to the median and interquartile range, also depicts the 5^{th}, 10^{th}, 90^{th} and 95^{th} percentiles. In computing these statistics, we weight each firm by its average output to ensure that our findings are not driven by more volatile small firms.

The figure shows that individual firms' profit shares fluctuate considerably over time. To interpret these numbers, we note that the average profit share is equal to 0.13 and, by construction, profit share deviations are centered at zero. Since the 5^{th} percentile of the distribution is equal to -0.33, this implies that 5% of firms experience losses that amount to 20% $(-0.33 + 0.13)$ of their output. At the other end of the distribution, 5% of firms experience gains that amount to 60% $(0.13 + 0.47)$ of their output.

These large fluctuations in firm profit shares are puzzling from the perspective of standard models of firm dynamics, in which capital and labor can freely adjust. Any homogeneous

Figure 2: Distribution of Output Growth Rates



Notes: The figure is a boxplot of the distribution of output growth rates $\log y_{it}/y_{it-1}$ in the data and that of a Gaussian distribution with the same mean (0.012) and standard deviation (0.484).

production function implies that the sum of payments to capital and labor is equal to a constant share of output, as long as there are no frictions that generate time-varying wedges between factor prices and marginal products. Hence, profit shares would be constant as well.

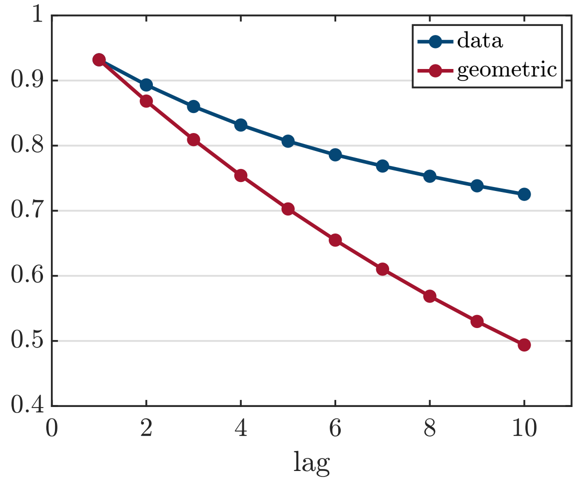
We next show that these fluctuations in profit shares arise because of large changes in output that are *not* accompanied by equally large changes in payments to capital and labor.

Firms Experience Large, Fat-Tailed and Transitory Changes in Output. To show that firms experience large and fat-tailed changes in output, Figure 2 depicts the distribution of annual output growth rates, $\log y_{it}/y_{it-1}$, and compares it with a Gaussian distribution with the same mean and variance. Relative to the Gaussian distribution, the inter-quartile range of output growth in the data is much lower: the 25^{th} percentile is equal to -0.14 and the 75^{th} is equal to 0.18. Thus, most firms experience relatively small changes in output from one year to the next. The standard deviation of output growth is high (0.48), however, because of occasionally large output changes: the 1^{st} percentile is -1.62 and the 99^{th} is 1.50. The ratio between the inter-quartile range and the standard deviation is a useful statistic to gauge the thickness of the tails of a distribution: the lower this ratio, the thicker the tails. This ratio is equal to 0.65 in the data and is therefore much lower than the 1.35 implied by the Gaussian distribution.[4]

To argue that firms experience transitory changes in output, Figure 3 plots the autocorrelogram of the logarithm of output, $\rho_k = \text{corr}\left(\log y_{it}, \log y_{it-k}\right)$, for lags $k = 1, \ldots, 10$ and compares it to the autocorrelogram of a process with geometric decay, ρ_1^k. The first-order

[4] An alternative statistic is the excess kurtosis, which is equal to 10.7 in the data.

Figure 3: Autocorrelation of Output



Notes: The figure plots the autocorrelogram of the logarithm of output ρ_k for lags $k = 1, \ldots, 10$ in the data and that of a process with geometric decay ρ_1^k.

autocorrelation of output is relatively low, $\rho_1 = 0.93$, and the autocorrelogram decays much more slowly with the horizon than for a process with geometric decay. These two observations suggest that the process for output is driven by a mix of transitory components, which reduce the first-order autocorrelation, and persistent components, which prevent the autocorrelation from decaying over longer horizons.

Capital and Labor Do Not Track Output Closely. We next show that fluctuations in output lead to large fluctuations in firm profits because capital and the wage bill do not track output closely. A fall in output is, thus, associated with a decrease in the profit share.

Table 2 reports results from regressions that relate changes in the wage bill, capital and profits to changes in output. The first two columns in Panel A show that the slope coefficients from regressing the growth rates of the wage bill and capital on the growth rate of output are equal to 0.40 and 0.16, respectively. That is, a 10% drop in output is associated with only a 4% drop in the firm's wage bill and a 1.6% drop in the capital stock. Though the pattern for capital is not surprising, given the evidence on capital adjustment costs (Cooper and Haltiwanger, 2006), the pattern for labor is relatively less known.[5]

We note that this imperfect comovement is only apparent at high frequency. Inputs and output comove much more in the cross-section: regressing the logarithm of the wage bill, $\log wl_{it}$, and capital, $\log k_{it}$, on the logarithm of output, $\log y_{it}$–i.e., a regression in levels as

[5] Donangelo et al. (2019) document similar patterns using data for U.S. firms.

Table 2: Comovement Between Capital, Labor, Profits and Output

	$\Delta \log wl$	$\Delta \log k$	$\Delta \pi / y$	$\Delta \hat{\pi} / y$
		A. All observations		
$\Delta \log y$	0.399	0.160	1.558	0.385
	(0.001)	(0.001)	(0.006)	(0.002)
		B. $\lvert \Delta \log y \rvert < 0.5$		
$\Delta \log y$	0.583	0.313	0.458	0.099
	(0.001)	(0.001)	(0.001)	(0.000)

Notes: The estimates in Panel A. (B.) are computed using 5,351,665 (4,517,994) observations. Standard errors reported in parentheses are clustered at the firm level. The regressions do not include firm or year fixed effects. Including these has a negligible effect. The independent variable is the annual growth rate of firm output, $\Delta \log y_{it}$. The dependent variables are: the growth rate of the wage bill, $\Delta \log wl_{it}$, the growth rate of capital, $\Delta \log k_{it}$, the change in the profit share, $\Delta \pi_{it}/y_{it}$, and the change in the counterfactual profit share that keeps the firm's labor share constant at its time-series mean, $\Delta \hat{\pi}_{it}/y_{it}$.

opposed to differences–yields elasticities of 0.92 and 0.84, respectively. Similarly, regressing the logarithm of the average firm wage bill, $\log \overline{wl_{it}}$, and capital, $\log \overline{k_{it}}$, on the logarithm of the average output, $\log \overline{y_{it}}$, yields elasticities of 0.97 and 0.93, respectively. The patterns we document are therefore more likely driven by frictions that generate short-term wedges that prevent capital and labor from perfectly comoving with output, rather than persistent wedges that systematically increase with firm size, such as markups.

The imperfect comovement of output and inputs in the first two columns of Table 2 implies that a firm's profit share, π_{it}/y_{it}, comoves positively with changes in output. To see this, we regress the change in a firm's profit share on the growth rate of its output. As the third column in Panel A shows, the slope coefficient is equal to 1.56, implying that a 10% drop in output is associated with a decline in the profit share of 0.16, a sizable amount given an average profit share of 0.13.

We next show that high-frequency movements in the firm's labor share account for a substantial fraction of the observed comovement between output growth and profit shares. To do so, we construct a counterfactual profit share that assumes labor is paid a constant share of output, equal to the time-series average of a firm's labor share. That is, we construct

$$\frac{\widehat{\pi_{it}}}{y_{it}} = \frac{\pi_{it}}{y_{it}} + \frac{wl_{it}}{y_{it}} - \overline{\frac{wl_{it}}{y_{it}}}.$$

As Table 2 shows, this counterfactual profit share comoves much less with output: the slope coefficient from regressing changes in this profit share on output growth falls to 0.39.

To address the concern that measurement error in output explains these patterns, Panel B of Table 2 reports results from a regression where we only include observations for which the growth rate of output is less than 50% in absolute value. Though the coefficients in the regressions of wage bill and capital growth on output growth are somewhat larger, once again, we find an incomplete pass-through of 0.58 and 0.31, respectively. The slope coefficient in a regression of changes in profit shares on output growth is equal to 0.46, smaller than when we include all observations, but nevertheless sizable. Changes in the labor share account for the bulk of this comovement: the slope coefficient in a regression of changes in the counterfactual profit shares on output growth falls to 0.10.

Since labor accounts for the bulk of a firm's expenses–the labor share in the data is 0.72–the observation that most variation in a firm's profit share over time is associated with movements in its labor share is not surprising. It suggests, however, that frictions that prevent the perfect comovement of labor and output are critical if one is to reproduce the observed volatility in firm profits.

Private Businesses Are Poorly Diversified. Lastly, to show that private businesses are poorly diversified, Table 3 reports statistics on the ownership structure of private businesses in Spain using the EFF data. In total, 12% of households are entrepreneurs. Of these, 93% own a single business and 7% own multiple businesses. Even multi-business owners disproportionately own one business: the share of their overall business wealth accounted for by their largest (main) business is 71%. Entrepreneurs are therefore exposed to business income risk stemming primarily from one business. We also find that most businesses are owned by a single entrepreneur: 71% of entrepreneurs own 100% of their business and, on average, the ownership share of an entrepreneur is 83%.

To summarize, private businesses experience large fluctuations in profits. Most of these fluctuations arise because firms do not fully adjust inputs, notably the wage bill, after large, fat-tailed and transitory changes in output.[6] Additionally, these firms are poorly diversified.

[6] In Appendix A, we show that these facts hold for other European countries, for both young and old firms, small and large firms, as well as across sectors.

Table 3: Ownership Structure of Private Businesses

fraction of households who are entrepreneurs	0.12
fraction of entrepreneurs who own exactly one business	0.93
share of business wealth from main business (multi-business owners)	0.71
share of main business that the entrepreneur owns	0.83
fraction of entrepreneurs who own 100% of main business	0.71

Notes: The table reports statistics on the ownership of Spanish private businesses based on the EFF data.

3 Model

We next interpret this evidence using a model of entrepreneurial dynamics in which firms face two sources of financial frictions. First, motivated by the evidence that private businesses are poorly diversified, we follow Quadrini (2000), Cagetti and De Nardi (2006) and Buera et al. (2011) in assuming that each firm is owned by a single entrepreneur. Second, we assume a collateral constraint that limits the firm's ability to borrow. We use the model to show that these two financial frictions lead to sizable productivity, output and wage losses, and that risk, rather than credit constraints, accounts for the bulk of these losses.

The model consists of a large number of households who differ in their entrepreneurial ability and choose whether to be workers or entrepreneurs. Entrepreneurs produce a homogeneous good with a decreasing returns to scale technology that uses capital and labor hired in competitive markets. We make two additional assumptions motivated by our empirical analysis. First, we assume that both capital and labor are chosen prior to the firm observing its productivity. This assumption allows us to parsimoniously capture the imperfect comovement between labor, capital and output.[7] As we show below, this assumption implies that wealth not only affects investment choices, but also employment decisions, consistent with the evidence in Ring (2023). Second, we assume that productivity is subject to both persistent and transitory shocks drawn from fat-tailed distributions, an assumption that allows us to capture the evidence on the distribution and dynamics of output growth.

[7] Such an assumption has been used both for capital (Gopinath et al., 2017; David et al., 2022a), employment (Boldrin et al., 2001; Arellano et al., 2019; Cooper et al., 2024; David et al., 2022b) and production (Eeckhout and Veldkamp, 2022). The time-to-build assumption for labor can also be interpreted as arising from adjustment costs, hiring or firing restrictions, or implicit insurance that firms provide to their more risk-averse workers (Guiso et al., 2005).

3.1 Environment

We assume a small open economy populated by a unit mass of households who can save and borrow at an exogenously given interest rate, r, and supply labor at equilibrium wage rate, W. There is no aggregate uncertainty. Each household i enters the period with entrepreneurial ability z_{it} and cash-on-hand m_{it}. Households can freely switch occupations each period; however, because inputs must be chosen before productivity shocks are realized, they must also choose their occupation before the realization of shocks.

Technology. Workers are endowed with one unit of time, so their cash-on-hand is

$$m_{it} = W + (1+r)a_{it},$$

where a_{it} is their wealth.

