Why Are Returns to Private Business Wealth So Dispersed?*

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Abstract

We use firm-level data from Orbis to document that average returns to private business wealth are dispersed, persistent, and negatively correlated with equity. We also show that firms experience large and fat-tailed changes in output that are not fully accompanied by changes in their capital stock and wage bill, and therefore generate large changes in firm profits. We interpret this evidence using a model of entrepreneurial dynamics in which return heterogeneity arises from both limited span of control, as well as from financial frictions which generate differences in marginal returns to wealth. The model matches the evidence on average returns and predicts that marginal returns are three fourths as dispersed as average returns, mostly reflecting risk as opposed to collateral constraints. Though financial frictions greatly depress individual firms' production choices and cash flows, they generate relatively modest productivity and output losses in the aggregate.

Keywords: inequality, entrepreneurship, rate of return heterogeneity, misallocation.

JEL classifications: E2, E44, G32

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1 Introduction

Recent work using administrative data from several countries has documented large and persistent differences in rates of return to wealth across households (Fagereng et al., 2020, Bach et al., 2020, Smith et al., 2022), which have been argued to be an important driver of wealth inequality (Benhabib et al., 2011, Benhabib et al., 2017). These differences are largely accounted for by heterogeneity in rates of return to private business wealth. For example, Fagereng et al. (2020) show that the mean annual return on financial wealth is 1%, with a standard deviation of 6%. In contrast, the mean annual return to private business wealth is 10%, with a standard deviation of 52%.

This evidence on returns to private business wealth is on *average returns*, that is, business income divided by the net worth of a business. Measured returns may therefore reflect returns to a fixed factor, such as entrepreneurial talent or market power. In addition, since private business owners may be financially constrained, average returns to wealth may also reflect financial frictions which generate differences in *marginal returns*. For example, collateral constraints may impede productive but poor entrepreneurs from borrowing and thus arbitraging rate of return differences. Similarly, since private business income is poorly diversified, differences in returns to wealth may reflect risk premia.

Our goal is to understand the role of financial frictions in accounting for the dispersion in average returns observed in the data. Specifically, we ask: how dispersed are marginal returns to private business wealth in an economy that reproduces the dispersion in average returns? What are the micro and macroeconomic consequences of these financial frictions? The answers to these questions have important implications for understanding the gains from policies that reallocate wealth towards private business owners (Itskhoki and Moll, 2019, Guvenen et al., 2022, Boar and Midrigan, 2022). If marginal returns are very dispersed, then there may be large productivity losses from misallocation and therefore large output and productivity gains from such policies.

Our paper uses micro data from Orbis on firm balance sheets and income statements for a number of European countries, for the period 1995-2018. We document that dispersion in average returns to private business wealth is large and persistent, and that average returns are negatively correlated with firm equity. We also show that firms in the data experience large and fat-tailed changes in output that are not fully accompanied by changes in their capital or wage bill and therefore generate large changes in firm profits. We interpret these facts through the lens of a model of entrepreneurship in which heterogeneity in average returns can arise from both a fixed factor, which we model as limited span of control, as well as from financial frictions due to borrowing constraints and uninsurable business risk. The model predicts that marginal returns are half as large and three fourths as dispersed as average returns, suggesting a potentially important role for financial frictions. Importantly, differences in marginal rates of return mostly reflect uninsurable business risk, as opposed to collateral constraints which play a negligible role due to firms' unwillingness to expand and take on more risk. Though financial frictions have sizable microeconomic consequences, depressing firm valuations and production, they have relatively modest macroeconomic consequences due to general equilibrium effects.

We interpret the empirical evidence through the lens of a relatively standard model of entrepreneurial dynamics in which firms face two sources of financial frictions. First, motivated by the evidence in Moskowitz and Vissing-Jørgensen (2002) that private business ownership is poorly diversified, we assume that each firm is entirely owned by a single entrepreneur who consumes the income generated by the business and can only insure the business risk by saving in a risk-free asset. Second, we assume a collateral constraint that limits firms' ability to borrow. Firms produce a homogeneous good with a decreasing returns to scale technology and differ in their productivity. Motivated by the evidence that output growth rates are fat-tailed, we assume that productivity shocks are drawn from a fat-tailed distribution. Additionally, motivated by the imperfect high-frequency comovement of capital and labor with output, we assume that inputs are chosen prior to the firm observing its productivity.

The time-to-build assumption implies that firms choose labor and capital to equate their expected marginal products with the respective factor prices, using their owner's stochastic discount factor to weigh future states. The negative covariance between firm productivity and the stochastic discount factor implies that firms underproduce relative to an environment with perfect risk sharing, depressing the labor share and the capital-output ratio.[1] In our economy with imperfect risk sharing there is an important distinction between expected marginal products under the physical and the risk-neutral probability measures. Though the former are elevated relative to factor prices, owing to the presence of risk, the latter are not, unless the collateral constraint binds. As the firm's equity grows, the owner's consumption becomes insulated from the fortunes of its business, leading to more risk-taking. Risk therefore generates a positive marginal return on equity.

[1]See Arellano et al. (2019) and Di Tella et al. (2022), who explore a similar mechanism to study the role of increased firm volatility in explaining aggregate fluctuations, as well as David et al. (2022b), who study the implications of this mechanism for the relationship between aggregate risk and the labor share.

We calibrate the model to match salient features of the micro data from Spain, a country for which the Orbis dataset has particularly good coverage. Matching the evidence requires that firms face large, both transitory and persistent, and fat-tailed shocks to their productivity. Such shocks lead to large changes in the firms' profit share, consistent with the evidence, and imply an important role for risk, even though business owners have only a moderate degree of risk aversion. The model matches well the dispersion and persistence in average rates of return in the data, which range from 5% at the median to 35% at the 95^{th} percentile. As in the data, average rates of return are negatively correlated with firm equity: the slope of a rank-rank regression of average returns against equity is -0.26.

We use the model to calculate how dispersed marginal returns are, to understand the sources of their dispersion, and to study the micro and macroeconomic implications of financial frictions. We find that expected marginal returns range from 2% at the 10^{th} percentile to 17% at the 95^{th} percentile and are three fourths as dispersed as average returns. This dispersion in expected marginal returns is largely driven by risk, as the dispersion in expected risk-neutral marginal returns is substantially smaller. This is in contrast to a model where only capital is subject to time-to-build frictions. Intuitively, because the labor share is high, frictions that impede the adjustment of labor to productivity shocks have much large effects on profits and therefore the amount of risk faced by entrepreneurs.

We illustrate the microeconomic implications of financial frictions by showing that the average business would be worth eight times more in their absence, mostly reflecting an increase in production and therefore cash flows, rather than lower discount rates. In contrast, the macroeconomic consequences are more modest due to general equilibrium forces. Eliminating financial frictions would increase aggregate productivity by 6% and, assuming that labor is in fixed supply, aggregate output by 8%. These modest effects reflect that the losses from the misallocation of capital and labor in our economy are relatively low, even though the model reproduces the weak rank-rank correlation of 0.24 between wealth and productivity in the data.[2] Intuitively, financial frictions generate relatively low losses from misallocation because though they prevent productive entrepreneurs from expanding, they do not generate large rank reversals, which Hopenhayn (2014) shows are necessary to generate large TFP losses. Our results therefore suggest that the macroeconomic gains from policies that increase the wealth share of productive entrepreneurs are relatively low, consistent with Boar and Midrigan (2022).

[2] See Moll (2014) who illustrates how the joint distribution of wealth and productivity shapes the losses from misallocation from financial frictions.

Lastly, we ask: is the dispersion in average returns to private business wealth observed in the data necessarily indicative of dispersion in marginal returns and therefore financial frictions? We answer this question by studying an alternative economy in which labor and capital are flexibly chosen and there are no collateral constraints, so that production choices are undistorted and marginal returns are equalized across firms. We recalibrate this economy and show that it also reproduces well the dispersion of average returns to private business wealth in the data. Intuitively, this model attributes a much larger role to the limited span of control and heterogeneity in productivity in explaining average rates of return. Such a model is, however, at odds with the data in that it predicts a much stronger negative relationship between average returns and equity: the slope of a rank-rank regression falls from -0.26 in our baseline model and in the data to -0.78. The broader lesson that emerges is that the correlation between average returns and equity, in conjunction with the dispersion in average returns, is informative about the severity of financial frictions, even though the dispersion in average returns on its own is not.

Related Work. Our paper studies the micro and macroeconomic consequences of financial frictions and is therefore related to the work of Buera et al. (2011), Midrigan and Xu (2014), Moll (2014), and Gopinath et al. (2017) who study the role of collateral constraints. In our framework risk, as opposed to collateral constraints, plays a more important role, thus relating our paper to Tan (2018), Robinson (2021) and David et al. (2022a), who study the role of risk in distorting firm investment. In contrast to these papers, we emphasize the importance of risk in labor choices, as do Arellano et al. (2019), Di Tella et al. (2022) and David et al. (2022b). However, our finding that the losses from misallocation in an economy that reproduces the dispersion in average returns in the data are modest is robust to eliminating the time-to-build assumption and thus risk premia altogether.

Our paper is also related to the work of Smith et al. (2019), who study whether pass-through business profits accrue to human capital, in addition to financial capital. Though we use a different methodology, we also find that business income partly reflects non-financial factors. Also related is the work of Karabarbounis and Neiman (2019) who attempt to decompose business income into profits, unmeasured investments and financial frictions. Although we focus on heterogeneity in returns across private businesses, and they focus on the aggregate time series, these researchers also find an important role for financial factors in explaining recent trends in income.

Lastly, our findings have implications for understanding the sources of wealth inequality and for the design of optimal tax policy. Our paper is therefore related to Benhabib et al. (2011) and Benhabib et al. (2017), who argue that heterogeneity in rates of return on savings is an important determinant of wealth inequality, as well as Guvenen et al. (2022), Brüggemann (2021), Itskhoki and Moll (2019), Boar and Knowles (2020), Gaillard and Wangner (2021) and Boar and Midrigan (2022), who analyze optimal tax policy in an environment with financially constrained businesses.

The remainder of the paper proceeds as follows. Section 2 presents a simple example to clarify the distinction between average and marginal returns. Section 3 discusses the data and the facts that motivate our quantitative analysis. Section 4 presents the model and the parameterization. Section 5 discusses the results. Section 6 studies a number of extensions. Finally, Section 7 concludes.

2 Average vs. Marginal Returns

We begin with a simple example to clarify the distinction between average and marginal rates of return, and to motivate our empirical and quantitative analysis. Assume a production technology that uses labor l and capital k to produce output y according to

$$y = f(z, k, l),$$

where z denotes the firm's productivity. Suppose that the firm can save and borrow at an interest rate r, faces a constant depreciation rate δ and hires labor at a competitive wage rate W. Letting b denote the amount the firm borrows and $a = k - b$ denote the firm's equity,[3] its income is given by output net of labor costs, capital depreciation and interest expenses

$$\pi = y - Wl - \delta k - rb, \tag{1}$$

or, equivalently,

$$\pi = ra + y - Rk - Wl,$$

where $R = r + \delta$ denotes the user cost of capital.

We first calculate the *average* return if the firm is unconstrained in its choice of capital and labor. The first order conditions for capital and labor are

$$f_k = R \qquad \text{and} \qquad f_l = W.$$

[3]Throughout the paper we use the terms equity, wealth and net worth interchangeably.

Equivalently, letting $\alpha_k = f_k k/y$ and $\alpha_l = f_l l/y$ denote the output elasticites with respect to capital and labor, it follows that capital and labor are paid

$$Rk = \alpha_k y \qquad \text{and} \qquad Wl = \alpha_l y.$$

The firm's average return is therefore

$$\frac{\pi}{a} = r + (1 - \alpha_k - \alpha_l)\,\frac{y(z)}{a}.$$

Since output is increasing in productivity, more productive firms earn higher returns, holding wealth constant.

Consider next the firm's *marginal* return on wealth, the change in firm income resulting from an incremental change in wealth. Differentiating (1) with respect to a gives

$$\frac{\partial \pi}{\partial a} = r + [f_k - R]\,\frac{\partial k}{\partial a} + [f_l - W]\,\frac{\partial l}{\partial a}.$$

If the firm is financially unconstrained, that is if $\partial k/\partial a = 0$ and $\partial l/\partial a = 0$, more wealth does not change the firm's production choices, so the marginal return is equal to r and is lower than the average return.

Consider next the case of a constrained firm. We assume for simplicity that the firm faces a collateral constraint $k \leq \lambda a$ or, equivalently, $b \leq \left(1 - \frac{1}{\lambda}\right) k$, where $\lambda \geq 1$ is the maximum leverage ratio. If the constraint binds, the marginal product of capital $f_k = \alpha_k y/k$ is above the user cost R, so the marginal return exceeds the interest rate r and is equal to

$$\frac{\partial \pi}{\partial a} = r + \left[\alpha_k \frac{y}{k} - R\right] \lambda.$$

In contrast, the average return is

$$\frac{\pi}{a} = r + \left[(1 - \alpha_l)\frac{y}{k} - R\right] \lambda,$$

where we use that $k = \lambda a$ for a constrained firm. We conclude that unless the firm operates a constant returns to scale technology as in Angeletos (2007) and Moll (2014), that is, unless $\alpha_k + \alpha_l = 1$, average returns overstate marginal returns. Intuitively, if $\alpha_k + \alpha_l < 1$, part of the firm's income accrues to a fixed factor (e.g. managerial talent as in Lucas, 1978 or market power as in Melitz, 2003), so the average return captures both financial frictions, as well as the return to the fixed factor.

Since marginal returns for private businesses are not readily observable in the data, in the remaining sections we use micro data and a richer dynamic quantitative model in which returns to wealth shape the entrepreneurs' savings choices and the joint distribution of wealth and productivity emerges endogenously. We use the model to measure how dispersed average returns are and to isolate the role of the fixed factor, risk premia and collateral constraints in accounting for the heterogeneity in the average returns observed in the data.

3 Data and Motivating Facts

In this section we describe the dataset we use and document several facts that motivate our modeling choices and inform our quantitative analysis. We document that differences in average returns for private businesses are large and persistent, and that returns are negatively correlated with wealth. We also document that firms experience large and fat-tailed changes in output that are not fully accompanied by changes in their capital or wage bill.

3.1 Data

The dataset we use is the historical product of Orbis, compiled by Moody's Bureau van Dijk. The data covers the period 1995-2018 and is compiled from national registers and other sources, and has harmonized information on annual balance sheets and income statements of privately and publicly traded firms (see Gopinath et al., 2017 and Kalemli-Ozcan et al., 2015 for a more detailed description of the data). We focus our empirical analysis on Spain, a country with excellent coverage of firms across the entire size distribution, as shown by Gopinath et al. (2017). However, as we show in the Appendix, all of our results hold for other countries.

Given our interest in private businesses, we restrict attention to partnerships and private limited companies. We also exclude firms that operate in Finance, Insurance and Real Estate, Public Administration and Defense. To minimize the concern that variables are measured with error, we exclude observations in the top and bottom 0.1% of the distribution of growth rates of value added, capital and wage bill, as well as the distribution of returns to wealth, capital-to-wealth ratio, capital-output ratio, the labor share, the profit share and wealth-to-value added ratio. Since our goal is to document the persistence of returns to wealth, we restrict the sample to firms for which we have at least 10 years of data. Our final sample consists of 228,394 firms which we observe for an average of 15.5 years. These firms represent 25% of all private businesses and account for 72% of the value added and 74% of the wealth of all private businesses in the original sample. As we show in the Appendix, none of our substantive findings change if we calculate statistics based on the entire sample of firms.

Table 1: Summary Statistics

	mean	p10	p25	p50	p75	p90
output	605	44	90	195	438	963
labor	431	34	70	150	330	710
capital	748	10	34	126	395	1,070
equity	953	6	41	148	475	1,362
income	74	-25	0	8	37	126
employment	15	2	3	6	12	25

Notes: Numbers are expressed in thousands of 2015 USD and are based on 3.5 million firm-year observations.

We next define the variables we use in the analysis. Our measure of the output y_{it} of firm i in year t is value added, which we compute as the difference between production (revenues + changes in inventories) and all non-labor costs, including taxes. Our measure of labor l_{it} is the firm's wage bill, including benefits. The capital stock k_{it} is the book value of property, plant, equipment and intangibles. We calculate equity a_{it} as the difference between the firm's total assets and total liabilities. Finally, we define income π_{it} as output net of labor, depreciation and interest expenses. All variables are inflation-adjusted.

Table 1 summarizes the distribution of these variables for our baseline sample. The average (median) firm has 15 (6) workers and has value added equal to 604 (195) thousand dollars, a wage bill of 431 (150) thousand dollars and income of 74 (8) thousand dollars. Though these firms are relatively large, the distribution of these variables is similar to that based on the full sample of firms, reported in the Appendix.

3.2 Facts

Dispersion and Persistence in Average Returns. We start by corroborating the findings of Fagereng et al. (2020) and Bach et al. (2020) about the dispersion and persistence of average returns to private business wealth, which we refer to for brevity as *returns*. Differently from these papers, our measure of returns is net of taxes. The first row of Table 2 reports moments of the cross-sectional distribution of annual returns. To compute these, we restrict the sample to firms with positive equity, which represent 92.5% of the firms in the baseline sample and weight all observations by the firm's equity. The mean rate of return is

Table 2: Average Rates of Return

	mean	std	p10	p25	p50	p75	p90	p95
π/a	0.081	0.217	-0.030	0.011	0.059	0.134	0.241	0.337
$\overline{\pi/a}$	0.078	0.117	0.000	0.031	0.071	0.114	0.169	0.219

Notes: All statistics are equity weighted.

8.1% and the distribution features substantial dispersion, with the standard deviation equal to 21.7%. Returns range from -3% at the 10^{th} percentile to 24% at the 90^{th}.[4]

To document how persistent returns are, we calculate for each firm the time series average of its return π_{it}/a_{it} (equity weighted) and denote it by $\overline{\pi/a}$. The second row of Table 2 reports the distribution of these time series averages across firms. They range from 0 at the 10^{th} percentile to 17% at the 90^{th} percentile and have a standard deviation of 12%. Since we observe firms for an average of 15 years, these numbers suggest that some firms earn consistently high returns over long periods of time.

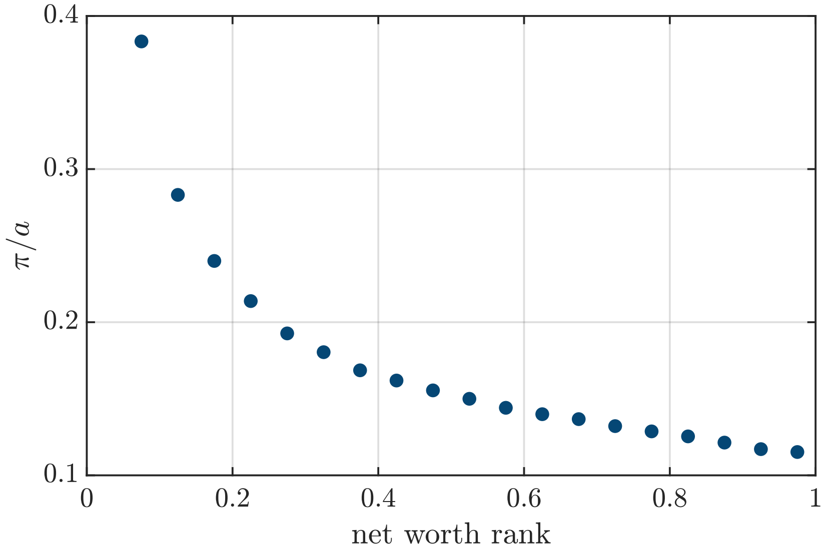
Average Returns and Equity Are Negatively Correlated. We next document that average rates of return are negatively correlated with firm equity.[5] Figure 1 presents a binned scatter plot of average rates of return against the firm's rank in the equity distribution. That is, we calculate for firms in each bin of the equity distribution the average return. The figure shows a clear negative relationship between the two. Firms at the bottom of the equity distribution have rates of return that average 40%, while those at the top have average returns of 10%. Notice that these numbers are larger than the mean returns reported in Table 2 because they are not weighted by equity.

A concern with the evidence in Figure 1 is that measurement error in equity can generate a spurious negative correlation. To alleviate this concern, we show in the Appendix that the pattern continues to hold when we rank firms by their previous period's equity. Thus, if measurement error generates this pattern, it must be strongly auto-correlated. We also show

[4] The distribution of pre-tax returns is even more dispersed, with the 10^{th} percentile equal to -4% and the 90^{th} equal to 33%.

[5] Though we do not explicitly target this correlation in our calibration, we use it below to evaluate alternative models.

Figure 1: Rates of Return and Equity



Notes: For visual clarity we exclude the bottom 5% of the net worth rank. These firms have an average return of 1.2.

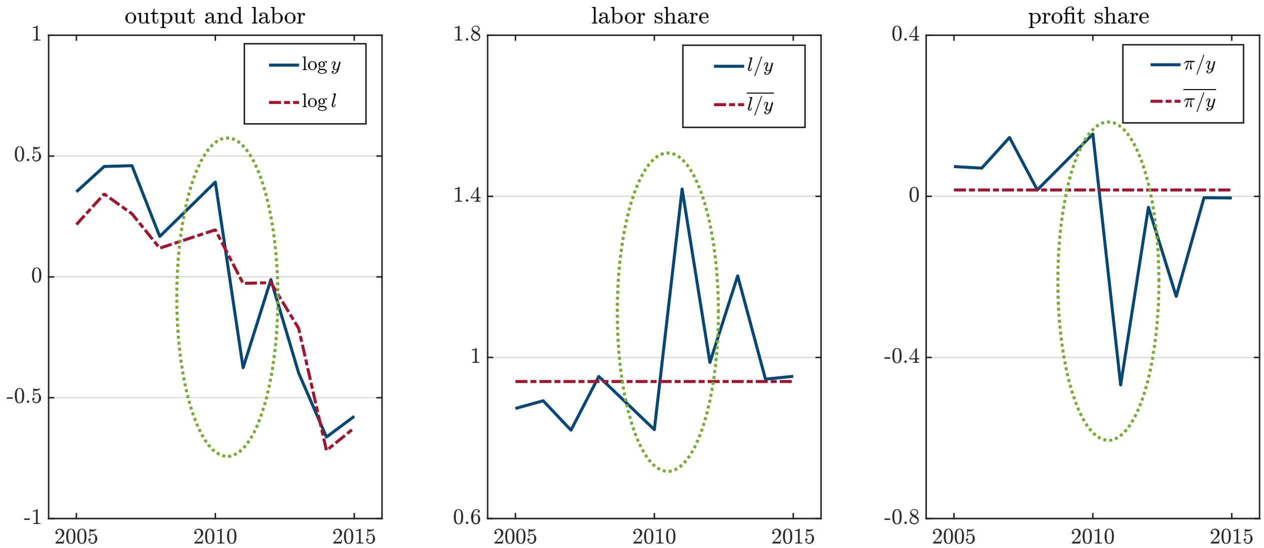
that measurement error would have to account for at least 40% of the dispersion in equity for us to observe the correlation in the data if in fact there were none. Finally, we note that Halvorsen et al. (2022) document a similar pattern using administrative data from Norway which is presumably less prone to measurement concerns.

Output Growth Rates Are Dispersed and Fat-Tailed. We next document that firms experience large and fat-tailed changes in output. To see this, Table 3 reports percentiles of the distribution of the growth rate of output $\log y_{it}/y_{it-1}$. For comparison, we also report in the second row of the table percentiles of a Gaussian distribution with the same variance. Relative to the Gaussian distribution, the distribution of output growth rates in the data features much heavier tails. For example, at the $1/10^{th}$ percentile the output growth rate is -2.78 in the data and -1.27 under a Gaussian distribution. Similarly, at the 99.9^{th} percentile the output growth rate is 2.68 in the data and 1.27 under a Gaussian distribution. Thus, even though most firms experience relatively small changes in output, as evidenced by the small interquartile range, a few firms experience extremely large changes in output. Even though we truncated the top and bottom 0.1% of output growth rates, the distribution has considerable excess kurtosis of 13.4.