Running a business requires a fraction $1-\phi$ of the time endowment, so entrepreneurs only earn labor income ϕW. They produce using a Cobb-Douglas technology

$$y_{it} = z_{it}\varepsilon_{it}\left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right)^{\eta},$$

where $\eta < 1$ is the span of control parameter, α is the elasticity of output to capital, z_{it} is the entrepreneurial ability and ε_{it} is an iid productivity shock. Ability evolves according to

$$\log z_{it} = \rho \log z_{it-1} + u_{it}.$$

To capture the fat-tailed distribution of output growth, we assume that the productivity shocks u_{it} and ε_{it} are drawn from mixtures of two normal distributions with mean zero. Specifically, u_{it} is drawn from a distribution with standard deviation σ_u with probability p and from a distribution with standard deviation $s\sigma_u$ with probability $1-p$. Similarly, ε_{it} is drawn from is drawn from a distribution with standard deviation σ_ε with probability p and from a distribution with standard deviation $s\sigma_\varepsilon$ with probability $1-p$. We thus constrain the relative volatility s and the mixture probability p to be the same for both shocks.[8]

Let b_{it} denote the debt of an entrepreneur at the start of a period. After the productivity shocks are realized, the entrepreneur's cash-on-hand,

$$m_{it} = \phi W + (1-\delta)k_{it} + y_{it} - W l_{it} - (1+r)b_{it},$$

is equal to labor income, undepreciated capital and output, net of labor costs and debt repayment. Alternatively, letting $a_{it} = k_{it} - b_{it}$ denote the entrepreneur's wealth and $\pi_{it} =$

[8] Allowing for different relative volatilities and mixture probabilities does not significantly improve the fit of the model, so we opted for the more parsimonious specification.

$y_{it} - Wl_{it} - Rk_{it}$ denote profits, equal to output net of payments to labor and the user cost of capital $R = r + \delta$, the entrepreneur's cash-on-hand is

$$m_{it} = \phi W + (1 + r)a_{it} + \pi_{it}.$$

Preferences. Households maximize expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\theta}}{1-\theta},$$

where c_{it} denotes consumption in period t, β is the discount factor, θ is the relative risk aversion and the expectation operator is over the idiosyncratic shocks.

Recursive formulation. Because households can freely change occupations each period, their state can be summarized by their cash-on-hand and entrepreneurial ability, regardless of their current occupation. Let $V^w(m, z)$ denote the value of a household with cash-on-hand m and entrepreneurial ability z who chooses to be a worker *next* period. Similarly, let $V^e(m, z)$ denote the value of a household who chooses to be an entrepreneur *next* period. Then, the value of the household is

$$V(m, z) = \max \{V^w(m, z), V^e(m, z)\}.$$

Let $o'(m, z)$ be an indicator that is equal to one if the household chooses to be an entrepreneur next period.

The value of choosing to be a worker is equal to

$$V^w(m, z) = \max_{c,\, a' \geq 0} \frac{c^{1-\theta}}{1-\theta} + \beta \mathbb{E}_{z'|z} V(m', z'),$$

subject to

$$c + a' = m$$

and

$$m' = W + (1 + r)a'.$$

Correspondingly, the value of choosing to be an entrepreneur is equal to

$$V^e(m, z) = \max_{c,\, a' \geq 0,\, l',\, k'} \frac{c^{1-\theta}}{1-\theta} + \beta \mathbb{E}_{z', \varepsilon'|z} V(m', z'),$$

subject to

$$c + a' = m$$

and

$$m' = \phi W + (1+r)a' + z'\varepsilon' \left(k'^{\alpha} l'^{1-\alpha}\right)^{\eta} - Wl' - Rk'.$$

We assume that the firm is subject to a collateral constraint that restricts the amount it can borrow to a fraction ξ of its capital stock. This constraint can be written as

$$k' \leq \frac{1}{1-\xi} a',$$

so that the firm's capital can be at most a multiple $1/(1-\xi)$ of the entrepreneur's wealth.

Equilibrium. We focus on a stationary equilibrium characterized by decision rules for saving $a'(m,z)$, consumption $c(m,z)$, labor demand $l'(m,z)$, capital demand $k'(m,z)$ and occupational choice $o'(m,z)$, an equilibrium wage W and a stationary distribution $F(m,z)$ over cash-on-hand and entrepreneurial ability, such that the decision rules solve the problem of households described above, the equilibrium wage ensures that the labor market clears

$$\int l'(m,z) o'(m,z) \mathrm{d}F(m,z) = \int \left(1 - o'(m,z) + o'(m,z)\phi\right) \mathrm{d}F(m,z),$$

and the time-invariant distribution $F(m,z)$ is consistent with individual choices.

3.2 Decision Rules

We next discuss how the production choices of entrepreneurs are shaped by financial frictions. Since labor and capital are chosen before observing productivity, the optimal input choices of a household i who will be an entrepreneur next period satisfy

$$\mathbb{E}_{it} c_{it+1}^{-\theta} \left[(1-\alpha)\eta \frac{y_{it+1}}{l_{it+1}} - W \right] = 0 \tag{1}$$

and

$$\mathbb{E}_{it} c_{it+1}^{-\theta} \left[\alpha\eta \frac{y_{it+1}}{k_{it+1}} - R \right] \geq 0. \tag{2}$$

The entrepreneur chooses labor and capital to equate their expected marginal products with the respective factor prices.[9] Since business income risk is not diversified, the owner uses its own stochastic discount factor to weight future states. The first order condition for capital holds with equality if and only if the collateral constraint does not bind.

The optimality condition for an entrepreneur's wealth accumulation is

$$c_{it}^{-\theta} = \beta (1 + r + \mu_{it}) \mathbb{E}_{it} c_{it+1}^{-\theta}, \tag{3}$$

[9] We use the notation $\mathbb{E}_{it}$ to denote the expectation operator conditional on current ability z_{it}.

where μ_{it} is the multiplier on the collateral constraint and satisfies

$$\mu_{it} = \frac{1}{1-\xi}\mathbb{E}_{it}\frac{c_{it+1}^{-\theta}}{\mathbb{E}_{it}c_{it+1}^{-\theta}}\left[\alpha\eta\frac{y_{it+1}}{k_{it+1}} - R\right].$$

Intuitively, since an additional unit of wealth allows the entrepreneur to use $1/(1-\xi)$ additional units of capital, the excess return on saving μ_{it} is equal to $1/(1-\xi)$ times the risk-adjusted expected difference between the marginal product of capital and its user cost. Henceforth, we use $\hat{\mathbb{E}}_{it} \equiv \mathbb{E}_{it}\frac{c_{it+1}^{-\theta}}{\mathbb{E}_{it}c_{it+1}^{-\theta}}$ to denote the expectation under the risk-neutral measure. With this notation, the optimality conditions for labor and capital can be written as

$$(1-\alpha)\,\eta\left(k_{it+1}^{\alpha}l_{it+1}^{1-\alpha}\right)^{\eta}\frac{1}{l_{it+1}}\hat{\mathbb{E}}_{it}\left(z_{it+1}\varepsilon_{it+1}\right) = W \tag{4}$$

and

$$\alpha\eta\left(k_{it+1}^{\alpha}l_{it+1}^{1-\alpha}\right)^{\eta}\frac{1}{k_{it+1}}\hat{\mathbb{E}}_{it}\left(z_{it+1}\varepsilon_{it+1}\right) = R + (1-\xi)\,\mu_{it}. \tag{5}$$

To gauge the effect that financial frictions have on production choices, we contrast the decision rules in our model with those in a setting in which entrepreneurial risk is perfectly diversified and there are no collateral constraints. In that case, the first-order conditions that determine the frictionless levels of labor l_{it+1}^{*} and capital k_{it+1}^{*} are

$$(1-\alpha)\,\eta\left(\left(k_{it+1}^{*}\right)^{\alpha}\left(l_{it+1}^{*}\right)^{1-\alpha}\right)^{\eta}\frac{1}{l_{it+1}^{*}}\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right) = W$$

and

$$\alpha\eta\left(\left(k_{it+1}^{*}\right)^{\alpha}\left(l_{it+1}^{*}\right)^{1-\alpha}\right)^{\eta}\frac{1}{k_{it+1}^{*}}\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right) = R.$$

Thus, absent financial frictions, factor prices are equal to the expected marginal product of capital and labor under the physical productivity measure.

Let τ_{it} denote the wedge between the expected marginal product of labor and the wage in our economy. This wedge is implicitly defined by

$$(1-\alpha)\,\eta\left(k_{it+1}^{\alpha}l_{it+1}^{1-\alpha}\right)^{\eta}\frac{1}{l_{it+1}}\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right) = \tau_{it}W. \tag{6}$$

The ratio of equations (6) and (4) implies that this wedge is equal to

$$\tau_{it} = \frac{\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)}{\hat{\mathbb{E}}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)} = \left(1 + \frac{\mathbb{COV}_{it}\left(c_{it+1}^{-\theta},\, z_{it+1}\varepsilon_{it+1}\right)}{\mathbb{E}_{it}c_{it+1}^{-\theta}\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)}\right)^{-1} \tag{7}$$

and captures the effects of undiversifiable business risk. This wedge, which we henceforth refer to as the *risk wedge*, is generally greater than one because of the negative covariance between the entrepreneur's marginal utility of consumption and productivity.[10]

[10]The wedge may be less than one for households with sufficiently high ability and low wealth. Their consumption decreases with cash-on-hand because additional cash-on-hand increases the probability of continuing as entrepreneurs and the returns to saving.

The capital choice is distorted not only by risk, but also by the collateral constraint. Letting ω_{it} denote the wedge that this constraint generates, we can write the first-order condition for capital as

$$\alpha\eta \left(k_{it+1}^{\alpha} l_{it+1}^{1-\alpha}\right)^{\eta} \frac{1}{k_{it+1}} \mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right) = \tau_{it}\omega_{it} R. \tag{8}$$

The ratio of equations (8) and (5) implies that ω_{it}, which we refer to as the *credit wedge*, is

$$\omega_{it} = 1 + (1-\xi)\frac{\mu_{it}}{R} \tag{9}$$

and is greater than one when the collateral constraint binds ($\mu_{it} > 0$).

Given the risk and credit wedges, τ_{it} and ω_{it}, the choices of labor and capital are therefore

$$l_{it+1} = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}} \left(\tau_{it}\right)^{-\frac{1}{1-\eta}} \left(\omega_{it}\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)\right)^{\frac{1}{1-\eta}} \tag{10}$$

and

$$k_{it+1} = \left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\tau_{it}\right)^{-\frac{1}{1-\eta}} \left(\omega_{it}\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)\right)^{\frac{1}{1-\eta}}. \tag{11}$$

Since the frictionless choices entail $\tau_{it} = \omega_{it} = 1$, the ratio of labor and capital to their frictionless counterparts is

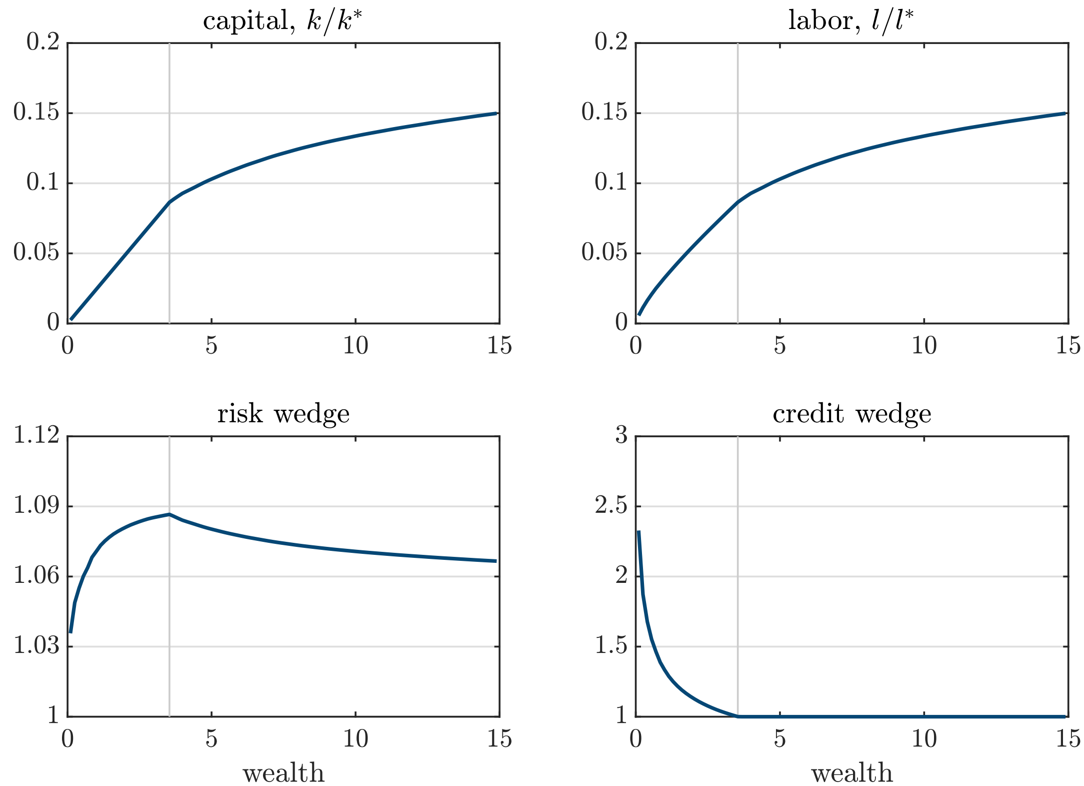
$$\frac{l_{it+1}}{l_{it+1}^{*}} = \left(\tau_{it}\right)^{-\frac{1}{1-\eta}} \left(\omega_{it}\right)^{-\frac{\alpha\eta}{1-\eta}} \qquad \text{and} \qquad \frac{k_{it+1}}{k_{it+1}^{*}} = \left(\tau_{it}\right)^{-\frac{1}{1-\eta}} \left(\omega_{it}\right)^{-1-\frac{\alpha\eta}{1-\eta}}.$$

Because the wedges are larger than one, in our model with financial frictions labor and capital choices are inefficiently low. Risk distorts capital and labor by the same amount, whereas the collateral constraint disproportionately distorts capital.