Table 3: Distribution of Output Growth Rates

	s.d.	p0.1	p0.5	p25	p75	p99.5	p99.9
Data	0.41	-2.78	-1.78	-0.12	0.15	1.68	2.68
Gaussian	0.41	-1.27	-1.06	-0.28	0.28	1.06	1.27

Figure 2: Example of a Firm



Capital and Labor Do Not Track Output Closely. We next show that fluctuations in output lead to large fluctuations in firm profits because capital and labor do not track output closely. A decline in output is therefore associated with a decrease in the profit share. We first illustrate this point by means of an example and then document the pattern more systematically.

The left panel of Figure 2 plots the logarithm of output and labor for an individual firm. We normalize units so that the logarithm of output centers around zero. Notice that this firm experiences a large decline in output from 2005 to 2015 and that labor tracks output fairly closely at low frequencies. Zooming in on higher frequencies, the sharp decline in output between 2010 and 2011 was accompanied by a modest decline in the wage bill. In fact, in 2011 the firm's wage bill substantially exceeded its output, leading to a labor share above unity (middle panel) and a profit share below zero (right panel).

Table 4 reports results from regressions that relate changes in labor, capital and profits to changes in output. The first two columns show that the slope coefficients from regressing the growth rates of labor and capital against the growth rate of output are equal to 0.37 and 0.15, respectively. That is, a 10% decline in output is associated with only a 3.7% drop in the firm's wage bill and a 1.5% decline in its capital stock. Though the pattern for capital is not surprising, given the evidence that investment is subject to large capital adjustment costs (Cooper and Haltiwanger, 2006), the pattern for labor is relatively less known.[6] We note that this imperfect pass-through is only apparent at high frequencies: inputs and output are much more strongly correlated in the cross-section, with an output elasticity of 0.93 for labor and 0.86 for capital.

The imperfect comovement of output and factor inputs imply that the firm's profit share π_{it}/y_{it} comoves positively with changes in output. To see this, we regress the change in a firm's profit share against the growth rate of its output. The slope coefficient is equal to 1.43, implying that a 10% drop in output is associated with a decline in the profit share of 0.14, a sizable amount since the average profit share for firms in our sample is only equal to 0.12. We next show that high-frequency movements in the firm's labor share account for a substantial fraction of the comovement between output growth and profit shares. To do so, we construct a counterfactual profit series under the assumption that the wage bill of the firm is always equal to a constant fraction s_i^l of its output, where s_i^l is the time-series average labor share of firm i. As the last column of Table 4 shows, this counterfactual profit share comoves much less with output: the slope coefficient is equal to 0.342.

To address the concern that measurement error in output explains these patterns, the second row of the table reports results from a regression where we only include firms for which the growth rate of output is less than 50% in absolute value. Though the coefficients in the regressions of labor and capital growth against output growth are somewhat larger, once again, we find an incomplete pass-through of 0.56 and 0.30, respectively. The slope coefficient in a regression of changes in profit shares against output growth is equal to 0.43, smaller than when we include all observations, but nevertheless sizable. Changes in the labor share once again account for the bulk of this comovement: absent changes in the labor share the slope coefficient in a regression of changes in profit shares on output growth falls to 0.07.

Since labor accounts for the bulk of a firm's expenses (the average firm in our sample has a labor share of 0.71), the observation that most variation in a firm's profit share over

[6] Donangelo et al. (2019) document similar patterns using data for US firms.

Table 4: Comovement Between Capital, Labor, Profits and Output

	$\Delta \log l$	$\Delta \log k$	$\Delta\pi/y$	$\Delta\hat{\pi}/y$
	A. All observations			
$\Delta \log y$	0.372	0.152	1.432	0.342
	(0.001)	(0.001)	(0.089)	(0.004)
	B. $\lvert\Delta \log y\rvert < 0.5$			
$\Delta \log y$	0.562	0.303	0.433	0.067
	(0.001)	(0.002)	(0.001)	(0.001)

Notes: The estimates in Panel A. (B.) are computed using 3.18 (2.81) million observations. Standard errors reported in parentheses are clustered at the firm level. The regressions do not include firm or year fixed-effects. Including these has a negligible effect on the reported coefficients.

time is associated with movements in its labor share is not surprising. It suggests, however, that frictions that prevent the perfect comovement of labor and output implicit in standard models of firm dynamics are critical if one is to reproduce the high-frequency fluctuations in firm profits observed in the data.

To summarize, we documented that there are large and persistent differences in average returns to business wealth, and that these returns are negatively correlated with firm equity. Additionally, firms experience large and fat-tailed changes in output that are only partially accompanied by changes in the wage bill and capital. Changes in output are therefore associated with large fluctuations in the firm's labor, capital and profit shares.

4 Model

Motivated by these observations, we next use a relatively standard model of entrepreneurial dynamics to quantify the role of financial in accounting for the dispersion in average returns and generating dispersion in marginal returns. As in Lucas (1978), firms produce a homogeneous good with a decreasing returns to scale technology and differ in their productivity.[7] Firms produce output using capital and labor hired in competitive markets. We assume that

[7]The firm's problem in this environment is equivalent to that of a monopolistically competitive firm that sells a differentiated variety and faces a constant demand elasticity.

firms face two sources of financial frictions. First, motivated by the evidence in Moskowitz and Vissing-Jørgensen (2002) that entrepreneurial investment is poorly diversified, we follow Quadrini (2000), Cagetti and De Nardi (2006) and Buera et al. (2011) in assuming that each firm is entirely owned by a single entrepreneur who can only partially insure the business income risk by saving in a risk-free asset. Second, we assume a collateral constraint that limits the firm's ability to borrow.

We make two additional assumptions that are motivated by our empirical analysis. First, we assume that both capital and labor are chosen prior to the firm observing its productivity. This time-to-build assumption allows us to capture in a parsimonious way the evidence that labor and capital growth comove weakly with output growth and has been used both for capital (Gopinath et al., 2017, David et al., 2022a) and employment (Boldrin et al., 2001, Arellano et al., 2019, Cooper et al., 2022, David et al., 2022b).[8] As we show below, this assumption implies that wealth not only affects investment choices, but also firms' employment decisions, consistent with the evidence in Ring (2022). Second, we assume that productivity shocks are drawn from a fat-tailed distribution, an assumption that allows us to capture the fat-tailed output growth we documented in the data.

4.1 Environment

We assume a small open economy populated by a unit mass of entrepreneurs who can save and borrow at a constant interest rate r. There is no aggregate uncertainty. We assume a fixed labor supply that is remunerated at an equilibrium wage rate W. An entrepreneur's lifetime utility is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\theta}}{1-\theta},$$

where c_t denotes consumption in period t, β is the discount factor and θ is the relative risk aversion. The budget constraint is

$$c_t + k_{t+1} - b_{t+1} = y_t - Wl_t + (1-\delta)\,k_t - (1+r)\,b_t,$$

where k_t and l_t are the amounts of capital and labor used to produce output y_t and b_t is the amount the firm borrows. We assume that capital depreciates at rate δ. Letting $a_t = k_t - b_t$ denote the entrepreneur's wealth, the budget constraint can be rewritten as

$$a_{t+1} - a_t = \pi_t - c_t,$$

[8]The time-to-build assumption for labor can be interpreted as arising from adjustment costs, hiring or firing restrictions, or perhaps reflecting implicit insurance that firms provide to their (more risk-averse) workers (Guiso et al., 2005).

where

$$\pi_t = y_t - Wl_t - Rk_t + ra_t$$

is the entrepreneur's income, defined as in the data.

We assume that the firm is subject to a collateral constraint

$$b_{t+1} \leq \xi k_{t+1}$$

that restricts the amount the firm can borrow to a fraction ξ of its capital stock. Equivalently, this constraint can be written as

$$k_{t+1} \leq \frac{1}{1-\xi} a_{t+1},$$

so that the firm's capital can be at most a multiple $1/(1-\xi)$ of its equity.

We assume a Cobb-Douglas production function

$$y_t = z_t \varepsilon_t \left(k_t^{\alpha} l_t^{1-\alpha}\right)^{\eta},$$

where $\eta < 1$ is the span of control parameter, α determines the elasticity of output to capital, z_t is the persistent productivity component and ε_t is an iid shock to firm productivity. The persistent component evolves according to

$$\log z_{t+1} = \rho \log z_t + u_{t+1}.$$

The innovations to firm productivity u_t and ε_t have standard deviations σ_u and σ_ε and are drawn from a Tukey g-h distribution, a flexible family of distributions that transforms a standard normal variable x according to

$$x \exp\left(\frac{h}{2}x^2\right)(1-2h)^{3/4},$$

where h is a parameter that governs the thickness of the tails. A higher h elongates the tails of the distribution relative to a standard normal ($h = 0$). As we show below, allowing for both persistent and transitory shocks is necessary to match the rate at which the autocorrelation of output and the standard deviation of its growth rate vary with the horizon.

4.2 Decision Rules

Since labor and capital are chosen before observing productivity, the optimal choices satisfy

$$\mathbb{E}_t c_{t+1}^{-\theta}\left[(1-\alpha)\,\eta \frac{y_{t+1}}{l_{t+1}} - W\right] = 0 \tag{2}$$

and

$$\mathbb{E}_t c_{t+1}^{-\theta}\left[\alpha\eta\frac{y_{t+1}}{k_{t+1}}-R\right]\geq 0. \tag{3}$$

The firm chooses labor and capital to equate their expected marginal products with the respective factor prices. Since business income risk is not diversified, the owner uses its own stochastic discount factor to weigh future states. The first order condition for capital holds with equality iff the collateral constraint does not bind.

The optimality condition for equity can be written as

$$c_t^{-\theta}=\beta\left(1+r+\mu_t\right)\mathbb{E}_t c_{t+1}^{-\theta}, \tag{4}$$

where μ_t is the multiplier on the collateral constraint and satisfies

$$\mu_t=\frac{1}{1-\xi}\mathbb{E}_t\frac{c_{t+1}^{-\theta}}{\mathbb{E}_t c_{t+1}^{-\theta}}\left[\alpha\eta\frac{y_{t+1}}{k_{t+1}}-R\right].$$

Intuitively, since an additional unit of wealth allows the firm to acquire $1/(1-\xi)$ additional units of capital, the excess return on saving is equal to $1/(1-\xi)$ times the risk-adjusted expected difference between the marginal product of capital and its user cost. Henceforth, we use $\hat{\mathbb{E}}_t\equiv\mathbb{E}_t\frac{c_{t+1}^{-\theta}}{\mathbb{E}_t c_{t+1}^{-\theta}}$ to denote the expectation under the risk-neutral measure.

Absent the collateral constraint, the optimal choices of labor and capital are

$$l_{t+1}=\left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}}\left(\frac{\left(1-\alpha\right)\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}}\left(\hat{\mathbb{E}}_t z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-\eta}}$$

and

$$k_{t+1}=\left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}}\left(\hat{\mathbb{E}}_t z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-\eta}}$$

and are a function of the risk-neutral expected productivity.

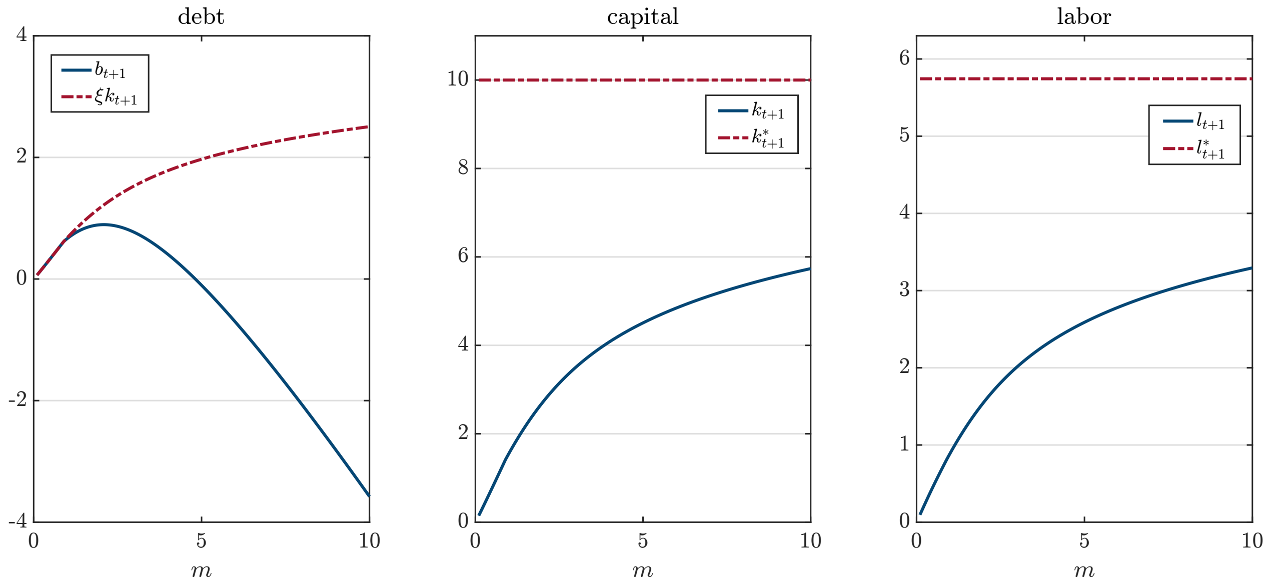
To see the impact risk has on the firm's input choices, we next contrast them with the optimal choices under full insurance, assuming no collateral constraints. In this case, the problem of the entrepreneur reduces to

$$\max_{k_{t+1},l_{t+1}}\quad -k_{t+1}+\frac{1}{1+r}\left(\mathbb{E}_t z_{t+1}\varepsilon_{t+1}\left(k_{t+1}^{\alpha}l_{t+1}^{1-\alpha}\right)^{\eta}-Wl_{t+1}+\left(1-\delta\right)k_{t+1}\right)$$

and input choices depend on expected productivity under the physical measure. The ratio of the firm's labor and capital to their frictionless counterparts is therefore

$$\frac{l_{t+1}}{l_{t+1}^{*}}=\frac{k_{t+1}}{k_{t+1}^{*}}=\left(\frac{\hat{\mathbb{E}}_t z_{t+1}\varepsilon_{t+1}}{\mathbb{E}_t z_{t+1}\varepsilon_{t+1}}\right)^{\frac{1}{1-\eta}}=\left(1+\frac{\mathbb{COV}_t\left(c_{t+1}^{-\theta},\ z_{t+1}\varepsilon_{t+1}\right)}{\mathbb{E}_t c_{t+1}^{-\theta}\mathbb{E}_t z_{t+1}\varepsilon_{t+1}}\right)^{\frac{1}{1-\eta}}$$

Figure 3: Decision Rules



Notes: Cash on hand m is defined as $a + \pi$.

and is less than unity because of the negative covariance between the owner's marginal utility of consumption and firm productivity.

The Euler equation (4) and the budget constraint allow us to characterize the entrepreneur's consumption-savings choice as a function of its cash on hand $m = a + \pi$ and the persistent productivity component z, $c(m, z)$ and $a'(m, z)$. The optimality conditions (2) and (3) allow us to characterize the firm's labor and capital choice $k'(a'(m, z), z)$ and $l'(a'(m, z), z)$ as a function of its savings and productivity. We illustrate these choices in Figure 3 as a function of cash on hand m, for a fixed level of productivity z. Both capital and labor increase with cash on hand because financial frictions are less severe for wealthier entrepreneurs. Financial frictions reduce capital and labor relative to their frictionless counterparts for two reasons. First, when the firm's cash on hand is low and therefore so are its savings $a'(m, z)$, the collateral constraint binds, as shown in the first panel that contrasts the firm's debt to its borrowing limit. Second, even for high levels of cash on hand, when the collateral constraint does not bind, the risk premium depresses input choices. This risk premium falls with cash on hand because the consumption of wealthier entrepreneurs is less sensitive to changes in business income, so the covariance between the owner's marginal utility of consumption and firm productivity decreases with wealth.

4.3 Implications for Rates of Return

We consider next the implications of financial frictions for rates of return. We decompose average and marginal returns into components arising from the fixed factor, risk premia and collateral constraints. We will quantify each of these components in our numerical analysis. Because part of the dispersion in rates of return reflects the ex-post realization of productivity shocks, we find it useful to focus on expected rates of return, which filter these out.

The entrepreneur's expected income is

$$\mathbb{E}_{t-1}\pi_t = ra_t + \mathbb{E}_{t-1}\left[z_t\varepsilon_t\left(k_t^{\alpha}l_t^{1-\alpha}\right)^{\eta} - Wl_t - Rk_t\right].$$

Consider first the implications for average returns. The first order conditions (2) and (3) imply that factor payments add up to

$$Wl_t + Rk_t = \eta\mathbb{E}_{t-1}y_t + \eta\frac{\mathbb{COV}_{t-1}\left(c_t^{-\theta}, z_t\varepsilon_t\right)}{\mathbb{E}_{t-1}c_t^{-\theta}} - \mu_{t-1}a_t.$$

Financial frictions reduce payments to labor and capital relative to the frictionless case, $\eta\mathbb{E}_{t-1}y_t$, through two channels, captured by the last two terms. First, the negative covariance between the marginal utility of consumption and productivity captures the risk premium. Second, the multiplier μ_{t-1} captures the collateral constraint. In turn, these two frictions increase expected profits relative to expected output

$$\mathbb{E}_{t-1}\pi_t = ra_t + (1-\eta)\,\mathbb{E}_{t-1}y_t - \eta\frac{\mathbb{COV}_{t-1}\left(c_t^{-\theta}, z_t\varepsilon_t\right)}{\mathbb{E}_{t-1}c_t^{-\theta}} + \mu_{t-1}a_t. \tag{5}$$

Aggregating (5) across all firms and scaling by aggregate wealth gives the aggregate return on wealth

$$\frac{\Pi_t}{A_t} = \underbrace{r}_{\text{risk-free rate}} + \underbrace{(1-\eta)\frac{Y_t}{A_t}}_{\text{fixed factor}} + \underbrace{\Omega_t}_{\text{finance frictions}} \tag{6}$$

which, as discussed in Section 2, reflects the risk-free return on wealth r, the fixed factor and the financial frictions.

Consider next the implications of financial frictions for marginal returns. Differentiating expected profits with respect to a_t gives the expected marginal return

$$\frac{\partial\mathbb{E}_{t-1}\pi_t}{\partial a_t} = r + \mathbb{E}_{t-1}\left[\alpha\eta\frac{y_t}{k_t} - R\right]\frac{\partial k_t}{\partial a_t} + \mathbb{E}_{t-1}\left[(1-\alpha)\,\eta\frac{y_t}{l_t} - W\right]\frac{\partial l_t}{\partial a_t},$$

which sums up the interest rate r, the expected difference between the marginal product of capital and its user cost multiplied by the marginal impact of wealth on capital, and the expected difference between the marginal product of labor and the wage rate multiplied by the marginal impact of wealth on labor. Marginal returns are affected by financial frictions via two channels. First, in the presence of risk the expected difference between the marginal products of factors and their user cost is positive because these expectations are evaluated with respect to the physical probability measures, not the risk-neutral ones. Second, even absent risk premia, if the collateral constraint binds, $\partial k_t/\partial a_t = 1/(1-\xi)$ and the expected marginal product of capital net of the user cost is positive.

We can isolate the role of the collateral constraint by calculating the risk-adjusted expected marginal return. The first order conditions (2) and (3) imply that the risk-adjusted expected marginal return is equal to

$$\frac{\partial \hat{\mathbb{E}}_{t-1}\pi_t}{\partial a_t} = r + \hat{\mathbb{E}}_{t-1}\left[\alpha\eta\frac{y_t}{k_t} - R\right]\frac{\partial k_t}{\partial a_t} = r + \mu_{t-1}$$

and thus only reflects the collateral constraint.

4.4 Parameterization

We next describe how we choose parameters for our quantitative analysis.

Assigned Parameters. We assume that a period in the model is one year and set the depreciation rate of capital $\delta = 0.10$ and the interest rate $r = 0.02$. We set the relative risk aversion $\theta = 2$, a commonly used value in the literature. In the robustness section below we report results for a lower value of $\theta = 0.5$.

Calibrated Parameters. We calibrate the discount factor, the elasticities of the production function, the maximum loan to value and the process for productivity to match moments in the firm level data from Spain. We report these moments in Table 5 and the calibrated parameter values in Table 6.

The discount factor β is pinned down by the average business wealth to output ratio, which is equal to 1.57 in the data and 1.55 in the model. The technology parameters are pinned down by the aggregate capital-output ratio (1.24 vs 1.27), the aggregate labor share (0.71 vs 0.74) and the aggregate income to output ratio (0.12 vs 0.14). The maximum loan to value ξ is pinned down by the 90^{th} percentile of the capital to wealth ratio (1.73 vs 1.72).

The persistence ρ_z and volatility σ_z and σ_e of the two productivity shocks are jointly pinned down by the autocorrelation of output at horizons one to three, the cross-sectional

Table 5: Targeted Moments

	Data	Model		Data	Model
s.d. $\log y_{it}$	1.26	1.31	aggregate a/y	1.57	1.55
s.d. $\log y_{it}/y_{it-1}$	0.41	0.37	aggregate k/y	1.24	1.27
s.d. $\log y_{it}/y_{it-2}$	0.52	0.51	aggregate l/y	0.71	0.74
s.d. $\log y_{it}/y_{it-3}$	0.60	0.62	aggregate π/y	0.12	0.14
iqr $\log y_{it}/y_{it-1}$	0.28	0.27	corr $\log y_{it}$, $\log y_{it-1}$	0.95	0.96
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	corr $\log y_{it}$, $\log y_{it-2}$	0.91	0.92
iqr $\log y_{it}/y_{it-3}$	0.52	0.54	corr $\log y_{it}$, $\log y_{it-3}$	0.88	0.89
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	p90 k/a	1.73	1.72

standard deviation of output, as well as the standard deviation of output growth rates at horizons one to three. Our model matches all these targets well. In addition to the speed at which the autocorrelation and volatility of growth rates changes with the horizon, the relative importance of transitory versus persistent shocks is pinned down by the extent to which the labor share for any given firm fluctuates over time relative to its time series mean. We thus also target the inter-quartile range of these deviations (0.12 vs 0.11). Finally, the parameter h that governs the thickness of the tails of the distribution of productivity shocks is pinned down by the inter-quartile range of the distribution of output growth rates. Intuitively, a fat-tailed distribution is characterized by a low inter-quartile range relative to the standard deviation, a feature that our model matches well.

The calibrated discount factor is $\beta = 0.916$. Because the capital-output ratio in the data is relatively low, the implied capital elasticity is low as well, $\alpha = 0.173$. One concern is that the capital-output ratio in the data, which reflects the book value of capital, is below the replacement cost. To address this, our robustness section targets a larger value of k/y from the EU-KLEMS data. The span of control parameter is $\eta = 0.948$, which is high compared to values of 0.85 or 0.90 typically used in the literature. As we discuss below, this is because risk plays an important role in our model and a substantial fraction of profits

Table 6: Parameter Values

β	0.916	discount factor	ρ_z	0.926	AR(1) z
α	0.173	capital elasticity	σ_z	0.041	std. dev. z shocks
η	0.948	span of control	σ_e	0.219	std. dev. e shocks
ξ	0.437	max loan to value	h	0.374	Tukey h parameter

accrue to capital income as opposed to managerial span of control. Absent risk, the value of η required to match the profit share in the data would be lower. The maximum loan to value is $\xi = 0.437$. The persistence and standard deviation of the persistent productivity component are 0.926 and 0.041, while the standard deviation of the transitory shock is 0.219. Notice that productivity is much less persistent than output, suggesting an important role for financial frictions in determining the dynamics of output. Intuitively, since wealth, which affects production choices, accumulates gradually, output can be very persistent even if productivity is not. Lastly, the Tukey h parameter is high and equal to 0.374, implying substantial excess kurtosis.