We illustrate these choices in Figure 4 as a function of wealth, a', for a fixed level of entrepreneurial ability, z.[11] Notice first that when the entrepreneur's wealth is sufficiently high–to the right of the vertical line in the figure–the collateral constraint does not bind ($\omega = 1$), so production choices are only distorted by risk ($\tau > 1$). In this region, additional wealth reduces the risk wedge because wealth insulates the consumption of the entrepreneur from changes in business income. As a consequence, labor and capital choices increase toward their frictionless levels. Second, when the entrepreneur's wealth is sufficiently low–to the left of the vertical line–the collateral constraint binds ($\omega > 1$), further reducing capital and labor.

[11] To derive these choices, we fix a level of ability, z, and trace out how the solutions to equations (1) and (2) vary with a', conditional on the household choosing entrepreneurship.

Figure 4: Decision Rules



Notes: The top panels plot labor, l', and capital, k', relative to their frictionless levels, as a function of wealth, a', and for a fixed level of entrepreneurial ability, z. The bottom panels plot the risk and credit wedges, τ and ω. The vertical lines mark the level of wealth for which the collateral constraint no longer binds.

In this region, additional wealth allows entrepreneurs to expand rapidly, reducing their credit wedge. Therefore, they end up taking more risk, so the risk wedge increases with wealth. A key takeaway from this figure is that entrepreneurs overcome collateral constraints relatively quickly, but distortions due to uninsurable risk persist even for wealthy entrepreneurs.

3.3 Parameterization

We next describe how we choose parameters for our quantitative analysis.

Assigned Parameters. A period in the model is one year. We set the depreciation rate $\delta = 0.10$, the interest rate $r = 0.02$ and the relative risk aversion $\theta = 2$. We set $\xi = 0.408$ to reproduce the aggregate debt to aggregate capital ratio of 0.408 in Orbis.[12]

[12]Since the allocation of wealth between business equity and personal wealth is indeterminate, our implicit assumption is that entrepreneurs hold, perhaps due to limited liability, the minimum business equity required to finance capital, so business equity is $(1-\xi)\,k_{it+1}$ and ξ is the ratio of aggregate debt to capital.

Estimated Parameters. We estimate the remaining parameters–the discount factor, the elasticities of labor and capital in the production function and the process for productivity–using the simulated method of moments. We target moments related to the prevalence and wealth of entrepreneurs, key aggregate ratios for private businesses, and the dynamics of output described in Section 2. Because the Orbis data does not provide information on the overall wealth of entrepreneurs, nor their share in the population, we calculate these statistics using data from the EFF.

Specifically, we choose the parameters $\boldsymbol{\vartheta} = (\beta, \alpha, \eta, \rho, \sigma_u, \sigma_\varepsilon, s, p, \phi)$ to minimize

$$\sqrt{\sum_{i=1}^{n} w_i \left(\frac{m_i(\boldsymbol{\vartheta}) - \bar{m}_i}{1 + \bar{m}_i} \right)^2}, \tag{12}$$

where $\bar{m}_i$ represents the empirical moments, $m_i(\boldsymbol{\vartheta})$ denotes moments implied by the model for a given set of parameters $\boldsymbol{\vartheta}$ and w_i are the assigned weights. We scale the difference between the empirical and model-implied moments by $1+\bar{m}_i$ to express the deviations in approximate percentage terms. This normalization prevents the denominator from approaching zero, which would overweight moments that are close to zero.

We target the moments listed in Table 4 and report the parameter estimates in Table 5. We assign a weight, w_i, that is three times lower for the last nine moments in Table 4, which are computed for three different horizons, to avoid overweighting them. As Table 4 shows, the model reproduces the targeted moments well, even though we only have nine parameters to fit 16 moments. The last row reports the value of the objective, which is 0.013. Because we normalize the weights so that they add up to one, this number implies that the moments in the model deviate from the empirical ones by approximately 1.3%.

Although all the parameters are jointly pinned down by all moments, we next provide some intuition for identification. The discount factor, β, is primarily pinned down by the wealth-to-income ratio of entrepreneurs, which is 12.5 in both the data and the model. The technology parameters, α and η, are primarily pinned down by the capital-output ratio (1.22 vs 1.21), the labor share (0.72 vs 0.71) and the profit share (0.13 vs 0.14). As we showed in Section 2, firms experience large, fat-tailed and transitory changes in output. The parameters of the productivity process ensure that the model reproduces this. The persistence, ρ, and volatilities, σ_u and σ_ε, of the two productivity shocks are jointly pinned down by the autocorrelation of output at horizons one to three, the cross-sectional standard deviation of output and the standard deviation of output growth rates at horizons one to three. In addition to the speed at which the autocorrelation and volatility of growth rates

Table 4: Targeted Moments

	data	model
fraction entrepr	0.12	0.13
wealth to income entrepr	12.5	12.5
capital-output ratio, k/y	1.22	1.21
labor share, wl/y	0.72	0.71
profit share, π/y	0.13	0.14
iqr $wl_{it}/y_{it} - \overline{wl_{it}/y_{it}}$	0.15	0.16
s.d. $\log y_{it}$	1.32	1.32
s.d. $\log y_{it}/y_{it-1}$	0.48	0.44
s.d. $\log y_{it}/y_{it-2}$	0.60	0.57
s.d. $\log y_{it}/y_{it-3}$	0.69	0.67
iqr $\log y_{it}/y_{it-1}$	0.32	0.27
iqr $\log y_{it}/y_{it-2}$	0.46	0.46
iqr $\log y_{it}/y_{it-3}$	0.58	0.63
corr $\log y_{it}$, $\log y_{it-1}$	0.93	0.94
corr $\log y_{it}$, $\log y_{it-2}$	0.89	0.91
corr $\log y_{it}$, $\log y_{it-3}$	0.86	0.87
value of objective	–	0.013

Notes: The table reports the fit of the estimation procedure. The targeted moments are in the column labeled "data" and the moments implied by parameter estimates are in the column labeled "model".

change with the horizon, the relative importance of transitory and persistent shocks also influences the extent to which the labor share for any given firm fluctuates over time relative to its time-series mean. We thus also target the inter-quartile range of these deviations (0.15 vs 0.16). The parameters s and p that govern the thickness of the tails of the distributions of productivity shocks are primarily pinned down by the inter-quartile range of the distribution of output growth rates. As discussed in Section 2, a fat-tailed distribution is characterized by a low inter-quartile range relative to the standard deviation, a feature that our model matches well. Finally, the parameter ϕ that determines the fraction of the time endowment required to run a business is primarily pinned down by the fraction of entrepreneurs.

The discount factor is $\beta = 0.948$. Because the capital-output ratio in the data is relatively low, the capital elasticity is also low, $\alpha = 0.172$. The span of control is $\eta = 0.966$, higher

Table 5: Parameter Values

		value	std. error
β	discount factor	0.948	0.004
α	capital elasticity	0.172	0.008
η	span of control	0.966	0.001
ρ	persistence z	0.977	0.001
σ_u	volatility z	0.009	0.000
σ_ε	volatility ε	0.094	0.002
s	relative volatility mixture	11.58	0.151
p	baseline probability mixture	0.913	0.001
ϕ	relative time endowment	0.977	0.002

Notes: The table reports the parameter estimates and the associated standard errors.

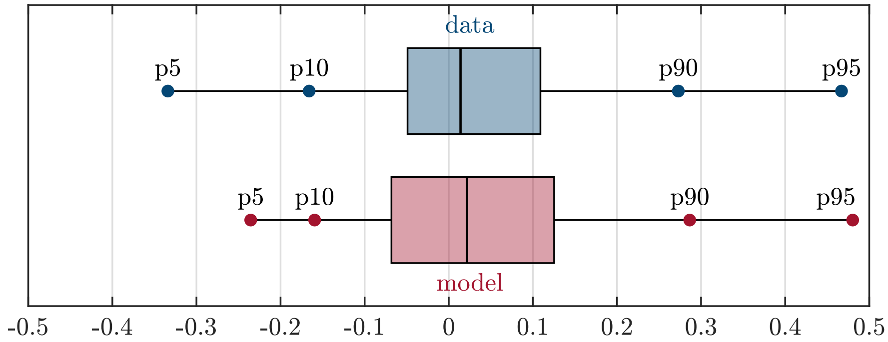
than the typical values of 0.85 or 0.90. As we explain below, this is because risk plays an important role in our model, so a substantial fraction of profits accrue to risk. The persistence and volatility of entrepreneurial ability are 0.977 and 0.009, while the volatility of the transitory productivity shock is 0.094. The parameter s is 11.58, so the second normal distributions of shocks are much more volatile. The probability p of drawing from the less volatile distributions is 0.913. Lastly, $\phi = 0.977$, so entrepreneurs keep most of their wage income when they run a business. Because most entrepreneurs are relatively unproductive and earn little business income, the opportunity cost required to match the fraction of entrepreneurs is low. Table 5 also reports the standard errors of the parameter estimates, computed by bootstrapping the moments in the data.[13] Because our sample of firms is large, the moments exhibit minimal variation across bootstrap samples, so the standard errors are small.

3.4 Model Fit: Untargeted Moments

We next argue that our model provides a reasonable account of how much risk private business owners are exposed to. To that end, we show that the model reproduces well the motivating facts in Section 2, as well as the extent to which firm profits comove with entrepreneurs' consumption.

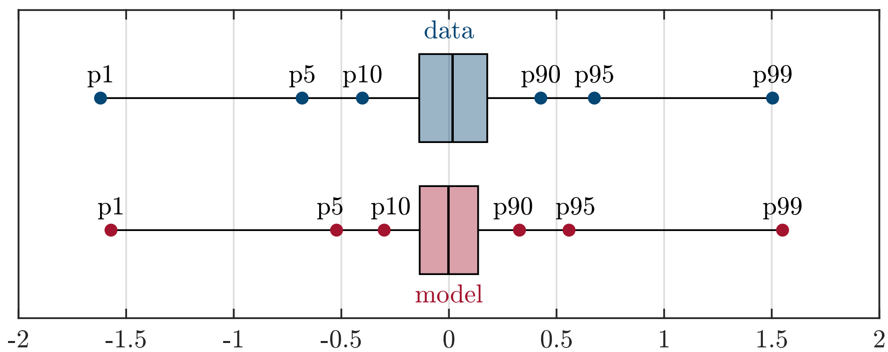
[13] Letting $\mathbf{W}$ be the diagonal weighting matrix, $\mathbf{V}$ be the bootstrapped variance-covariance matrix of the empirical moments, and $\mathbf{D}$ the Jacobian with typical element $D_{ij} = \frac{\partial m_i(\boldsymbol{\vartheta})}{\partial \vartheta_j}$, the standard errors are the square root of the diagonal elements of the matrix $\left(\mathbf{D}^\top \mathbf{W} \mathbf{D}\right)^{-1} \mathbf{D}^\top \mathbf{W} \mathbf{V} \mathbf{W}^\top \mathbf{D} \left(\mathbf{D}^\top \mathbf{W} \mathbf{D}\right)^{-1}$.

Figure 5: Implications for the Distribution of Profit Share Deviations



Notes: The figure is a boxplot of the distribution of profit share deviations $\pi_{it}/y_{it} - \overline{\pi_{it}/y_{it}}$. The 1^{st} and 99^{th} percentiles are -1.66 and 1.27 in the data and -1.92 and 0.83 in the model. All statistics are weighted by the time-series average of each firm's output.

Figure 6: Implications for the Distribution of Output Growth Rates



Notes: The figure is a boxplot of the distribution of output growth $\log y_{it}/y_{it-1}$ in the data and in the model.

Figure 5 depicts the distribution of profit share deviations from their time-series mean in the data and in the model. These two distributions line up closely, so, as in the data, firms in the model experience large fluctuations in profits. For example, the 10^{th} percentile of these deviations is -0.17 in the data and -0.16 in the model. Since the average profit share is 0.13 in the data and 0.14 in the model, this implies that more than 10% of firms experience losses in a given period. Moreover, the extremely large deviations at the bottom of the distribution–the 1^{st} percentile is equal to -1.66 in the data and -1.92 in the model–suggest that some firms experience extremely large losses in both the model and the data.

Figure 6 compares the distribution of output growth rates in the model and in the data. Although we only targeted the inter-quartile range and the standard deviation, the figure shows that the model reproduces the entire distribution of output growth rates.