Untargeted Moments. We next evaluate the model's ability to reproduce several untargeted moments. As Table 7 shows, the model reproduces well the volatility and persistence of labor and capital in the data. For example, focusing on employment, the standard deviation of growth rates is 0.30 in the data and 0.36 in the model, the inter-quartile range is 0.19 vs 0.23, and the autocorrelation is 0.97 vs 0.96.

Importantly, the model also reproduces well the low comovement of the growth rates of employment and output. Recall that in the data a regression of $\Delta \log l_{it}$ on $\Delta \log y_{it}$ gives a coefficient of 0.56 when we restrict the sample to observations with $|\Delta \log y_{it}| \leq 0.5$. The corresponding regression coefficient in the model is 0.54. Given that the labor share is high in both the data and the model (0.71 vs. 0.74), the majority of movements in profits are due to fluctuations in the labor share. Specifically, the slope coefficient from regressing the change in profit shares on the growth of output is 0.43 in the data and 0.29 in the model. Thus, if anything, the model understates the extent to which profit shares comove with output growth. As in the data, most of this comovement is due to changes in the labor share: when

Table 7: Untargeted Moments

	Data	Model		Data	Model
s.d. $\Delta \log l_{it}$	0.30	0.36	s.d. $\Delta \log k_{it}$	0.54	0.36
iqr $\Delta \log l_{it}$	0.19	0.23	iqr $\Delta \log k_{it}$	0.25	0.23
autocorr $\log l_{it}$	0.97	0.96	autocorr $\log k_{it}$	0.96	0.96
slope $\Delta \log l_{it}$ on $\Delta \log y_{it}$	0.56	0.54	slope $\Delta \log k_{it}$ on $\Delta \log y_{it}$	0.30	0.54
slope $\Delta \pi_{it}/y_{it}$ on $\Delta \log y_{it}$	0.43	0.29	slope $\Delta \hat{\pi}_{it}/y_{it}$ on $\Delta \log y_{it}$	0.07	-0.06

we subtract movements in the labor share from changes in the profit share, the correlation with output growth rates is nearly zero. It is thus reassuring that our model reproduces the comovement between the wage bill and output, despite the parsimony of the time-to-build assumption on labor, and therefore generates fluctuations in profits for the same reason that they occur in the data.

As is well understood, the correlation between wealth and productivity is a crucial determinant of the severity of financial frictions (Moll, 2014). Using the parameters α and η reported above, we calculate productivity in the model as the Solow residual in the production function and find that the rank correlation between wealth and productivity in the data is quite low, 0.24. Reassuringly, the model matches this correlation perfectly.

4.5 Distribution of Average Returns

We conclude this section by showing that the model reproduces the distribution of average rates of return in the data and their correlation with equity even though we have not explicitly targeted these statistics. Panel A of Table 8 contrasts the cross-sectional, equity weighted, distribution of π/a in the data and in the model. The average return is 8.1% in the data and 9.2% in the model. Recall from equation (6), that the average return reflects the risk-free rate, the fixed factor and the financial frictions. Under our parameterization, the risk-free rate is equal to 2%, the fixed factor $\eta Y/A$ is equal to 3.3%, so financial frictions account for the remaining 3.8% of the average return. Therefore, financial frictions are as important as the fixed factor in accounting for aggregate profits.

Importantly, the distribution of average returns in the model is very dispersed, with a

Table 8: Distribution of Average Returns

	mean	std	p10	p25	p50	p75	p90	p95
				A. π/a, equity weighted				
Data	0.081	0.217	-0.030	0.011	0.059	0.134	0.241	0.337
Model	0.092	0.166	-0.001	0.021	0.047	0.121	0.244	0.350
				B. $\overline{\pi/a}$, equity weighted				
Data	0.078	0.117	0.000	0.031	0.071	0.114	0.169	0.219
Model	0.092	0.070	0.024	0.037	0.073	0.126	0.185	0.227

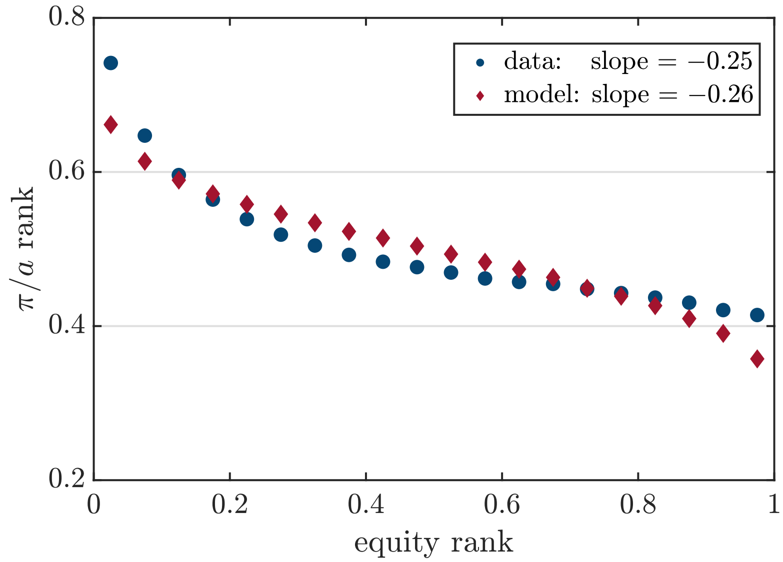
standard deviation of 16.6%, only slightly smaller than the 21.7% in the data. Though the model fails to generate the negative returns at the bottom of the distribution, it reproduces all other percentiles closely, especially the top ones. For example, the 90^{th} percentile of the distribution of returns is 24% in both the data and the model. Consider next the persistence of these returns. Panel B of Table 8 reports the distribution of the time series (equity weighted) average return $\overline{\pi/a}$ for each firm. Since we observe firms for an average of 15 years in the data, we calculate these averages over 15 years in the model. Once again, the model reproduces well the distribution of these averages. For example, the 90^{th} percentile of the distribution of $\overline{\pi/a}$ is 16.9% in the data and 18.5% in the model.

Figure 4 shows a binscatter of average returns against equity. We first calculate, for each firm, its rank in the equity distribution as well as its rank in the π/a distribution. We then bin firms according to their equity rank and report, for each equity bin, the average π/a rank. As in the data, firms at the bottom of the equity distribution have higher returns than firms at the top. Specifically, the slope of a rank-rank regression is equal to -0.25 in the data and -0.26 in the model.

5 Microeconomic and Aggregate Implications

We now use the model to study how dispersed are marginal returns, understand the sources of their dispersion and the microeconomic and macroeconomic consequences of financial fric-

Figure 4: Average Returns and Equity



Notes: The figure plots the rank-rank correlation between average rates of return and equity for 20 bins.

tions. We show that marginal returns are quite dispersed, mostly reflecting risk premia, and have sizable consequences for individual firms' production choices and business valuations. Nevertheless, financial frictions generate relatively modest productivity losses in the aggregate. We also show that even without financial frictions the model generates the same amount of dispersion in average returns as in the data, suggesting that dispersion in average returns is not necessarily indicative of financial frictions.

5.1 Distribution of Marginal Returns

We start by calculating the distribution of expected average returns, $\mathbb{E}_{t-1}\pi_t/a_t$, which allows us to isolate the role of transitory shocks in driving the cross-sectional dispersion of realized average returns. The first row of Table 9 shows that expected average returns are dispersed, with a standard deviation of 8.4%.[9] Recall from Table 8 that the standard deviation of realized average returns is 16.6%, suggesting that transitory shocks account for approximately half of the observed dispersion in realized average returns.

The second row shows the distribution of expected marginal returns, $\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$. As discussed in Section 4.3, dispersion in expected marginal returns arises due to financial frictions. Expected marginal returns are approximately three fourths as dispersed as expected

[9] As earlier, all the statistics we compute are equity weighted.

Table 9: Dispersion in Marginal Returns

	mean	std	p10	p25	p50	p75	p90	p95
$\mathbb{E}_{t-1}\pi_t/a_t$	0.095	0.084	0.025	0.036	0.065	0.125	0.205	0.263
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.051	0.060	0.020	0.021	0.025	0.051	0.115	0.175
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.021	0.003	0.020	0.020	0.020	0.020	0.020	0.026

Notes: All statistics are equity weighted.

average returns: their standard deviation is equal to 6%. Importantly, most of this dispersion reflects risk, as opposed to collateral constraints. To see this, the last row of Table 9 reports the distribution of expected marginal returns calculated under the risk-neutral measure, $\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$. As discussed in Section 4.3, these risk-adjusted returns exceed the interest rate r only if the collateral constraint binds. The majority of firms in our model are unconstrained as risk-adjusted returns are equal to the interest rate even at the 90^{th} percentile.

In summary, we find that marginal returns are three fourths as dispersed as average returns and mostly reflect compensation for risk rather than collateral constraints, suggesting a potentially important role for financial frictions.

5.2 Valuation of Private Businesses

We next evaluate the microeconomic consequences of financial frictions by studying their implications for the income and valuation of firms.

Entrepreneur's Valuation of the Business. We first calculate the price at which the owner of firm i is willing to sell their business. Specifically, let V_i denote the lifetime value of the business owner. If the entrepreneur were to sell the business at price p_{1i} and forfeit all claims to its future profits as well as the current equity, it would simply consume out of its wealth p_{1i} and consequently enjoy lifetime utility

$$\hat{V}(p_{1i}) = \frac{\left(1 - \beta^{\frac{1}{\theta}}(1+r)^{\frac{1}{\theta}-1}\right)^{-\theta} p_{1i}^{1-\theta}}{1-\theta}.$$

The price p_{1i} is then defined as the implicit solution to $V_i = \hat{V}(p_{1i})$. The first row of Table 10 reports the equity weighted distribution of p_{1i}, scaled by the book value of equity a_i. The ratio of the entrepreneur's valuation of its business to the book value of equity ranges from 1.2 at the 10^{th} percentile to 3.3 at the 90^{th} percentile, with an average of 2.1. That is, the average entrepreneur values their business by approximately twice as much as the book value of equity. Notice that this valuation reflects the entrepreneur's stochastic discount rates, which are higher than the risk-free rate r because of collateral constraints and risk, as well as the lower cash-flows from underproduction. We next decompose the contribution of discount rates and cash-flows in determining this valuation.

Present Value of Income Flows, Discounted at r. To isolate the contribution of higher discount rates, we let p_{2i} denote the present value of the expected income flows π_{it} that firm i generates under the status quo allocations, but discounted at rate r

$$p_{2i} = (1+r)\,a_i + \mathbb{E}\sum_{s=0}^{\infty}\frac{1}{(1+r)^s}\pi_{is}.$$

Relative to p_1, this statistic keeps the stochastic process for income flows unchanged, but discounts them at the lower interest rate r. The second row of Table 10 shows that the average ratio of p_2 to equity is 4.7 and ranges from 1.4 at the 10^{th} percentile to 9.4 at the 90^{th} percentile. That this valuation is approximately twice as large as the entrepreneur's valuation of the business shows that the implicit discount rates are large.