Table 6: Implications for Comovement with Output

	$\Delta \log wl$	$\Delta \log k$	$\Delta\pi/y$	$\Delta\hat{\pi}/y$
Data	0.58	0.31	0.46	0.10
Model	0.55	0.61	0.42	0.06

Notes: The table reports the slope coefficients from regressing the variables listed at the top of each column on the output growth rate $\Delta \log y$. The estimates in the data are based on the 4,517,994 observations for which $|\Delta \log y_{it}| < 0.5$. We apply the same restriction to the model-generated data.

Importantly, as shown in Table 6, the model also reproduces well the low comovement of the growth rates of the wage bill and output. In the data, a regression of $\Delta \log wl$ on $\Delta \log y$ gives a coefficient of 0.58 when we restrict the sample to observations with $|\Delta \log y_{it}| \leq 0.5$. The corresponding regression coefficient in the model is 0.55. Given that the labor share is high in both the data and the model (0.72 vs. 0.71), the majority of movements in profits are due to fluctuations in the labor share. Specifically, the slope coefficient from regressing the change in profit shares $\Delta\pi/y$ on the growth of output $\Delta \log y$ is 0.46 in the data and 0.42 in the model. Most of this comovement is due to changes in the labor share: eliminating these reduces the correlation between profit shares and output growth to 0.10 in the data and 0.06 in the model. It is thus reassuring that our model reproduces the comovement between the wage bill and output, despite the parsimony of the time-to-build assumption on labor, and thus generates fluctuations in profits for the same reason as in the data.

In our estimation, we targeted moments that characterize the persistence and volatility of output only. As Table 7 shows, the model also reproduces the persistence and volatility of capital and labor inputs. For example, focusing on the wage bill, the standard deviation of growth rates is 0.36 in the data and 0.32 in the model, the inter-quartile range is 0.22 vs. 0.23, and the autocorrelation is 0.96 vs. 0.97.

The discussion above suggests that our model accurately captures the amount of business risk that entrepreneurs face. We conclude this section by arguing that the model also successfully captures the extent to which entrepreneurs' consumption is exposed to this risk. To that end, we regress changes in consumption, Δc_{it}, on changes in profit, $\Delta\pi_{it}$, in both the model and the data. We use the EFF to calculate consumption as the annualized value of average monthly spending on consumer goods reported by entrepreneurs and profits as

Table 7: Additional Moments

	data	model
s.d. $\Delta \log wl_{it}$	0.36	0.32
iqr $\Delta \log wl_{it}$	0.22	0.23
autocorr $\log wl_{it}$	0.96	0.97
s.d. $\Delta \log k_{it}$	0.60	0.36
iqr $\Delta \log k_{it}$	0.28	0.27
autocorr $\log k_{it}$	0.96	0.97
slope Δc_{it} on $\Delta \pi_{it}$	0.02	0.02

Notes: The first six data moments are calculated using the Orbis data. The last data moment is calculated using the EFF and is based on 799 observations. The change in consumption, Δc_{it}, and profits, $\Delta \pi_{it}$, is over three years in both the data and the model. We include year fixed effects in the regression.

the annual profit from the business. Because the survey is conducted every three years, we compute changes in consumption and profits over three years in both the model and the data. The last row of Table 7 reports the slope coefficient of this regression, which is 0.02 in both the data and the model, suggesting that consumption comoves little with profits. In the model, this is because entrepreneurs can use their wealth and labor income, as well as switch occupations, to mitigate the impact of large declines in profits. Furthermore, through their choice of labor and capital, they can effectively choose their exposure to risk and thus limit the extent to which productivity shocks translate into consumption.

4 Aggregate Implications of Financial Frictions

We now turn to quantifying the aggregate implications of uninsurable business income risk and credit constraints. We show that uninsurable risk leads to considerable losses in productivity, output and the equilibrium wage. In contrast, credit constraints have relatively small effects. To establish this, we derive a mapping that traces out how the distribution of risk and credit wedges shapes the productivity losses from misallocation and distorts output and the equilibrium wage. We use this mapping to show that the bulk of the losses from financial frictions are accounted for by risk rather than credit wedges.

4.1 Losses from Misallocation

To calculate the productivity losses from misallocation, we follow the Hsieh and Klenow (2009) approach and compare the aggregate productivity in our economy with that achievable by a planner that optimally allocates inputs across firms. Letting Y_{t+1} denote aggregate output[14]

$$Y_{t+1} = \int \left(\mathbb{E}_{it} z_{it+1} \varepsilon_{it+1} \left(k_{it+1}^{\alpha} l_{it+1}^{1-\alpha}\right)^{\eta}\right) \mathrm{d}i,$$

and $L_{t+1} = \int l_{it+1} \mathrm{d}i$ and $K_{t+1} = \int k_{it+1} \mathrm{d}i$ the total amount of labor and capital used in production, aggregate productivity Z_{t+1} is

$$Z_{t+1} = \frac{Y_{t+1}}{\left(K_{t+1}^{\alpha} L_{t+1}^{1-\alpha}\right)^{\eta}} = \int \mathbb{E}_{it} z_{it+1} \varepsilon_{it+1} \left(\left(n_{it+1}^{k}\right)^{\alpha} \left(n_{it+1}^{l}\right)^{1-\alpha}\right)^{\eta} \mathrm{d}i,$$

where $n_{it}^{l} \equiv \frac{l_{it+1}}{L_{t+1}}$ and $n_{it}^{k} \equiv \frac{k_{it+1}}{K_{t+1}}$ are the shares of labor and capital allocated to each firm in period t. Importantly, aggregate productivity depends only on these shares and not on the aggregate levels of labor and capital.

To find the efficient allocation, we solve the problem of a planner who chooses the shares n_{it}^{l} and n_{it}^{k} to maximize aggregate productivity. As in the baseline, the planner chooses these shares before observing the shocks. We isolate the intensive margin losses from misallocation by restricting the planner to only reallocate capital and labor across those households $i \in \Omega_t = \{i : o_{it+1} = 1\}$ who are entrepreneurs in the baseline economy. As we show below, the extensive margin distortions arising from inefficient occupational choice are small.

The problem of the planner is then

$$\max_{n_{it}^{l}, n_{it}^{k}} \int_{\Omega_t} \mathbb{E}_{it} z_{it+1} \varepsilon_{it+1} \left(\left(n_{it}^{k}\right)^{\alpha} \left(n_{it}^{l}\right)^{1-\alpha}\right)^{\eta} \mathrm{d}i$$

subject to

$$\int_{\Omega_t} n_{it}^{k} \mathrm{d}i \leq 1 \qquad \text{and} \qquad \int_{\Omega_t} n_{it}^{l} \mathrm{d}i \leq 1.$$

The planner chooses these shares to equate the expected marginal product of labor and capital across entrepreneurs, which implies that

$$n_{it}^{l} = n_{it}^{k} = \frac{\left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)\right)^{\frac{1}{1-\eta}}}{\int_{\Omega_t} \left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)\right)^{\frac{1}{1-\eta}} \mathrm{d}i} = \frac{\left(z_{it}\right)^{\frac{\rho}{1-\eta}}}{\int_{\Omega_t} \left(z_{it}\right)^{\frac{\rho}{1-\eta}} \mathrm{d}i} \equiv n_{it}.$$

[14] We note that though there is uncertainty about the level of output, y_{it+1}, of any given firm, by the law of large numbers idiosyncratic shocks average out and aggregate output is deterministic.

We use the notation n_{it} to refer to the efficient size of firm i, which is a function of entrepreneurial ability alone. The efficient level of aggregate productivity, Z_{t+1}^P, is

$$Z_{t+1}^P = \left(\int_{\Omega_t} \left(\mathbb{E}_{it} \left(z_{it+1} \varepsilon_{it+1} \right) \right)^{\frac{1}{1-\eta}} \mathrm{d}i \right)^{1-\eta}.$$

In contrast, the allocations in equations (10) and (11) imply that in our baseline model aggregate productivity, Z_{t+1}, satisfies

$$\frac{Z_{t+1}}{Z_{t+1}^P} = \frac{\int_{\Omega_t} \tau_{it}^{-\frac{\eta}{1-\eta}} \omega_{it}^{-\frac{\alpha\eta}{1-\eta}} n_{it} \mathrm{d}i}{\left(\int_{\Omega_t} \tau_{it}^{-\frac{1}{1-\eta}} \omega_{it}^{-\frac{1-(1-\alpha)\eta}{1-\eta}} n_{it} \mathrm{d}i \right)^{\alpha\eta} \left(\int_{\Omega_t} \tau_{it}^{-\frac{1}{1-\eta}} \omega_{it}^{-\frac{\alpha\eta}{1-\eta}} n_{it} \mathrm{d}i \right)^{(1-\alpha)\eta}}. \tag{13}$$

That is, the productivity losses from misallocation are a function of the distribution of risk and credit wedges weighted, as in Hopenhayn (2014), by the efficient firm size, n_{it}. We note that when the wedges are equalized across firms, i.e., $\tau_{it} = \tau_t$ and $\omega_{it} = \omega_t$, $Z_{t+1} = Z_{t+1}^P$ and productivity is maximized. Dispersion in wedges reduces Z_{t+1} below Z_{t+1}^P, leading to productivity losses from the misallocation of inputs.

We first summarize the distribution of risk and credit wedges in Table 8. The first two columns report percentiles of the two distributions, weighted by efficient firm size. The risk wedge is larger and more dispersed than the credit wedge, ranging from 1.27 at the 10^{th} percentile to 1.61 at the 90^{th} percentile. In contrast, the credit wedge is equal to one, except at the very top–the 90^{th} percentile is 1.02. This does not imply that all firms in our economy experience very large risk distortions or are unconstrained. As the unweighted distributions in the last two columns show, the risk wedge is only equal to 1.12 at the 90^{th} percentile, while the credit wedge is positive for more than half of the firms. Risk wedges are thus important precisely for high-ability entrepreneurs who have a large efficient scale. In contrast, many of the firms that are collateral-constrained are relatively unproductive and have a low efficient scale. To understand why this is the case, recall from Figure 4 that the credit wedge is high for relatively poor entrepreneurs, but quickly falls to one. In contrast, the risk wedge decays much more slowly with wealth. Thus, as productive entrepreneurs accumulate wealth, they quickly overcome collateral constraints but remain exposed to risk.

We next report the productivity losses from misallocation in the top row of Table 9. Aggregate productivity is 10.8% lower relative to the efficient one. We then ask: how much of these losses are due to risk versus credit constraints? To isolate the role played by risk, the second column of the table reports the aggregate productivity losses implied by equation (13) when we eliminate the credit wedge by setting $\omega_{it} = 1$. To isolate the role played by credit

Table 8: The Distribution of Risk and Credit Wedges

	weighted		unweighted	
	risk	credit	risk	credit
10^{th} pct.	1.27	1.00	1.03	1.00
25^{th} pct.	1.36	1.00	1.04	1.00
50^{th} pct.	1.46	1.00	1.05	1.02
75^{th} pct.	1.55	1.00	1.08	1.05
90^{th} pct.	1.61	1.02	1.12	1.08

Notes: The table reports percentiles of distribution of risk wedges, τ_{it}, and credit wedges, ω_{it}, weighted by efficient firm size, n_{it}, in the first two columns and unweighted in the last two columns.

Table 9: Aggregate Implications

	total	due to risk	due to credit
misallocation, $-\log Z/Z^P$	0.108	0.110	0.002
labor wedge, τ^L	1.128	1.117	1.000
capital wedge, τ^K	1.149	1.117	1.011
output losses, $-\log Y/Y^P$	0.158	0.154	0.004
wage losses, $-\log W/W^P$	0.278	0.264	0.004

Notes: The table reports aggregate productivity, output and wage losses relative to the efficient allocation, and the aggregate labor and capital wedges, τ^L and τ^K. In the second and third columns, these objects are calculated under the assumption that $\omega_{it} = 1$ and $\tau_{it} = 1$, respectively.

constraints, the third column reports the aggregate productivity losses when we eliminate the risk wedge by setting $\tau_{it} = 1$ in equation (13). As the table shows, the aggregate productivity losses due to risk are 11%, as large as the overall losses. In contrast, the aggregate productivity losses due to credit constraints are small, only 0.2%.

4.2 Aggregate Wedges

In addition to distorting aggregate productivity, financial frictions depress the overall demand for labor and capital. We can summarize these effects by computing the wedges between the aggregate marginal product of capital and labor and their respective prices. Letting the aggregate labor wedge, τ_t^L, and the aggregate capital wedge, τ_t^K, be implicitly defined by

$$(1-\alpha)\,\eta\frac{Y_{t+1}}{L_{t+1}} = \tau_t^L W \qquad \text{and} \qquad \alpha\eta\frac{Y_{t+1}}{K_{t+1}} = \tau_t^K R,$$

we can show that these wedges are given by

$$\tau_t^L = \frac{\int_{\Omega_t} \tau_{it}^{-\frac{\eta}{1-\eta}} \omega_{it}^{-\frac{\alpha\eta}{1-\eta}} n_{it}\mathrm{d}i}{\int_{\Omega_t} \tau_{it}^{-\frac{1}{1-\eta}} \omega_{it}^{-\frac{\alpha\eta}{1-\eta}} n_{it}\mathrm{d}i} \qquad \text{and} \qquad \tau_t^K = \frac{\int_{\Omega_t} \tau_{it}^{-\frac{\eta}{1-\eta}} \omega_{it}^{-\frac{\alpha\eta}{1-\eta}} n_{it}\mathrm{d}i}{\int_{\Omega_t} \tau_{it}^{-\frac{1}{1-\eta}} \omega_{it}^{-\frac{1-(1-\alpha)\eta}{1-\eta}} n_{it}\mathrm{d}i},$$

and also depend on the weighted distribution of risk and collateral wedges. Table 9 shows that the aggregate wedges are large: they are akin to a tax on labor of 12.8% and on capital of 14.9%. Once again, these distortions are primarily driven by risk rather than collateral constraints. When we isolate the role of risk by setting $\omega_{it} = 1$ in the expressions for τ_t^L and τ_t^K, the implicit tax on labor and capital falls to only 11.7%. In contrast, when we isolate the role of credit constraints by setting $\tau_{it} = 1$, the implicit tax on capital is very small, 1.1%, and labor is undistorted.

4.3 Output and Wages

We gauge the implications of financial frictions for aggregate output and wages by comparing the aggregate output, Y_{t+1}, and the equilibrium wage, W, in our baseline economy to the efficient level of output, Y_{t+1}^P, and the associated marginal product of labor. In keeping with the assumption that the planner cannot change who becomes an entrepreneur, the efficient level of output is attained by a planner who chooses the amount of capital, K_{t+1}^P, to maximize the discounted value of output and undepreciated capital, net of the investment cost,

$$-K_{t+1}^P + \frac{1}{1+r}\left(Z_{t+1}^P\left(\left(K_{t+1}^P\right)^\alpha L_{t+1}^{1-\alpha}\right)^\eta + (1-\delta)\,K_{t+1}^P\right),$$

taking as given the aggregate labor supply, L_{t+1}, and the efficient level of productivity, Z_{t+1}^P, derived in Section 4.1. The solution to this problem implies

$$Y_{t+1}^P = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} \left(Z_{t+1}^P\right)^{\frac{1}{1-\alpha\eta}} L_{t+1}^{\frac{(1-\alpha)\eta}{1-\alpha\eta}}.$$

In constrast, output in our model is equal to

$$Y_{t+1} = \left(\frac{\alpha\eta}{\tau_t^K R}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} Z_{t+1}^{\frac{1}{1-\alpha\eta}} L_{t+1}^{\frac{(1-\alpha)\eta}{1-\alpha\eta}},$$

so the output losses from financial frictions,

$$\frac{Y_{t+1}}{Y_{t+1}^P} = \left(\tau_t^K\right)^{-\frac{\alpha\eta}{1-\alpha\eta}} \left(\frac{Z_{t+1}}{Z_{t+1}^P}\right)^{\frac{1}{1-\alpha\eta}},$$

are due to lower productivity and a lower capital intensity induced by the capital wedge.

The aggregate labor wedge implies that the wage in our economy is lower than the marginal product of labor. That is,

$$W = (1-\alpha)\,\eta \frac{Y_{t+1}}{L_{t+1}} \frac{1}{\tau_t^L}.$$

Letting W^P denote the marginal product of labor under the planner's allocation, the wage losses from financial frictions are further magnified by the labor wedge, so

$$\frac{W}{W^P} = \left(\tau_t^L\right)^{-1} \left(\tau_t^K\right)^{-\frac{\alpha\eta}{1-\alpha\eta}} \left(\frac{Z_{t+1}}{Z_{t+1}^P}\right)^{\frac{1}{1-\alpha\eta}}.$$

The last two rows of Table 9 quantify these losses. Output is 15.8% lower in our economy relative to the efficient allocation. The wage losses are nearly twice as large, 27.8%, reflecting the large labor distortions. Once again, the bulk of these losses is due to risk, which generates output and wage losses of 15.4% and 26.4%, respectively. In contrast, collateral constraints generate output and wage losses of only 0.4%.

4.4 Extensive Margin Distortions

We have so far focused on the intensive margin distortions by restricting the planner to reallocate labor and capital only among the households who are entrepreneurs in the baseline economy. We now allow the planner to also choose who becomes an entrepreneur. Specifically, the planner chooses the ability cutoff, $\bar{z}$, such that all agents $i \in \Omega_t^P = \{i : z_{it} \geq \bar{z}\}$ become entrepreneurs in period $t+1$. The planner allocates the resulting labor supply

$$L_{t+1}^P = 1 - (1-\phi) \int_{\Omega_t^P} \mathrm{d}i$$

and capital, K_{t+1}^P, efficiently, so aggregate productivity is

$$Z_{t+1}^P = \left(\int_{\Omega_t^P} \left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)\right)^{\frac{1}{1-\eta}} \mathrm{d}i \right)^{1-\eta}.$$

In choosing the cutoff $\bar{z}$ the planner recognizes that a lower $\bar{z}$ increases the entrepreneurship rate and therefore aggregate productivity, but it comes at the cost of a lower labor supply.

We note that these allocations coincide with the equilibrium allocations that would prevail in the absence of financial frictions discussed in Section 3.2.

We find that the extensive margin distortions are small. The planner reduces the entrepreneurship rate from 13.2% to 1.2%. However, since the marginal entrepreneurs have low ability, fixing the extensive margin distortions leads to small gains in the aggregate. The productivity, output and wage losses in our baseline model relative to the unconstrained planner are 10.8%, 16% and 27.8%, respectively, very close to those in Table 9.

4.5 On The Role of Credit Constraints

We next show that our result–that uninsurable risk, not credit constraints, is responsible for most of the production losses–does not hinge on the exact functional form or parameterization of the collateral constraint. To that end, we consider two alternative economies. In the first, entrepreneurs cannot borrow at all, providing an upper bound on how important credit constraints may be. In the second, there is no limit to how much entrepreneurs may borrow to finance capital and the collateral constraint does not distort production choices at all, making this lower bound on how important credit constraints may be.

Table 10 reports, for each of these economies, the distribution of risk and credit wedges, measures of production distortions and their aggregate effects on productivity, output and wages. Consider first the economy with no borrowing. As Panel A shows, though the credit wedges, ω_{it}, become larger and more dispersed compared to our baseline economy, they remain smaller and less dispersed than the risk wedges. Consequently, as Panel B shows, most of the losses from misallocation are, once again, due to the risk and not the credit wedge (8.3% vs 1.4%). Moreover, the overall losses from misallocation increase from 10.8% in our baseline to only 11.7%. Thus, completely eliminating firms' ability to borrow has modest aggregate effects, as further illustrated in Panel C by the small changes in productivity, output and wages relative to the baseline.

Consider next the economy without a limit on firm credit. As Panel A of Table 10 shows, the risk wedges are the only source of distortions and they are nearly as large and dispersed as in our baseline economy. Consequently, as Panels B and C show, removing credit frictions has negligible effects on misallocation, productivity, output and wages.

Thus, the extent to which firms can borrow to finance capital has little effect on macroeconomic variables. Intuitively, the presence of risk leads entrepreneurs to operate at a small scale and to accumulate wealth for precautionary reasons. Thus, they can self-finance most

Table 10: The Role of Collateral Constraints

A. Weighted Distribution of Wedges

	no borrowing		no credit limit	
	risk	credit	risk	credit
10^{th} pct.	1.21	1.06	1.28	1.00
50^{th} pct.	1.36	1.09	1.46	1.00
90^{th} pct.	1.48	1.13	1.63	1.00

B. Aggregate Production Distortions

	no borrowing			no credit limit		
	total	due to risk	due to credit	total	due to risk	due to credit
misallocation	0.117	0.083	0.014	0.109	0.109	0.000
labor wedge	1.115	1.124	1.000	1.131	0.131	1.000
capital wedge	1.233	1.124	1.439	1.131	0.131	1.000

C. Productivity, Output and Wages

	no borrowing	no credit limit
$\Delta \log Z$, rel. to baseline	-0.009	0.000
$\Delta \log Y$, rel. to baseline	-0.025	0.003
$\Delta \log W$, rel. to baseline	-0.013	0.001

Notes: Panel A reports the weighted distribution of risk and credit wedges in two economies: one with $\xi = 0$ ("no borrowing") and one with $\xi = 1$ ("no credit limit"). Panel B reports the losses from misallocation and aggregate wedges. Panel C reports productivity, output and wages relative to the baseline economy.

of their desired capital, which keeps credit wedges small, even in an economy without credit.

5 Why Is Risk So Important?

In our baseline model, we introduced three ingredients that are necessary to reproduce the patterns in the data: fat-tailed shocks to productivity, transitory shocks to productivity and the assumption that labor inputs are chosen before productivity shocks are realized. We next show that each of these ingredients is key to our conclusion that uninsurable risk drives most macroeconomic losses from financial frictions. To that end, we consider three counterfactual economies in which we eliminate each ingredient in isolation. We re-estimate each of these economies and revisit our decomposition of productivity, output and wage losses between risk and credit wedges. In contrast to our baseline model, in each of these counterfactual economies, the importance of the risk wedges falls substantially while that of credit wedges rises. In fact, credit wedges emerge as the primary driver of these losses.

5.1 Role of Fat-Tailed Shocks

We first analyze a counterfactual economy where $s = p = 1$, so both the persistent and transitory productivity shocks are drawn from normal distributions. Tables 11 and 12 report the targeted moments and the estimated parameters. As discussed in Section 2, the tails of the shock distribution are primarily identified by the extent to which the inter-quartile range of output growth is lower than its standard deviation. We therefore use an objective function that assigns a weight of zero to the inter-quartile range moments.[15] As Table 11 shows, the model reproduces the targeted moments well, but implies values for the inter-quartile range that are much larger than in the data. Since the absence of fat-tailed shocks limits downside risk, the model requires a higher discount factor to match entrepreneurial wealth and a lower span of control to match the profit share.

The second column of Table 13 shows that removing fat-tailed shocks reverses our conclusion regarding the relative importance of risk and credit constraints for macroeconomic aggregates. The losses from misallocation fall significantly, from 10.8% in the baseline to 2.1%, with credit constraints now accounting for the majority (1.5%), compared to just 0.3% for risk. We observe a similar pattern for the wage and output losses: these are much smaller and a smaller share is accounted for by risk.

5.2 Role of Transitory Shocks

We next analyze a counterfactual economy where $\sigma_\varepsilon = 0$, so that there are no transitory shocks to productivity. Tables 11 and 12 report the targeted moments and the estimated parameters. As discussed in Sections 2 and 3.3, transitory shocks are primarily identified by how quickly the auto-correlation of output decays with the horizon, how rapidly the volatility of changes in output increases with the horizon, and the extent to which individual firms' labor shares fluctuate over time. Consequently, we do not target the autocorrelation and

[15]In Appendix B, we report results from an alternative estimation strategy, where we use the same weights as in the baseline, thus targeting all moments. Our conclusions below are robust under this approach.

Table 11: Targeted Moments: Alternative Models

	data	no fat tails	no transitory shocks	flexible labor
fraction entrepr	0.12	0.12	0.11	0.12
wealth to income entrepr	12.5	12.5	12.4	12.5
capital-output ratio, k/y	1.22	1.22	1.25	1.22
labor share, wl/y	0.72	0.73	0.74	0.72
profit share, π/y	0.13	0.13	0.11	0.14
iqr $wl_{it}/y_{it} - \overline{wl_{it}/y_{it}}$	0.15	0.15	*0.04*	0.00
s.d. $\log y_{it}$	1.32	1.32	1.32	1.32
s.d. $\log y_{it}/y_{it-1}$	0.48	0.42	0.46	0.50
s.d. $\log y_{it}/y_{it-2}$	0.60	0.60	*0.70*	0.59
s.d. $\log y_{it}/y_{it-3}$	0.69	0.74	*0.90*	0.68
iqr $\log y_{it}/y_{it-1}$	0.32	*0.47*	0.34	0.32
iqr $\log y_{it}/y_{it-2}$	0.46	*0.71*	*0.61*	0.45
iqr $\log y_{it}/y_{it-3}$	0.58	*0.89*	*0.86*	0.58
corr $\log y_{it}, \log y_{it-1}$	0.93	0.95	0.94	0.93
corr $\log y_{it}, \log y_{it-2}$	0.89	0.90	*0.86*	0.90
corr $\log y_{it}, \log y_{it-3}$	0.86	0.85	*0.78*	0.87
value of objective	–	0.053	0.056	0.041

Notes: The table reports the fit of the estimation procedure across counterfactual models. We do not target the italicized moments, but report their implied values at the estimated parameters. The last row reports the value of the original objective in equation (12). For reference, this is equal to 0.013 in the baseline estimation.

volatility of output changes beyond a one-year horizon, nor labor share deviations from their time-series means.[16] As Table 11 shows, the model matches the targeted moments well, but overstates the decline in the output autocorrelation and the increase in the volatility of output growth with the horizon. Moreover, it predicts much smaller fluctuations in firms' labor shares. Since entrepreneurs now face less risk, the model requires a higher discount factor than in the baseline to match the entrepreneurial wealth and a lower span of control to match the profit share. Additionally, the model requires a lower autocorrelation for the persistent component of productivity to match the autocorrelation of output.