Present Value of Riskless Income Flows, Discounted at r. We next calculate the value of the business if it had access to full insurance and faced no collateral constraints. In this case, the entrepreneur would be able to expand production and earn higher expected profits π_{it}^* corresponding to the capital and labor choices k_{it}^* and l_{it}^* in Section 4.2. The value of the business, denoted by p_{3i}, would then be equal to

$$p_{3i} = (1+r)\,a_i + \mathbb{E}\sum_{s=0}^{\infty}\frac{1}{(1+r)^s}\pi_{is}^*.$$

A comparison of p_{2i} and p_{3i} allows us to gauge the role of frictions in depressing the firm's capital and labor choices and, therefore, expected profits. The last row of Table 10 shows

Table 10: Valuation of Private Businesses

	mean	p10	p25	p50	p75	p90
p_1/a	2.1	1.2	1.4	1.8	2.4	3.3
p_2/a	4.7	1.4	1.9	3.0	5.4	9.4
p_3/a	17.4	2.0	3.3	7.4	17.7	39.0

Notes: All statistics are equity weighted.

that the average ratio of p_3 to equity is 17.4 and ranges from 2 at the 10^{th} percentile to 39 at the 90^{th} percentile. That the ratio of p_3 to p_2 is approximately 4 on average suggests that financial frictions play a sizable role in distorting microeconomic production choices.

We conclude that financial frictions play an important role in determining firm valuations, both by increasing discount rates and especially by depressing capital and labor. Because p_3 represents the value at which a firm would sell if the owner were able to access frictionless financial markets, for example by incorporating the business as in Boar and Midrigan (2019) and Peter (2021), while p_1 represents the actual value of the business to its owners, the sizable gap between these two valuations suggests that financial frictions, and especially imperfect risk sharing, have large microeconomic consequences.

5.3 Macroeconomic Implications

We next evaluate the macroeconomic consequences of financial frictions in our model. To do so, we note that dispersion in marginal returns to wealth in our model reflects differences in the marginal product of capital and labor across producers, that is, *misallocation*. Moreover, a high average level of marginal returns to wealth reflects a low aggregate capital-output ratio and a low labor share which depress output.

To quantify the amount of misallocation, we compare aggregate TFP in our economy with that in an economy without financial frictions. In the absence of frictions aggregate TFP would be equal to

$$Z_t^* = \frac{Y_t^*}{\left(K_t^\alpha L_t^{1-\alpha}\right)^\eta} = \left(\int \left(\mathbb{E}_{t-1}\left(z_{it}\varepsilon_{it}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{1-\eta},$$

reflecting that the efficient capital and labor choices are also made before the realization of the productivity shocks. In contrast, in our economy, aggregate TFP is

$$Z_t = \frac{Y_t}{\left(K_t^{\alpha} L_t^{1-\alpha}\right)^{\eta}} = \left(\int \left(\mathbb{E}_{t-1}\left(z_{it}\varepsilon_{it}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tau_{it}^{-\frac{1}{1-\eta}}\right)^{1-\eta},$$

where $\tau_{it} = \left(\left(\tau_{it}^{k}\right)^{\alpha}\left(\tau_{it}^{l}\right)^{1-\alpha}\right)^{\eta}$ reflects the wedge in the first order conditions for capital and labor induced by lack of risk-sharing and collateral constraints, and where the wedges τ_{it}^{k} and τ_{it}^{l} are equal to

$$\tau_{it}^{k} = \frac{\mathbb{E}_{t-1} y_{it}/k_{it}}{Y_t/K_t} \qquad \text{and} \qquad \tau_{it}^{l} = \frac{\mathbb{E}_{t-1} y_{it}/l_{it}}{Y_t/L_t}.$$

The first row of Table 11 shows that absent financial frictions aggregate productivity would be 5.7% higher compared to the baseline. Thus the losses from misallocation are relatively modest compared to those calculated by Hsieh and Klenow (2009), reflecting that financial frictions do not generate large rank reversals which, as Hopenhayn (2014) points out, are necessary to generate large TFP losses.

Consider next the implications for output. Since we normalize the aggregate labor supply to unity, aggregate output in our economy is equal to

$$Y_t = Z_t^{\frac{1}{1-\alpha\eta}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha\eta}{1-\alpha\eta}}.$$

Because the capital-output ratio in our economy is equal to 1.27, whereas absent financial frictions it would be equal to $\alpha\eta = 1.37$, output in the absence of financial frictions would be 8.4% higher, a gap that reflects both higher TFP as well as capital deepening. Finally, financial frictions also depress the labor share, which in our model is equal to 0.736 and absent financial frictions would be equal to 0.784. This decline in the labor share further reduces the equilibrium wage, which would be 15.5% higher absent financial frictions.

We therefore conclude that in the aggregate financial frictions have relatively modest effects on output and productivity, despite their large consequences for individual firms. This is because our macroeconomic calculations factor in the positive general equilibrium effect on the wage from removing financial frictions. In contrast, our business valuation calculations keep the wage that firms face unchanged.

Lastly, we show that risk, rather than the collateral constraint, is responsible for the bulk of the output and productivity losses from financial frictions. The second and third rows of Table 11 explore the consequences of removing the firms' ability to borrow ($\xi = 0$) and of eliminating the collateral constraint ($\xi = 1$). Notice that productivity would fall by 0.93% relative to our baseline economy if firms were unable to borrow, output would fall by 3.7%

Table 11: Macroeconomic Implications

	Z	Y	W
No financial frictions	5.73	8.39	15.51
No borrowing, $\xi = 0$	-0.93	-3.73	-2.03
No collateral constraint, $\xi = 1$	0.02	0.15	0.02

Notes: All numbers are percent deviations from the baseline model.

and wages by 2%. Eliminating the collateral constraint would, in contrast, have a negligible effect. We thus once again conclude that risk, as opposed to collateral constraints, accounts for the bulk of distortions due to financial frictions.

5.4 Are Average Returns Informative About Marginal Returns?

We next argue that dispersion in average returns is not necessarily evidence of dispersed marginal returns. We do this by studying three alternative economies in which we sequentially reduce the severity of financial frictions, and thus the dispersion in marginal returns. We show that all of these economies can reproduce the dispersion in average returns in the data, but have very different implications for the distribution of marginal returns. Namely, we study *(i)* an economy where labor is flexibly chosen, that is after productivity is realized, but capital is still chosen in advance as in Midrigan and Xu (2009) and Gopinath et al. (2017); *(ii)* an economy where both labor and capital are flexibly chosen, as in Buera et al. (2011); and *(iii)* an economy with flexible inputs and no collateral constraints.

We calibrate each of these models to match the same set of targets as in Table 5. As we show in the Appendix, we match these targets well, with three exceptions. First, the labor share is constant when labor is flexibly chosen, so the model cannot reproduce the time series fluctuations in individual firms' labor share we see in the data. Second, the model without a collateral constraint overstates the 90^{th} percentile of the capital to equity ratio, which is equal to 1.96 in the model and 1.73 in the data. More importantly, all these models imply that labor comoves one-to-one with output, so they predict much smaller fluctuations in profit shares compared to the data.

The discount factor β required to match the wealth to output ratio increases from 0.916 in the baseline model to 0.927 in the model with flexible labor, 0.936 in the model with flexible labor and capital, and 0.937 when we also eliminate the collateral constraint. Making financial frictions less severe reduces the incentives to save, so a higher discount factor is needed to reproduce the wealth accumulation in the data. The span of control parameter η required to match the profit share falls from 0.948 in the baseline model to 0.931 in the model with flexible labor, 0.917 in the model with flexible labor and capital, and 0.904 when we also eliminate the collateral constraint. Intuitively, profits in our baseline model accrue both to the fixed factor, $1-\eta$, as well as to wealth due to the presence of financial frictions. Reducing the severity of financial frictions increases the importance of the fixed factor.

Table 12 reports moments of the distribution of expected average returns and long-term returns implied by these models. For comparison, the top row of each panel reproduces the results from the baseline model. As the table shows, all models imply similar dispersion and persistence in average rates of return, suggesting that dispersion in average returns is not indicative of firms facing financial frictions and thus high marginal rates of return. Intuitively, as we reduce the severity of financial frictions, the importance of the fixed factor and heterogeneity in productivity increases, so the model can reproduce the dispersion in average returns without relying on financial frictions. We note, however, that the model with flexible inputs and no collateral constraints, in which wealth does not influence production choices, is greatly at odds with the data in that it predicts a much stronger negative relationship between average returns and equity: the slope of a rank-rank regression falls from -0.26 in our baseline model and in the data to -0.78.

Table 13 reports the distribution of the expected marginal returns across alternative models, computed both under the physical and risk-neutral probabilities. When labor is flexibly chosen, marginal returns are almost as dispersed as in our baseline, with a standard deviation of 6.3%. Importantly, most of this dispersion is now due to the collateral constraint: as Panel B of Table 13 shows, the risk-neutral expected marginal returns have a standard deviation of 5.4%. When both factors are flexible, the dispersion in expected marginal returns falls to 3.9% and, once again, mostly reflects the collateral constraint. Of course, the model with flexible factors and no collateral constraint predicts no dispersion in marginal returns.

The results above reveal that assuming that labor is chosen before observing productivity, a feature needed to match the evidence on the comovement of output and labor, greatly amplifies the importance of risk in generating dispersion in marginal returns. This is because labor represents a large fraction of firm output, so an economy with flexible labor choices

Table 12: Distribution of Expected Average Returns

	mean	std	p10	p50	p90
			A. $\mathbb{E}_{t-1}\pi_t/a_t$		
Baseline	0.095	0.084	0.025	0.065	0.205
Labor flexible	0.088	0.104	0.022	0.049	0.196
Both flexible	0.088	0.082	0.025	0.059	0.187
No frictions	0.085	0.134	0.022	0.039	0.192
			B. $\overline{\pi/a}$		
Baseline	0.092	0.070	0.024	0.073	0.185
Labor flexible	0.086	0.079	0.022	0.055	0.189
Both flexible	0.087	0.069	0.026	0.064	0.182
No frictions	0.081	0.125	0.022	0.039	0.195

Notes: All statistics are equity weighted. The "No frictions" economy is one in which there are no collateral constraints and both inputs are chosen flexibly.

generates much smaller fluctuations in income, which dampens risk premia. Figure 5 illustrates this point by contrasting the optimal choice of capital in the baseline model to that in the economy with flexible labor. We choose units for productivity so that in both of these economies the frictionless optimal level of capital is equal to 10. Notice that a firm chooses a much higher level of capital in the flexible labor model, owing to a lower risk premium. As the right panel of the figure shows, because the firm desires a higher level of capital when labor is flexible, the collateral constraint binds in a larger region of the state space and, consequently, the risk-adjusted return is much higher.

6 Extensions

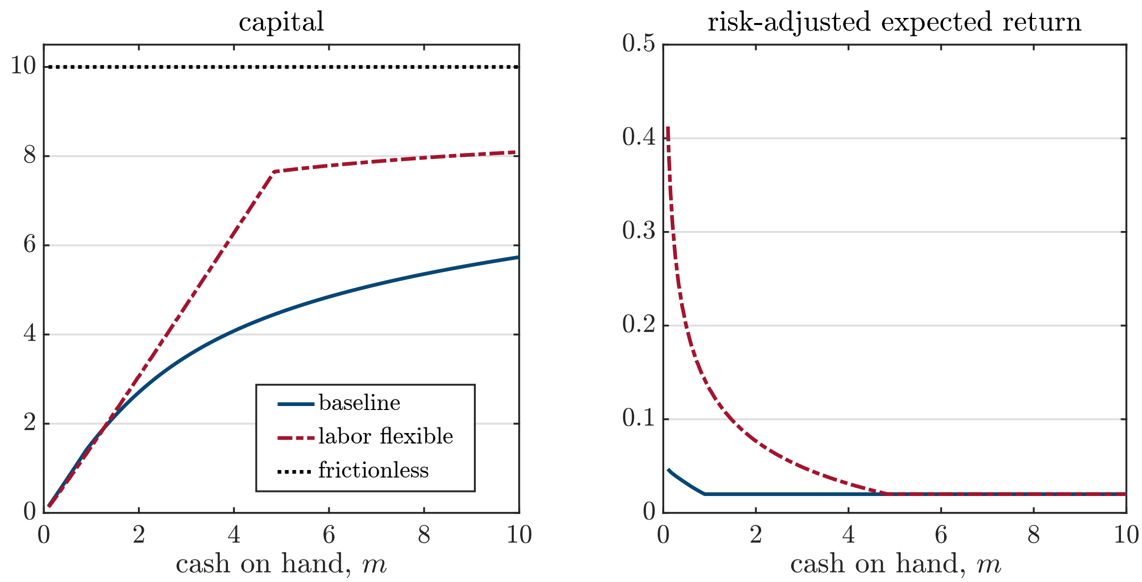
We next investigate the role of fat-tailed and transitory shocks, that of entrepreneurs' attitudes towards risk and address the concern that the high average returns in the data may reflect that the book value of capital is lower than the replacement value. We also show that

Table 13: Distribution of Expected Marginal Returns

	mean	std	p10	p50	p90
			A. Baseline		
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.051	0.060	0.020	0.025	0.115
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.021	0.003	0.020	0.020	0.020
			B. Labor flexible		
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.042	0.063	0.020	0.020	0.092
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.038	0.054	0.020	0.020	0.076
			C. Both flexible		
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.035	0.039	0.020	0.021	0.067
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.033	0.036	0.020	0.020	0.061

Notes: All statistics are equity weighted.