[16] In Appendix B we report results from an alternative estimation strategy where we target all moments and show that our conclusions below are robust.

Table 12: Parameter Values: Alternative Models

		no fat tails	no transitory shocks	flexible labor
β	discount factor	0.956	0.971	0.955
α	capital elasticity	0.185	0.213	0.205
η	span of control	0.926	0.950	0.904
ρ	persistence z	0.979	0.940	0.980
σ_u	volatility z	0.053	0.028	0.017
σ_ε	volatility ε	0.148	–	0.028
s	relative volatility mixture	–	6.919	10.37
p	baseline probability mixture	–	0.911	0.910
ϕ	relative time endowment	0.962	0.968	0.944

Notes: The table reports the parameter estimates across the three alternative models.

Table 13: Aggregate Implications: Alternative Models

	baseline	no fat tails	no transitory shocks	flexible labor
misallocation, $-\log Z/Z^P$				
total	0.108	0.021	0.121	0.033
due to risk	0.110	0.003	0.004	0.001
due to credit	0.002	0.015	0.109	0.024
output losses, $-\log Y/Y^P$				
total	0.158	0.057	0.229	0.096
due to risk	0.154	0.012	0.013	0.024
due to credit	0.004	0.046	0.224	0.076
wage losses, $-\log W/W^P$				
total	0.278	0.093	0.243	0.096
due to risk	0.264	0.052	0.045	0.024
due to credit	0.004	0.046	0.224	0.076

Notes: The table reports the misallocation, output and wage losses across counterfactual models.

The third column of Table 13 shows that in this economy, macroeconomic losses from financial frictions remain substantial but, importantly, are primarily driven by credit constraints rather than risk. For example, the losses from misallocation are 12.1%, larger than the 10.8% in the baseline. Credit constraints alone generate productivity losses of 10.9%, while risk alone only generates losses of 0.4%.

The role of risk diminishes because profit shares fluctuate less. Meanwhile, credit constraints become more important because lower risk encourages firms to operate at a larger scale, thus making them more likely to hit the collateral constraint. Additionally, the reduced persistence of productivity prevents productive firms from self-financing, further amplifying the impact of collateral constraints (Midrigan and Xu, 2014, Moll, 2014).

5.3 Role of Labor Chosen Before Productivity Is Realized

Lastly, we analyze a counterfactual economy where labor is chosen after productivity shocks are realized. Appendix B characterizes this economy. Here, we highlight that the credit wedge remains defined as in the baseline model, while the risk wedge is

$$\tau_{it} = \left(1 + \frac{\mathbb{COV}_{it}\left(c_{it+1}^{-\theta}, \left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right)}{\mathbb{E}_{it}c_{it+1}^{-\theta}\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}\right)^{-1},$$

where the productivity term, $z_{it+1}\varepsilon_{it+1}$, is raised to the power $1/\left(1-(1-\alpha)\eta\right)$ reflecting that labor can respond ex-post to productivity shocks.

We estimate this model using the same strategy as in the baseline. Tables 11 and 12 report the targeted moments and the estimated parameters. While the model matches the targeted moments well, it fails to generate fluctuations in firms' labor shares around their time-series means because the labor share is constant in this setting.

As the last column of Table 13 shows, removing the labor friction reduces the macroeconomic losses, which are now primarily driven by credit constraints.[17] The productivity losses fall to 3.3%, much lower than the 10.8% in the baseline. Risk alone generate losses of only 0.1%, while credit constraints generate losses of 2.4%. Intuitively, risk now only distorts the capital choice. Since the capital share in production is smaller than the labor share, the assumption that only capital is subject to time-to-build reduces business income volatility. This, in turn, makes it optimal for firms to expand, increasing the role of credit constraints.

[17] These losses are calculated relative to the allocations achievable by a planner who can also choose labor after the realization of productivity shocks. Appendix B provides further details on this planner's problem.

To conclude, removing either of these three key ingredients substantially reduces the role of risk wedges because models without these features predict much less downside business income risk. For example, in the three counterfactual economies, the first percentile of firm profit share deviations from their time-series mean ranges from -0.21 to -0.43, far smaller in absolute value than in the data (-1.66) or the baseline model (-1.92). Thus, accurately assessing the impact of risk on production requires a model that replicates the observed fluctuations in firm profit shares over time.

6 Implications for Profits and Returns

Beyond its implications for misallocation and production distortions, our findings have implications for two other prominent lines of work. The first studies the distribution of income between labor, capital and rents. This literature decomposes firm profits between economic profits from limited span of control or markups, unmeasured inputs or financial frictions (Farhi and Gourio, 2018, Karabarbounis and Neiman, 2019).[18] Our model predicts that profits primarily reflect compensation for risk. The second studies the distribution of returns to wealth (Fagereng et al., 2020, Bach et al., 2020, Smith et al., 2022) and its role for wealth inequality (Benhabib et al., 2011, Benhabib et al., 2017, Halvorsen et al., 2022). This literature highlights an important role for differences in returns to private business wealth. Our model predicts that these returns largely reflect compensation for risk, rather than collateral constraints, in contrast to what workhorse models of entrepreneurship that abstract from the three ingredients we discussed in Section 5 predict.

To illustrate these points, we begin by describing how risk and credit wedges influence profit shares and rates of return. Since part of profit dispersion stems from ex-post productivity shocks, we focus on expected profits, which filter out these fluctuations. An entrepreneur's expected profits are given by

$$\mathbb{E}_{it}\pi_{it+1} = \mathbb{E}_{it}\left(y_{it+1} - Wl_{it+1} - Rk_{it+1}\right),$$

or, using the optimality conditions for labor and capital,

$$\mathbb{E}_{it}\pi_{it+1} = (1-\eta)\,\mathbb{E}_{it}y_{it+1} - \eta\frac{\mathbb{COV}_{it}\left(c_{it+1}^{-\theta},\, y_{it+1}\right)}{\mathbb{E}_{it}c_{it+1}^{-\theta}} + \mu_{it}a_{it+1}.$$

In a frictionless setting, expected profits would be a constant fraction, $1-\eta$, of expected output. Financial frictions affect profits through two channels, captured by the last two

[18]See also Smith et al. (2019) who study whether pass-through business profits accrue to human capital, in addition to financial capital.

Table 14: Implications for Profits and Returns

	total	due to risk	due to credit	due to span of control
profit shares				
mean	0.146	0.110	0.003	0.034
standard deviation	0.055	0.056	0.010	–
excess returns				
mean	0.022	0.022	0.001	–
standard deviation	0.055	0.054	0.008	–

Notes: The first column reports the average and standard deviation of profit shares and excess returns. The subsequent columns report the contributions of risk, collateral constraints and limited span of control by setting the wedges ω_{it} and τ_{it} to one, in turn. We weight firms by expected output when computing profit shares and by wealth when computing excess returns.

terms. First, the negative covariance between the marginal utility of consumption and output captures the risk premium. Second, the multiplier μ_{it} captures the collateral constraint. Using the definitions of risk and credit wedges, τ_{it} and ω_{it}, from equations (7) and (9), we can write a firm's profit share as

$$\frac{\mathbb{E}_{it}\pi_{it+1}}{\mathbb{E}_{it}y_{it+1}} = \underbrace{1-\eta}_{\text{span of control}} + \underbrace{\eta\left(1-\frac{1}{\tau_{it}}\right)}_{\text{risk}} + \underbrace{\frac{1}{1-\xi}\left(\omega_{it}-1\right)R\frac{a_{it+1}}{\mathbb{E}_{it}y_{it+1}}}_{\text{credit constraints}}.$$

This expression allows us to decompose profit shares into the parts arising from limited span of control, risk and credit constraints.

The top of Table 14 reports the mean and standard deviation of profit shares, both overall and for each of the three components. Risk accounts for 75% of the aggregate profit share (0.11/0.146), with the remaining share attributed to the span of control. Risk also drives most of the variation in profit shares. The standard deviation of profit shares is 5.5%, as large as when we only account for risk wedges. By contrast, credit wedges contribute little to dispersion, generating a standard deviation of just 1%.

We next discuss the model's implication for rates of return on wealth in excess of the risk-free rate. An entrepreneur's expected return to saving an additional unit of wealth is

$$r_{it} = r + \frac{\partial \mathbb{E}_{it}\pi_{it+1}}{\partial a_{it+1}},$$

which consists of the risk-free rate, r, and an excess return that captures the effect of the additional unit of wealth on expected profits. This excess return is equal to

$$\frac{\partial \mathbb{E}_{it}\pi_{it+1}}{\partial a_{it+1}} = \left(\alpha\eta\mathbb{E}_{it}\frac{y_{it+1}}{k_{it+1}} - R\right)\frac{\partial k_{it+1}}{\partial a_{it+1}} + \left((1-\alpha)\,\eta\mathbb{E}_{it}\frac{y_{it+1}}{l_{it+1}} - W\right)\frac{\partial l_{it+1}}{\partial a_{it+1}}.$$

The two terms on the right-hand side represent the difference between the expected marginal product of each factor and its price, multiplied by the marginal impact of wealth on the firm's optimal choice of that factor. Using the definitions of risk and credit wedges, τ_{it} and ω_{it}, we can express the excess return as

$$\frac{\partial \mathbb{E}_{it}\pi_{it+1}}{\partial a_{it+1}} = (\tau_{it}\omega_{it} - 1)\,R\frac{\partial k_{it+1}}{\partial a_{it+1}} + (\tau_{it} - 1)\,W\frac{\partial l_{it+1}}{\partial a_{it+1}}.$$

This decomposition separates the effects of risk and credit constraints, which can be isolated by setting ω_{it} and τ_{it} equal to one, respectively.

The bottom of Table 14 shows that the average excess return of 2.2% is almost entirely accounted for by risk. Similarly, risk accounts for most of the dispersion in excess returns: their standard deviation is 5.5%, nearly as large as that accounted for by risk alone.

7 Robustness

We next show that our conclusion that uninsurable business risk has important macroeconomic costs is robust to reducing the coefficient of risk aversion and to assuming that, in addition to private businesses, output is also produced by undistorted corporate firms.

7.1 Role of Risk Aversion

We first examine whether the important role we found for risk is driven by our assumption that the relative risk aversion is equal to two. To that end, we re-estimate a version of the model in which $\theta = 0.5$, at the lower end of values typically used. Tables 15 and 16 report the targeted moments and the estimated parameters. The model with lower risk aversion also matches the targeted moments well, but requires a higher discount factor and a lower span of control than in the baseline.

As Table 17 shows, the model generates smaller–yet still sizable–aggregate losses. Productivity losses amount to 6.4%, output losses to 9.2% and wage losses to 16.7%–approximately three-fifths of the losses in the baseline. Importantly, as in the baseline, these losses are primarily driven by risk rather than collateral constraints. Thus, while households' attitudes

Table 15: Targeted Moments: Robustness

	data	lower risk aversion	corporate firms
fraction entrepr	0.12	0.13	0.13
wealth to income entrepr	12.5	12.5	12.5
capital-output ratio, k/y	1.22	1.22	1.22
labor share, wl/y	0.72	0.72	0.73
profit share, π/y	0.13	0.13	0.12
iqr $wl_{it}/y_{it} - \overline{wl_{it}/y_{it}}$	0.15	0.16	0.15
s.d. $\log y_{it}$	1.32	1.33	1.32
s.d. $\log y_{it}/y_{it-1}$	0.48	0.48	0.48
s.d. $\log y_{it}/y_{it-2}$	0.60	0.60	0.60
s.d. $\log y_{it}/y_{it-3}$	0.69	0.70	0.70
iqr $\log y_{it}/y_{it-1}$	0.32	0.30	0.28
iqr $\log y_{it}/y_{it-2}$	0.46	0.46	0.46
iqr $\log y_{it}/y_{it-3}$	0.58	0.60	0.61
corr $\log y_{it}$, $\log y_{it-1}$	0.93	0.93	0.93
corr $\log y_{it}$, $\log y_{it-2}$	0.89	0.90	0.90
corr $\log y_{it}$, $\log y_{it-3}$	0.86	0.87	0.86
value of objective	–	0.005	0.007

Notes: The table reports the moments in the economies with $\theta = 0.5$ and with corporate firms.

towards risk indeed shape macroeconomic aggregates, the role of risk remains both significant and dominant, even for moderate levels of risk aversion.