Figure 5: Decision Rules When Labor Is Flexible



our empirical findings for Spain are robust across countries.

6.1 Role of Fat-Tailed Shocks

We show that our conclusion that risk plays an important role is, to a large extent, driven by the fat-tailed nature of the shocks, an essential feature needed to match the distribution of output growth rates in the data. To do that we study an alternative economy in which productivity shocks are drawn from a Gaussian distribution and calibrate the model to match the same targets as in Table 5. As we show in the Appendix, this model fails to match the fat tails of the distribution of output growth rates, as measured by the ratio of the standard deviation to the interquartile range. We also note that the model requires a higher discount factor, $\beta = 0.931$, as opposed to 0.916 in the baseline, as well as a lower span of control parameter, $\eta = 0.934$, as opposed to 0.948 in the baseline.

As Panel A of Table 14 shows, the model with Gaussian shocks generates a similar distribution of expected average returns as the baseline model. In contrast, expected marginal returns are smaller and less dispersed, as shown in Panel B. For example, the standard deviation of expected marginal return falls from 6% in the baseline model to 3.7% in the model with Gaussian shocks. Moreover, these marginal returns now reflect to a much larger extent binding collateral constraints. As shown in Panel C, the standard deviation of risk-adjusted expected marginal returns increases from 0.3% in our baseline to 2.3% in the model without fat-tailed shocks.

6.2 Role of Transitory Shocks

We next show that without transitory shocks the model can also match the distribution of average returns, but predicts lower and less dispersed marginal returns, which now reflect binding collateral constraints.

As earlier, we recalibrate the model without transitory shocks to match the original targets in the data and report the results in the Appendix. Not surprisingly, the model without transitory shocks overstates the rate at which the standard deviation of output growth rates increases with the horizon and the rate at which the autocorrelation of output decays. Additionally, the model is also no longer able to match the large volatility of a firm's labor share around its time series mean. Since firms now face considerably less risk, the model requires a higher discount factor, $\beta = 0.931$, and a lower span of control, $\eta = 0.928$, to match the aggregate wealth and income to output ratios in the data.

Panels A and B of Table 14 show that the model without transitory shocks generates the same distribution of average returns as in the baseline, but predicts smaller and less dispersed

Table 14: Distribution of Expected Returns

	mean	std	p10	p50	p90
			A. Average		
Baseline	0.095	0.084	0.025	0.065	0.205
Gaussian	0.085	0.074	0.025	0.058	0.188
No transitory shocks	0.087	0.089	0.024	0.054	0.193
Lower risk aversion	0.086	0.068	0.026	0.063	0.176
Scaled capital	0.071	0.058	0.024	0.052	0.145
			B. Marginal		
Baseline	0.051	0.060	0.020	0.025	0.115
Gaussian	0.036	0.037	0.020	0.021	0.082
No transitory shocks	0.038	0.051	0.020	0.020	0.080
Lower risk aversion	0.038	0.041	0.020	0.021	0.076
Scaled capital	0.044	0.043	0.020	0.027	0.092
			C. Risk-adjusted		
Baseline	0.021	0.003	0.020	0.020	0.020
Gaussian	0.028	0.023	0.020	0.020	0.047
No transitory shocks	0.032	0.031	0.020	0.020	0.064
Lower risk aversion	0.020	0.001	0.020	0.020	0.020
Scaled capital	0.022	0.007	0.020	0.020	0.020

Notes: All statistics are equity weighted.

expected marginal returns. For example, the standard deviation of expected marginal returns is 5.1% in this model and 6% in the baseline. As shown in Panel C, collateral constraints are now responsible for a larger share of the dispersion in marginal returns: the standard deviation of the risk-adjusted returns is 3.1%, much larger than the 0.3% in the baseline.

6.3 Role of Preferences for Risk

Motivated by the important role we found for risk, we next investigate the extent to which our results are driven by the coefficient of relative risk aversion. The coefficient of relative risk aversion may be lower if the entrepreneur has access to other sources of insurance that we do not explicitly model. We now consider an economy with $\theta = 0.5$. We find that even though the dispersion in marginal returns falls, it is nevertheless large and, once again, mostly accounted for by risk rather than collateral constraints.

We recalibrate the model with $\theta = 0.5$ to match the same targets as in the baseline and report the results in the Appendix. With a lower coefficient of risk aversion the model requires a higher discount factor $\beta = 0.959$, and a lower span of control $\eta = 0.934$.

As Panel A of Table 14 shows, the model generates the same distribution of expected average returns as the baseline. Expected marginal returns are, however, one-third less dispersed than in the baseline: the standard deviation falls from 6% in the baseline to 4.1% with a lower risk aversion. As in our baseline model, risk-adjusted expected marginal returns are much smaller than the returns computed under the physical measure: their standard deviation is equal to 0.1%. Thus, even though the entrepreneurs' attitudes towards risk shape the overall dispersion in marginal returns, our conclusion that the bulk of this dispersion is due to risk premia as opposed to collateral constraints is robust.

6.4 Book Value of Capital

One concern is that the book value of capital in the Orbis data is too low compared to its replacement value, leading us to understate the overall amount of wealth that firms have and thus overstate average returns. Indeed, in the Orbis data for Spain, the aggregate capital-output ratio is only 1.43.[10] In contrast, the corresponding capital-output ratio in the EU-KLEMS data for Spain is 1.86, 30% larger.

To assess the quantitative importance of this potential bias, we scale up each firm's capital stock in the Orbis data by 30% and increase its equity to reflect the higher value of the firm's capital. We recalculate the targets in Table 5 and recalibrate the model to match the updated values. We report the results of the calibration in the Appendix. We note that with a higher capital stock and equity, the model requires a higher discount factor, $\beta = 0.932$, a higher span of control, $\eta = 0.964$, and a higher capital elasticity, $\alpha = 0.217$.

[10] This value is larger than the target we use in Table 5 because it includes all firms, including publicly traded ones.

Table 15: Distribution of Average Returns, Scaled Capital

	mean	std	p10	p25	p50	p75	p90	p95
				A. π/a, equity weighted				
Data	0.064	0.174	-0.032	0.008	0.048	0.108	0.199	0.282
Model	0.068	0.146	-0.012	0.017	0.035	0.088	0.181	0.267
				B. $\overline{\pi/a}$, equity weighted				
Data	0.063	0.094	-0.005	0.023	0.058	0.093	0.141	0.184
Model	0.068	0.052	0.021	0.029	0.053	0.092	0.135	0.166

Table 15 reports the distribution of average returns resulting from scaling up the capital stock, in the data and in the recalibrated model. Compared to Table 8, average returns in the data are smaller on average and less dispersed: the average return is now equal to 6.4%, lower than 8.1% previously, and the standard deviation is 17.4%, slightly lower than 21.7% previously. Long-run average returns are also slightly less dispersed: the standard deviation is 9.4%, lower than 11.7% previously. The recalibrated model matches these numbers well, especially at the top of the distribution.

Since agents now are more patient and save more compared to the baseline, the recalibrated model predicts less dispersion in both average and marginal returns. For example, as shown in Panels A and B of Table 14, the standard deviation of expected average returns is 5.8%, whereas that of marginal returns is 4.3%. As in our baseline model, most of the dispersion in marginal returns is due to risk, not collateral constraints: the standard deviation of risk-adjusted expected marginal returns is 0.7%.

Our conclusion that firms' capital and labor choices are considerably distorted by risk is therefore not an artifact of the low book value of capital in the Orbis data.

6.5 Evidence From Other Countries

In Section 3 we used data from Spain to show that average rates of return are dispersed, persistent, and negatively correlated with equity, that the distribution of output growth rates displays fat tails and that both labor and capital track output imperfectly at high frequencies. In this section, we document that these findings hold more generally. We focus on five other countries for which Orbis has relatively good coverage: Italy, France, Norway, Portugal and Slovakia.

Table A.2 shows that average rates of return are dispersed and persistent in all these countries. Table A.3 shows that the slope coefficient in a rank-rank regression of average returns on equity ranges from -0.31 to -0.25, numbers similar to that we find for Spain. Table A.4 shows that the standard deviation of output growth rates is higher than the interquartile range, and that the kurtosis of the output growth rate distribution ranges from 16 to 20 across countries. Lastly, Table A.5 shows that the coefficient of a regression of changes in the logarithm of the wage bill agains log output ranges from 0.42 to 0.58, and that of changes in log capital against log output ranges from 0.22 to 0.34.

7 Conclusion

In this paper we ask: What accounts for the heterogeneity in average returns to private business wealth? To answer this question, we first use micro data from Orbis on firm level balance sheets and income statements to document that differences in average returns for privately held businesses are large, persistent and negatively correlated with equity. We also document that firms experience large, fat-tailed, and partly transitory changes in output that are not accompanied by equally sized changes in their capital stock and wage bill. This implies that fluctuations in output are accompanied by large changes in firm profits.

We then study a model of entrepreneurial dynamics that is quantitatively consistent with this evidence. The model accounts well for the dispersion in average returns in the data. We use the model to back out the distribution of marginal returns to saving in the business. We find that marginal returns are large and dispersed, considerably depressing the valuation of firms. Marginal returns are three fourths as dispersed as average returns, mostly reflecting risk, as opposed to collateral constraints. Though financial frictions greatly depress individual firms' production choices, cash flow and firm values, they generate relatively modest TFP and output losses in the aggregate, suggesting that the gains from policies that increase the wealth share of productive entrepreneurs are relatively low.

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Appendix

A Data

A.1 Summary Statistics

Table A.1 reports summary statistics for the full sample of firms.

Table A.1: Summary Statistics, Full Sample

	mean	p10	p25	p50	p75	p90
output	455	29	62	142	331	749
labor	328	22	48	111	253	561
capital	556	5	19	75	273	789
equity	693	-2	18	84	302	948
income	51	-26	-1	5	27	94
employment	12	1	2	4	10	21

Notes: Numbers are expressed in thousands of 2015 USD and are based on 6.6 million firm-year observations.

A.2 Measurement Error

Figure A1 shows that the negative relationship between returns and net worth holds when we rank firms by their net worth in the previous year. Though the slope is less steep relative to Figure 1 which ranks firms by the current equity, the negative relationship between returns and equity is evident here as well. We also note that a similar negative relationship is apparent when we rank firms by other characteristics that are correlated with equity, such as capital, labor or output. Thus, for measurement error to explain the negative relationship we document, it must be correlated across time as well as with other measures of firm size.