7.2 Corporate Firms

In the baseline model, we assumed that all production is carried out by private businesses. Here, we extend the framework to include financially unconstrained corporate firms alongside private businesses. Although assuming corporate firms are entirely unconstrained may seem inconsistent with findings from the corporate finance literature (e.g. Hennessy and Whited, 2007, Ottonello and Winberry, 2020), this assumption allows us to recover a lower bound on the aggregate impact of financial frictions. We find that the macroeconomic losses are only slightly lower than in our baseline and, as before, mostly driven by risk.

Table 16: Parameter Values: Robustness

		lower risk aversion	corporate firms
β	discount factor	0.971	0.971
α	capital elasticity	0.171	0.169
η	span of control	0.934	0.963
ρ	persistence z	0.979	0.980
σ_u	volatility z	0.012	0.008
σ_ε	volatility ε	0.086	0.086
s	relative volatility mixture	13.54	12.03
p	baseline probability mixture	0.933	0.913
ϕ	relative time endowment	0.970	0.993

Notes: The table reports the parameter estimates in the economies with $\theta = 0.5$ and with corporate firms.

Table 17: Aggregate Implications: Robustness

	baseline	lower risk aversion	corporate firms
misallocation, $-\log Z/Z^P$			
total	0.108	0.064	0.105
due to risk	0.110	0.064	0.101
due to credit	0.002	0.000	0.001
output losses, $-\log Y/Y^P$			
total	0.158	0.092	0.135
due to risk	0.154	0.090	0.129
due to credit	0.004	0.001	0.003
wage losses, $-\log W/W^P$			
total	0.278	0.167	0.172
due to risk	0.264	0.164	0.168
due to credit	0.004	0.001	0.003

Notes: The table reports the misallocation, output and wage losses in the model with relative risk aversion $\theta = 0.5$ and in the model with a corporate sector.

A representative corporate firm with productivity Z_c produces using the same technology as private businesses,

$$Y_c = Z_c \left(K_c^{\alpha} L_c^{1-\alpha}\right)^{\eta},$$

but faces no frictions in hiring capital and labor.[19] In this setting, misallocation arises from two sources: within the entrepreneurial sector, as in the baseline, and across sectors. Because corporate firms face no financial frictions, this sector is too large relative to the efficient allocations. For example, the employment share of corporate firms is

$$n_c^l = \frac{Z_c^{\frac{1}{1-\eta}}}{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{1-\alpha\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}},$$

where τ^L, $\tau^K > 1$ represent the labor and capital wedges in the entrepreneurial sector, as defined in Section 4.2. The presence of corporate firms reduces overall capital and labor wedges relative to the baseline. For example, the economy-wide labor wedge is

$$n_c^l + \tau^L \left(1 - n_c^l\right),$$

which is smaller than the labor wedge τ^L in the entrepreneurial sector.

We estimate this model using the same strategy as in the baseline. For any given set of parameters, we set the productivity of corporate firms, Z_c, so that the model exactly matches the 58.7% output share of public firms in Orbis. Tables 15 and 16 report the estimation results. The model with corporate firms matches the data well, with similar parameter values as in the baseline. The only notable difference is that this model requires a higher discount factor. This is because the presence of unconstrained corporate firms bids up the wage, reducing entrepreneurs' optimal firm size and their incentives to accumulate wealth to overcome financial frictions.

Table 17 reports the economy-wide productivity, output and wage losses from financial frictions. Productivity and output losses are only slightly smaller than in the baseline economy. For example, misallocation losses amount to 10.5%, very similar to the 10.8% in the economy without corporate firms. As discussed above, the presence of corporate firms reduces the aggregate labor wedge, leading to smaller wage losses–three-fifths of the baseline. Most importantly, as in the baseline economy without corporate firms, macroeconomic losses are primarily driven by risk rather than credit constraints. For example, risk generates losses from misallocation of 10.1%, whereas credit constraints generate losses of only 0.1%.

[19] See Appendix B for a detailed description of this economy.

8 Conclusion

In this paper, we argue that private business owners' inability to diversify the risk associated with their business has significant macroeconomic consequences–much larger than those stemming from limited access to credit. We build this argument by leveraging microdata from Orbis and a model of entrepreneurial dynamics. We document that firms experience large fluctuations in their profit shares that are due to large, fat-tailed and transitory changes in output that are not fully accompanied by changes in their inputs. We interpret this evidence using a model consistent with these facts. In the model, firms can reduce their exposure to risk by operating at a smaller scale. This leads to large losses from misallocation and inefficiently low levels of aggregate output and wages. The bulk of these losses are accounted for by risk and not by credit frictions that firms face.

Our findings have important implications for policy design. For example, an immediate implication is that policies that increase firms' access to credit would have a limited impact, unless they also improve risk sharing. More broadly, while a large body of work studies redistributive wealth and income tax policies in economies with credit-constrained private businesses,[20] broadening the scope of this work to include a large role for distortions stemming from uninsurable risk, as does Di Tella et al. (2024), is an exciting avenue for future work.

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[20]See, for example, Guvenen et al. (2023), Brüggemann (2021), Itskhoki and Moll (2019), Boar and Knowles (2024) and Boar and Midrigan (2023).

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Online Appendix
Not for Publication

A Data

We use two datasets: Orbis and the Spanish Survey of Household Finances. In this section we describe each of them and show that our motivating facts in Section 2 hold for countries other than Spain.

A.1 Orbis

The main data source we use is the historical product of Orbis available at the NBER. Orbis is a large firm-level data set compiled by Moody's Bureau van Dijk Electronic Publishing (BvD). Bvd collects data from various sources, such as national business registries, and harmonizes them into an internationally comparable format. The Orbis data contain annual balance sheet and income statements together with information on the use of inputs and detailed industry identifiers for both private and public firms.

A.1.1 Variable Definition

Our analysis uses the following variables:

1. **Capital**: we define physical capital as the sum of tangible assets (`TFAS`) and intangible assets (`IFAS`)

   `Capital = TFAS + IFAS.`

2. **Equity**: we define equity as the difference between total assets (`TOAS`) and total liabilities (`CULI`+`NCLI`)

   `Equity = TOAS - (CULI + NCLI).`

3. **Wage bill**: we define the wage bill as the firm's total wage bill, inclusive of pension costs (`STAF`).

4. **Output**: we define output as value added, which we compute as the sum of after-tax profit and losses (`PLAT`), depreciation (`DEPR`), wage bill (`STAF`), net of financial profit and losses (`FIPL`). Because Orbis defines `PLAT` as the sum of operating revenues (`OPRE`) and financial profit and losses (`FIPL`), net of the costs of goods sold (`COST`), other operating expenses (`OOPE`) and taxes (`TAXA`), our definition of output amounts to

   `Output = PLAT + DEPR - FIPL + STAF = OPRE - COST - OOPE - TAXA + DEPR + STAF.`

Because the two cost measures include the cost of labor as well as depreciation, we add these to the definition of output in order to capture the difference between operating revenues and the cost of production, excluding the cost of labor and depreciation.

5. **Profit**: we define profit in a way that is consistent with our model as output net of labor costs and the user cost of capital. We thus calculate profits in the Orbis data as

   $$\texttt{Profit = PLAT - FIPL - 0.02} \times \texttt{Capital = Output - STAF - DEPR - 0.02} \times \texttt{Capital}$$

   where the last term assumes an interest rate of 2%, as in the model, to calculate the interest cost of capital.

6. **Debt**: since our notion of debt in the model is financial liabilities net of financial assets, we define debt as the difference between capital `Capital` and firm equity `Equity`, whenever this difference is positive. Otherwise, we set it equal to zero.

   We deflate nominal variables using country-specific CPI deflators from the World Bank World Development Indicators, using 2015 as the base year.

A.1.2 Sample Selection

Given our interest in private businesses, we restrict attention to partnerships and private limited companies. Most of the private firms in Spain are private limited companies ("Sociedad de Responsabilidad Limitada" or S.L.). This is a separate legal entity, so its owners are only liable for the company's debt up to the amount of their capital contribution. A private limited company must have at least one shareholder, but there is no maximum limit. The remaining, much smaller share of firms in the sample, are general partnerships ("Sociedad Colectiva" or S.C.) whose owners face unlimited personal liability for the company's debt. A general partnership must have at least two partners, but there is no maximum limit.

We exclude firms that operate in finance, insurance and real estate, public administration, defense and education. Additionally, we exclude firms with negative values of output, capital, wage bill and depreciation, as well as firms with missing values of output capital, wage bill, equity, profits and depreciation. To minimize the concern that variables are measured with error, we exclude observations in the top and bottom 0.1% of the distribution of the debt-to-capital ratio, the capital-to-output ratio, the wage bill-to-output ratio, the profit-to-output ratio, as well as the annual growth rate of output, capital and the wage bill. Lastly, because we compute growth rates over horizons of up to 3 years, we restrict attention to firms that have at least 4 years of data. Table A.1 reports the impact these sample restrictions have on the sample size.

Table A.1: Sample Selection

	Number of observations	Percent of sample
Full sample	13,103,529	100
Only private firms	11,449,985	87.4
No FIRE, public admin, defense and educ	10,005,535	76.4
No negative observations	9,227,697	70.4
No missing observations	7,036,146	53.7
Drop top and bottom 0.1%	6,972,670	53.2
At least 4 years	6,298,358	48.1

Notes: The table records the number of observations and the corresponding percentage of the full sample at every step of the sample selection. An observation is a firm-year pair.

A.2 Spanish Survey of Household Finances (EFF)

The survey is conducted every three years and each wave samples approximately 6,000 households. A subset of these households is surveyed across multiple waves, creating a panel component. The sample for each wave is chosen to be representative of the population and it oversamples the wealthy to ensure that it adequately captures the distribution of wealth.

A.2.1 Variable Definition

Our analysis uses the following variables:

1. **Entrepreneur**: Dummy variable equal to one when the household owns a business ran or managed by a member of the household, and the household reports positive business wealth. Specifically, a household is considered to run or manage a business if the reference person answers YES to the survey question: "*Does your household own (even if not fully) any business run or managed by a member of your household*?"[21]. To compute the business wealth of a household, we sum over the values of all the businesses that they own.

2. **Business wealth**: This variable is provided in the EFF summary data and is constructed as the sum of the value of each business related to self-employment owned by

[21]The survey question refers to businesses not listed on the stock market and in whose management one or more household members participate directly. They may be self-employed persons, owners or members of a family business, sole proprietors of a business, fee-earning professionals, or managing partners of a non-family jointly-owned business.

the household. The value of the business is the answer to the survey question: "*What is the current value of the business after deducting any outstanding debt associated with that business?*".

3. **Wealth (net worth)**: This variable is equal to the net worth variable provided in the EFF summary data plus the value of automobiles.

4. **Total income**: This variable is provided in the EFF summary data and is the total income that the household obtained in the year previous to the survey wave.

5. **Business income**: This variable is the annual profit from the business (net of the annual losses arising from the business), for each of the businesses owned by the household. It does not include the amount received by the household as remuneration for their work. The value of this variable is the answer to the survey question: "*What is the annual profit before tax provided by this business to your household*".

6. **Consumption**: This variable is the annualized value of average monthly spending on consumer goods reported by the household. Specifically, it is the annualized value of the answer to the survey question: "*What is your household's total average spending on consumer goods in a month including food?*"

A.2.2 Sample Selection

We use data for the 2008-2020 survey waves. We restrict the sample to households in which the reference household member is between 22 and 79 years old. We convert all nominal variables in 2020 Euros.

A.3 Evidence From Other Countries

In this section we revisit the main facts reported in Section 2 for five other countries for which the Orbis data has relatively good coverage: Italy, France, Norway, Portugal and Slovakia.

Profit Shares Fluctuate Considerably. Table A.2 reports moments of the distribution of profit share deviations for each country.

Firms Experience Large, Fat-Tailed and Transitory Changes in Output. Table A.3 reports the standard deviation, inter-quartile range and the kurtosis of the distribution of output growth rates.

Table A.2: Distribution of Profit Share Deviations in Other Countries

	p5	p10	p25	p50	p75	p90	p95
Spain	-0.33	-0.17	-0.05	0.01	0.11	0.27	0.47
Italy	-0.29	-0.15	-0.05	0.01	0.10	0.26	0.45
France	-0.23	-0.13	-0.04	0.01	0.07	0.17	0.29
Norway	-0.27	-0.15	-0.05	0.01	0.09	0.20	0.32
Portugal	-0.37	-0.19	-0.05	0.02	0.13	0.31	0.54
Slovakia	-0.33	-0.17	-0.05	0.01	0.11	0.27	0.41

Notes: All statistics weighted by the time-series average of each firm's output.

Table A.3: Distribution of Output Growth Rates in Other Countries

	s.d.	iqr	kurtosis
Spain	0.48	0.32	13.7
Italy	0.53	0.33	13.2
France	0.36	0.24	16.0
Norway	0.41	0.27	16.1
Portugal	0.57	0.37	12.8
Slovakia	0.61	0.41	11.3

Notes: The table reports statistics of the distribution of output growth rates $\log y_{it}/y_{it-1}$.

Capital and Labor Do Not Track Output Closely. Table A.4 reports the slope coefficients of regressions of the growth rate of labor and capital againts the growth rate of output.