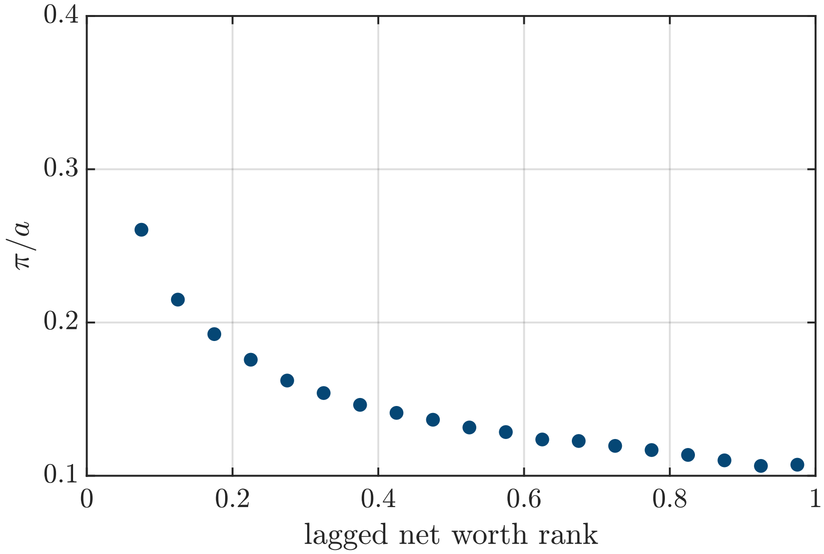
To bound the amount of measurement error that would lead us to interpret a zero or positive correlation between returns and net worth as negative, let $\hat{a}$ denote measured net worth and assume that it is equal to

$$\hat{a} = a\mu_a,$$

where a denotes true net worth and μ_a is multiplicative measurement error that is uncorrelated with a. The measured return $\hat{r}$ is then equal to

$$\hat{r} = \frac{\pi}{\hat{a}} = \frac{\pi}{a\mu_a}.$$

Figure A1: Rates of Return and Lagged Equity



Notes: As in the baseline, we exclude the bottom 5% of the lagged net worth rank, which have an average return on equity of 0.4.

The covariance between the logarithm of measured returns and measured net worth is

$$Cov\left(\log \hat{r}, \log \hat{a}\right) = Cov\left(\log \frac{\pi}{a} - \log \mu_a, \log a + \log \mu_a\right) = Cov\left(\log \frac{\pi}{a}, \log a\right) - Var\left(\log a\right).$$

The first term is the true covariance between the logarithm of returns and net worth and the second is the variance of measurement error. If returns and net worth are in fact uncorrelated, then the negative of the covariance between the logarithm of measured returns and measured net worth captures the variance of the measurement error and the square root of that is the standard deviation of measurement error. In our sample the latter represents approximately 40% of the standard deviation of the logarithm of net worth, so for us to interpret a zero or positive relationship between returns and net worth as negative it must be that at least 40% of the dispersion in net worth in the data arises as a result of measurement error.

A.3 Facts on Average Returns in Other Countries

In this section we revisit the main facts reported in Section 3 for five other countries in the Orbis data: Italy, France, Norway, Portugal and Slovakia. We also show that our results are robust to not restricting the Orbis sample to firms with at least ten years of data.

Dispersion and Persistence in Average Returns. Panel A of Table A.2 reports moments of the cross-sections distribution of returns, π/a. Panel B of the table reports the distribution of long-term returns, $\overline{\pi/a}$.

Table A.2: Average Rates of Return in Other Countries

	mean	p10	p25	p50	p75	p90	p95
				A. π/a			
Italy	0.06	-0.06	0.00	0.04	0.12	0.24	0.35
France	0.14	-0.02	0.04	0.11	0.21	0.36	0.50
Norway	0.18	-0.01	0.04	0.12	0.25	0.48	0.68
Portugal	0.08	-0.02	0.01	0.06	0.14	0.25	0.34
Slovakia	0.08	-0.03	0.00	0.02	0.11	0.27	0.43
Spain*	0.08	-0.04	0.01	0.05	0.13	0.25	0.37
				B. $\overline{\pi/a}$			
Italy	0.06	-0.04	0.00	0.05	0.11	0.18	0.25
France	0.13	0.00	0.05	0.11	0.18	0.30	0.41
Norway	0.18	0.02	0.06	0.12	0.25	0.42	0.64
Portugal	0.08	-0.01	0.03	0.06	0.12	0.19	0.25
Slovakia	0.07	-0.02	0.00	0.03	0.12	0.24	0.35
Spain*	0.08	-0.01	0.02	0.07	0.11	0.19	0.25

Notes: All statistics are equity weighted. Spain* refers to the unrestricted data for Spain.

Average Returns and Equity Are Negatively Correlated. Table A.3 reports the rank-rank slope of a regression of returns π/a on equity a.

Output Growth Rates Are Dispersed and Fat-Tailed. Table A.4 reports the standard deviation, inter-quartile range and the kurtosis of the distribution of output growth rates.

Capital and Labor Do Not Track Output Closely. Table A.5 reports the slope coefficients of regressions of the growth rate of labor and capital againts the growth rate of output.

Table A.3: Rates of Return and Equity in Other Countries

	rank-rank slope
Italy	-0.25
France	-0.27
Norway	-0.31
Portugal	-0.28
Slovakia	-0.27
Spain*	-0.32

Notes: Spain* refers to the unrestricted data for Spain.

Table A.4: Distribution of Output Growth Rates in Other Countries

	s.d.	iqr	kurtosis
Italy	0.41	0.27	17.5
France	0.29	0.21	19.7
Norway	0.34	0.24	19.3
Portugal	0.47	0.31	16.3
Slovakia	0.49	0.34	14.9
Spain*	0.50	0.33	13.0

Notes: Spain* refers to the unrestricted data for Spain.

B Parameterization of Alternative Models

Tables B.1 and B.2 report the targeted moments and calibrated parameter values for the alternative models discussed in Section 5.4.

Tables B.3 and B.4 report the targeted moments and calibrated parameter values for the alternative models discussed in Section 6.

Tables B.5 and B.6 report the targeted moments and calibrated parameter values for the economy with a higher level of capital-output ratio discussed in Section 6.4.

Table A.5: Comovement Between Capital, Labor and Output in Other Countries

	$\Delta \log l$	$\Delta \log k$
Italy	0.576	0.253
	(0.001)	(0.002)
France	0.549	0.258
	(0.001)	(0.002)
Norway	0.510	0.222
	(0.003)	(0.006)
Portugal	0.424	0.293
	(0.002)	(0.004)
Slovakia	0.429	0.341
	(0.006)	(0.011)
Spain*	0.588	0.318
	(0.001)	(0.001)

Notes: The sample is restricted to observations for which $|\Delta \log y| < 0.5$. Standard errors in parentheses are clustered at the firm level. Spain* refers to the unrestricted data for Spain.

Table B.1: Targeted Moments, Remove Frictions

	Data	Baseline	Labor Flexible	Both Flexible	No Frictions
s.d. $\log y_{it}$	1.26	1.31	1.26	1.26	1.26
s.d. $\log y_{it}/y_{it-1}$	0.41	0.37	0.38	0.40	0.38
s.d. $\log y_{it}/y_{it-2}$	0.52	0.51	0.52	0.52	0.51
s.d. $\log y_{it}/y_{it-3}$	0.60	0.62	0.62	0.61	0.61
iqr $\log y_{it}/y_{it-1}$	0.28	0.27	0.31	0.28	0.28
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	0.41	0.40	0.42
iqr $\log y_{it}/y_{it-3}$	0.52	0.54	0.50	0.51	0.53
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	0	0	0
corr $\log y_{it}$, $\log y_{it-1}$	0.95	0.96	0.95	0.95	0.95
corr $\log y_{it}$, $\log y_{it-2}$	0.91	0.92	0.92	0.92	0.92
corr $\log y_{it}$, $\log y_{it-3}$	0.88	0.89	0.88	0.88	0.88
aggregate a/y	1.57	1.55	1.57	1.56	1.58
aggregate k/y	1.24	1.27	1.24	1.24	1.24
aggregate l/y	0.71	0.74	0.75	0.75	0.75
aggregate π/y	0.12	0.14	0.14	0.14	0.13
p90 k/a	1.73	1.72	1.73	1.73	1.96

Table B.2: Parameter Values, Remove Frictions

		Baseline	Labor Flexible	Both Flexible	No Frictions
β	discount factor	0.916	0.927	0.936	0.937
α	capital elasticity	0.173	0.198	0.187	0.165
η	span of control	0.948	0.931	0.917	0.904
ξ	max loan to value	0.437	0.421	0.420	–
ρ_z	AR(1) z	0.926	0.935	0.949	0.962
σ_z	std. dev. z shocks	0.041	0.053	0.040	0.043
σ_e	std. dev. e shocks	0.219	0.087	0.021	0.016
h	Tukey h parameter	0.374	0.417	0.333	0.401

Table B.3: Targeted Moments, Extensions

	Data	Baseline	No Fat Tails	No Transitory Shocks	Lower $\theta = 0.5$
s.d. $\log y_{it}$	1.26	1.31	1.26	1.26	1.26
s.d. $\log y_{it}/y_{it-1}$	0.41	0.37	0.37	0.41	0.38
s.d. $\log y_{it}/y_{it-2}$	0.52	0.51	0.52	0.61	0.52
s.d. $\log y_{it}/y_{it-3}$	0.60	0.62	0.63	0.74	0.62
iqr $\log y_{it}/y_{it-1}$	0.28	0.27	0.43	0.28	0.27
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	0.63	0.48	0.41
iqr $\log y_{it}/y_{it-3}$	0.52	0.54	0.78	0.66	0.53
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	0.12	0.03	0.12
corr $\log y_{it}$, $\log y_{it-1}$	0.95	0.96	0.96	0.95	0.95
corr $\log y_{it}$, $\log y_{it-2}$	0.91	0.92	0.91	0.88	0.92
corr $\log y_{it}$, $\log y_{it-3}$	0.88	0.89	0.87	0.83	0.88
aggregate a/y	1.57	1.55	1.57	1.57	1.57
aggregate k/y	1.24	1.27	1.24	1.24	1.24
aggregate l/y	0.71	0.74	0.75	0.75	0.75
aggregate π/y	0.12	0.14	0.13	0.14	0.14
p90 k/a	1.73	1.72	1.72	1.72	1.72

Table B.4: Parameter Values, Extensions

		Baseline	No Fat Tails	No Transitory Shocks	Lower $\theta = 0.5$
β	discount factor	0.916	0.931	0.931	0.959
α	capital elasticity	0.173	0.177	0.184	0.173
η	span of control	0.948	0.934	0.928	0.934
ξ	max loan to value	0.437	0.420	0.420	0.420
ρ_z	AR(1) z	0.926	0.935	0.907	0.944
σ_z	std. dev. z shocks	0.041	0.036	0.054	0.053
σ_e	std. dev. e shocks	0.219	0.118	–	0.362
h	Tukey h parameter	0.374	–	0.322	0.436

Table B.5: Targeted Moments, Scaled Capital

	Data	Model		Data	Model
s.d. $\log y_{it}$	1.26	1.27	aggregate a/y	1.95	1.94
s.d. $\log y_{it}/y_{it-1}$	0.41	0.39	aggregate k/y	1.61	1.62
s.d. $\log y_{it}/y_{it-2}$	0.52	0.52	aggregate l/y	0.71	0.71
s.d. $\log y_{it}/y_{it-3}$	0.60	0.61	aggregate π/y	0.12	0.13
iqr $\log y_{it}/y_{it-1}$	0.28	0.28	corr $\log y_{it}$, $\log y_{it-1}$	0.95	0.95
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	corr $\log y_{it}$, $\log y_{it-2}$	0.91	0.92
iqr $\log y_{it}/y_{it-3}$	0.52	0.53	corr $\log y_{it}$, $\log y_{it-3}$	0.88	0.88
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	p90 k/a	1.75	1.73

Table B.6: Parameter Values, Scaled Capital

β	0.932	discount factor	ρ_z	0.930	AR(1) z
α	0.217	capital elasticity	σ_z	0.031	std. dev. z shocks
η	0.964	span of control	σ_e	0.255	std. dev. e shocks
ξ	0.454	max loan to value	h	0.376	Tukey h parameter