A.4 Additional Empirical Results

In this section we report additional statistics from the Orbis data. Table A.5 summarizes the distribution of output growth rates for young (age ≤ 5) and old (age > 5) firms.

Tables A.6, A.7 and A.8 report our motivating facts on the deviations of profit shares,

Table A.4: Capital, Labor and Output Comovement in Other Countries

	$\Delta \log wl$	$\Delta \log k$
Spain	0.583	0.313
	(0.001)	(0.001)
Italy	0.655	0.273
	(0.001)	(0.001)
France	0.587	0.255
	(0.001)	(0.001)
Norway	0.528	0.224
	(0.002)	(0.005)
Portugal	0.453	0.319
	(0.001)	(0.003)
Slovakia	0.483	0.337
	(0.004)	(0.006)

Notes: The sample is restricted to observations for which $|\Delta \log y| < 0.5$. Standard errors in parentheses are clustered at the firm level.

Table A.5: Distribution of Output Growth by Firm Age

	all	young	old
mean	0.012	0.110	-0.014
standard deviation	0.484	0.550	0.462
interquartile range	0.316	0.418	0.294
kurtosis	13.67	10.30	15.10

Notes: The column labeled "young" reports moments of the distribution of output growth for Spanish firms whose age is 5 years or younger. The column labeled "old" reports moments for firms that are older than 5 years.

the distribution of output growth and the comovement between output and inputs for the five largest sectors in Spain in terms of total output.

Table A.9 shows that the comovement between output and inputs is similar for small and

Table A.6: Distribution of Profit Share Deviations by Sector

	p5	p10	p25	p50	p75	p90	p95
Manufacturing	-0.35	-0.18	-0.05	0.02	0.12	0.28	0.47
Construction	-0.43	-0.19	-0.04	0.02	0.13	0.35	0.60
Wholesale and retail trade	-0.32	-0.17	-0.06	0.02	0.11	0.26	0.43
Accommodation and food	-0.31	-0.17	-0.05	0.01	0.10	0.25	0.41
Professional activities	-0.33	-0.16	-0.05	0.01	0.09	0.27	0.46

Notes: All statistics weighted by the time-series average of each firm's output.

Table A.7: Distribution of Output Growth Rates by Sector

	s.d.	iqr	kurtosis
Manufacturing	0.40	0.27	17.1
Construction	0.60	0.43	10.0
Wholesale and retail trade	0.44	0.28	16.0
Accommodation and food	0.41	0.26	17.4
Professional activities	0.55	0.36	11.1

Notes: The table reports statistics of the distribution of output growth rates $\log y_{it}/y_{it-1}$.

large firms. We define a firm to be large if it is in the top decile of the distribution of value added, and small otherwise.

B Model

In this section, we discuss in more detail the model with flexible labor in Section 5 and the extension with corporate firms in Section 7. We also report results from alternative calibration strategies of the economies discussed in Section 5.

Table A.8: Capital, Labor and Output Comovement by Sector

	$\Delta \log wl$	$\Delta \log k$
Manufacturing	0.591	0.343
	(0.002)	(0.003)
Construction	0.546	0.325
	(0.001)	(0.003)
Wholesale and retail trade	0.668	0.330
	(0.002)	(0.004)
Accommodation and food	0.580	0.258
	(0.003)	(0.005)
Professional activities	0.593	0.267
	(0.003)	(0.005)

Notes: The sample is restricted to observations for which $|\Delta \log y| < 0.5$. Standard errors in parentheses are clustered at the firm level.

Table A.9: Capital, Labor and Output Comovement by Firm Size

	$\Delta \log wl$	$\Delta \log k$
Small	0.582	0.304
	(0.001)	(0.002)
Large	0.581	0.368
	(0.002)	(0.004)

Notes: The sample is restricted to observations for which $|\Delta \log y| < 0.5$. Standard errors in parentheses are clustered at the firm level.

B.1 A Model With Flexible Labor

When labor is chosen after the realization of productivity shocks, the entrepreneur equates the marginal product of labor to the wage,

$$(1-\alpha)\,\eta z_{it+1}\varepsilon_{it+1}\left((k_{it+1})^{\alpha}\,(l_{it+1})^{1-\alpha}\right)^{\eta}\frac{1}{l_{it+1}} = W,$$

so in choosing its capital before productivity is realized it recognizes that labor is a function of the realization of productivity. Consequently, the first-order condition for capital is

$$\alpha\eta\left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}}(k_{it+1})^{\frac{\alpha\eta}{1-(1-\alpha)\eta}-1}\frac{\mathbb{E}_{it}\left(c_{it+1}\right)^{-\theta}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}{\mathbb{E}_{it}\left(c_{it+1}\right)^{-\theta}}=R+(1-\xi)\,\mu_{it},$$

so the risk wedge is a function of the covariance of consumption with $(z_{it+1}\varepsilon_{it+1})^{\frac{1}{1-(1-\alpha)\eta}}$. The risk wedge is therefore

$$\tau_{it}=\left(1+\frac{\mathbb{COV}_{it}\left(c_{it+1}^{-\theta},\,(z_{it+1}\varepsilon_{it+1})^{\frac{1}{1-(1-\alpha)\eta}}\right)}{\mathbb{E}_{it}c_{it+1}^{-\theta}\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}\right)^{-1},$$

and the collateral wedge is

$$\omega_{it}=1+(1-\xi)\,\frac{\mu_{it}}{R}.$$

To calculate the aggregate productivity losses from misallocation, we follow the same approach as in the baseline model. Since the planner can also choose labor after the realization of productivity, the share of labor allocated to each firm is a function of the predetermined amount of capital

$$n_{it}^{l}=\frac{(z_{it+1}\varepsilon_{it+1})^{\frac{1}{1-(1-\alpha)\eta}}(k_{it+1})^{\frac{\alpha\eta}{1-(1-\alpha)\eta}}}{\int(z_{it+1}\varepsilon_{it+1})^{\frac{1}{1-(1-\alpha)\eta}}(k_{it+1})^{\frac{\alpha\eta}{1-(1-\alpha)\eta}}\,\mathrm{d}i}.$$

In turn, the share of capital allocated to each firm is

$$n_{it}^{k}=\frac{\left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}}{\int\left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\mathrm{d}i}$$

and the resulting efficient level of productivity is

$$Z_{t+1}^{P}=\left(\int\left(\mathbb{E}_{it}\left(z_{it+1}\varepsilon_{it+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\mathrm{d}i\right)^{1-\eta}.$$

The productivity losses from misallocation are therefore

$$\frac{Z_{t+1}}{Z_{t+1}^{P}}=\frac{\int\left(\tau_{it}\omega_{it}\right)^{-\frac{\alpha\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i}{\left(\int\left(\tau_{it}\omega_{it}\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i\right)^{\alpha\eta}\left(\int\left(\tau_{it}\omega_{it}\right)^{-\frac{\alpha\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i\right)^{(1-\alpha)\eta}}$$

and depend on the distribution of risk and credit wedges, weighted by the efficient capital share of firms. Since labor is undistorted in this economy, the aggregate labor wedge is $\tau_t^L=1$. The aggregate capital wedge τ_t^K is

$$\tau_t^{K}=\frac{\int\left(\tau_{it}\omega_{it}\right)^{-\frac{\alpha\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i}{\int\left(\tau_{it}\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}}n_{it}^{k}\mathrm{d}i},$$

and the aggregate output wage losses satisfy

$$\frac{Y_{t+1}}{Y_{t+1}^P} = \frac{W}{W^P} = \left(\tau_t^K\right)^{-\frac{\alpha\eta}{1-\alpha\eta}} \left(\frac{Z_{t+1}}{Z_{t+1}^P}\right)^{\frac{1}{1-\alpha\eta}}.$$

B.2 A Model With Corporate Firms

We assume that private businesses produce alongside a representative corporate firm with technology

$$Y_c = Z_c \left(K_c^{\alpha} L_c^{1-\alpha}\right)^{\eta}$$

who faces no frictions and therefore sets capital equal to

$$K_c = \left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} Z_c^{\frac{1}{1-\eta}}$$

and labor equal to

$$L_c = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}} Z_c^{\frac{1}{1-\eta}},$$

so the output of the corporate sector is

$$Y_c = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} Z_c^{\frac{1}{1-\eta}}.$$

In contrast, the total capital and labor used by the entrepreneurial sector are[22]

$$K = \left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\tau^K\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}$$

and

$$L = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}} \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{1-\alpha\eta}{1-\eta}} Z^{\frac{1}{1-\eta}},$$

so output produced in the entrepreneurial sector is

$$Y = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\tau^e\right)^{-1} Z^{\frac{1}{1-\eta}}.$$

We can write therefore write an aggregate production function that expresses aggregate output $Y_a = Y_c + Y$ as a function of the total amount of capital $K_a = K_c + K$ and labor $L_a = L_c + L$ as

$$Y_a = Z_a \left(K_a^{\alpha} L_a^{1-\alpha}\right)^{\eta},$$

[22] We omit all time subscripts, as aggregate variable are time-invariant.

where aggregate productivity is

$$Z_a = \frac{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}{\left(Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}\right)^{\alpha\eta} \left(Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{1-\alpha\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}\right)^{(1-\alpha)\eta}}.$$

The aggregate capital and labor wedges τ_a^K and τ_a^L are implicitly defined by

$$\alpha\eta\frac{Y_a}{K_a} = \tau_a^K R \qquad \text{and} \qquad (1-\alpha)\,\eta\frac{Y_a}{L_a} = \tau_a^L W$$

and are equal to

$$\tau_a^K = \frac{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}$$

and

$$\tau_a^L = \frac{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{(1-\alpha)\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}{Z_c^{\frac{1}{1-\eta}} + \left(\tau^K\right)^{-\frac{\alpha\eta}{1-\eta}} \left(\tau^L\right)^{-\frac{1-\alpha\eta}{1-\eta}} Z^{\frac{1}{1-\eta}}}.$$

B.3 The Importance of Risk: Alternative Estimation

In this section we present an alternative estimation of two of the three models discussed in Section 5. Here, we re-estimate the economies without fat-tailed and without transitory shocks to match the exact same moments as in the baseline estimation and revisit our decomposition of productivity, output and wage losses attributed to risk and credit wedges. The results we discuss in Section 5 are robust to this alternative estimation strategy. Tables B.1 and B.2 report the results of the estimation and Table B.3 shows the decomposition between risk and credit wedges.

Table B.1: Targeted Moments: Alternative Models

	data	no fat tails	no transitory shocks
fraction entrepr	0.12	0.09	0.09
wealth to income entrepr	12.5	12.4	12.4
capital-output ratio, k/y	1.22	1.27	1.26
labor share, wl/y	0.72	0.78	0.75
profit share, π/y	0.13	0.07	0.10
iqr $wl_{it}/y_{it} - \overline{wl_{it}/y_{it}}$	0.15	0.12	0.04
s.d. $\log y_{it}$	1.32	1.31	1.32
s.d. $\log y_{it}/y_{it-1}$	0.48	0.43	0.39
s.d. $\log y_{it}/y_{it-2}$	0.60	0.58	0.58
s.d. $\log y_{it}/y_{it-3}$	0.69	0.70	0.73
iqr $\log y_{it}/y_{it-1}$	0.32	0.33	0.31
iqr $\log y_{it}/y_{it-2}$	0.46	0.51	0.51
iqr $\log y_{it}/y_{it-3}$	0.58	0.65	0.66
corr $\log y_{it}$, $\log y_{it-1}$	0.93	0.95	0.96
corr $\log y_{it}$, $\log y_{it-2}$	0.89	0.90	0.91
corr $\log y_{it}$, $\log y_{it-3}$	0.86	0.86	0.86
value of objective	–	0.027	0.038

Notes: The table reports the fit of the estimation procedure across alternative models when we target the exact same set of moments as in the estimation of the baseline model.

Table B.2: Parameter Values: Alternative Models

		no fat tails	no transitory shocks
β	discount factor	0.971	0.961
α	capital elasticity	0.183	0.204
η	span of control	0.981	0.953
ρ	persistence z	0.974	0.966
σ_u	volatility z	0.023	0.036
σ_ε	volatility ε	0.118	–
s	relative volatility mixture	–	3.944
p	baseline probability mixture	–	0.950
ϕ	relative time endowment	0.984	0.968

Notes: The table reports the parameter estimates across alternative models when we target the exact same set of moments as in the estimation of the baseline model.

Table B.3: Aggregate Implications: Alternative Models

	no fat tails	no transitory shocks
misallocation, $-\log Z/Z^P$		
total	0.056	0.081
due to risk	0.004	0.001
due to credit	0.041	0.076
output losses, $-\log Y/Y^P$		
total	0.103	0.162
due to risk	0.013	0.006
due to credit	0.093	0.159
wage losses, $-\log W/W^P$		
total	0.126	0.172
due to risk	0.049	0.024
due to credit	0.093	0.159

Notes: The table reports the misallocation, output and wage losses across alternative models evaluated at the estimates obtained when we target the same set of moments as in the estimation of the baseline model.