

Appendix for “Inventories, Markups, and Real Rigidities in  
Menu Cost Models”

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## 1. Robustness

We report on a number of additional experiments that we have conducted. The goal of these experiments is to isolate the role of several assumptions on technology that we have made, and study the robustness of our results with respect to the process for monetary policy, the rate at which inventories depreciate, and the nature of capital adjustment costs. In all these experiments, unless otherwise specified, we leave the model's parameters other than those subject to investigation unchanged, with the exception of the capital adjustment costs, which are chosen in each experiment so that the model reproduces the relative variability of investment in the data.

### A. Extensions of the Original Model

We next consider several perturbations of the assumptions on technology and the labor market frictions we have made in the original model.

#### *Eliminating Decreasing Returns*

We have assumed above that intermediate good producers face decreasing returns to scale, the size of which is governed by  $\gamma = 0.9$ . Panel 1 of Table A1 shows that the model's predictions change little if we assume constant returns to scale by setting  $\gamma = 1$ . Marginal costs of production increase a bit less rapidly now, and firms face a somewhat stronger incentive to invest in inventories. This effect is fairly small quantitatively, and the model predicts an elasticity of inventories to sales of 0.49 (0.34 in the data and 0.25 in the baseline parameterization), and an elasticity of inventory investment to output of 0.16 (0.12 in the data and 0.09 in the baseline parameterization).

#### *Increasing Returns and Variable Capital Utilization*

Existing research has proposed a number of mechanisms that ensure that the marginal cost of production reacts gradually to monetary shocks. One approach, in addition to simply eliminating capital from the production function, is to assume increasing returns to scale, by increasing  $\gamma$  above unity. An alternative approach is to allow firms to vary the intensity with which they utilize the available stock of capital. Here we show that such approaches also have counterfactual implications for the dynamics of inventories. Our results are thus robust to the exact mechanism that gives rise to sticky marginal costs.

We first assume increasing returns to scale by setting  $\gamma = 1.25$ . As Panel 1 of Table A1

shows, markups are less countercyclical now, since costs of production increase more gradually. This implies, however, a stronger intertemporal substitution motive: the elasticity of inventories to sales is now equal to 1.19, much greater than in the data.

We then introduce variable capital utilization by modifying the production function to

$$y(s^t) = \left( l(s^t)^\alpha [u(s^t) k(s^{t-1})]^{1-\alpha} \right)^\gamma, \quad (1)$$

where  $u(s^t)$  is the degree of capital utilization. We assume, as Dotsey and King (2006) do, that a greater rate of utilization raises the rate at which capital depreciates, so that

$$k(s^t) = (1 - \delta(u(s^t))) k(s^{t-1}) + x(s^t),$$

where  $\delta' > 0$ . We choose the second derivative of  $\delta(\cdot)$  to ensure that the capital services used,  $u(s^t) k(s^t)$ , are two-thirds as volatile as the labor input, and almost ten times more volatile than in the model without variable utilization. Once again we find that the model produces much more volatility of inventories than in the data: the elasticity of inventories to sales is equal to 1.57, since costs of production increase much more gradually.

### ***Role of Wage Rigidities***

We next show that wage rigidities are also important for the ability of our model to account for the data. To see this, we set  $\lambda_w = 0$  in our baseline parameterization and eliminate wage rigidities. The marginal cost is now very volatile in response to monetary shocks, and overshoots initially as a consequence of the sharp increase in the real wage (due to wealth effects and the greater disutility from work) and also due to the gradual adjustment of the stock of capital. Firms choose to sell out of existing inventories in anticipation of future cost declines and inventory investment counterfactually decreases. As Table A1 shows, the model with flexible wages predicts that inventories are strongly countercyclical, with an elasticity of inventories to sales equal to -1.2 and an elasticity of inventory investment to output of -0.68. Once again, the model's counterfactual properties can be traced back to its implications for markups: now that costs sharply increase following a monetary expansion, the drop in markups is extremely large. Since the marginal cost overshoots the increase in the money

stock, the drop in markups accounts for more than 100% (in fact almost 300%) of the increase in consumption.

### *Convex Ordering Costs*

Our paper’s results are reminiscent of the finding that New Keynesian models with capital produce highly volatile investment fluctuations. As we discuss below, the typical solution that researchers have adopted to slow down the variability of investment is to introduce convex capital adjustment costs. Such a solution works equally well in the context of inventories. To see this, we modify the distributor’s problem by assuming a convex cost of ordering:

$$\max_{P_i(s^t), z_i(s^t)} \sum_{t=0}^{\infty} \int_{s^t} Q(s^t) \left( P_i(s^t) q_i(s^t) - \Omega(s^t) y_i(s^t) - \Omega(s^t) \frac{\eta}{2} [y_i(s^t) - \bar{y}]^2 \right) ds^t \quad (2)$$

where  $\eta$  determines the size of the ordering cost and  $\bar{y}$  is a constant. Given the adjustment cost, the distributor’s marginal cost of acquiring inventories is equal to  $\Omega(s^t) (1 + \eta [y_i(s^t) - \bar{y}])$  and is thus increasing in the amount the distributor orders.

Consider, in Table A1, the effect of introducing such ordering costs in the model where labor is the only factor of production and in which the marginal cost of production is very sticky. We choose  $\eta = 0.95$  so as to match the variability of the inventory stock in the data. The model accounts well, almost by construction, for the volatility of inventories and inventory investment. Importantly, the model’s ability to match the data is an outcome of the fact that markups, defined as the ratio of the price,  $P_i(s^t)$ , to the marginal cost of orders,  $\Omega(s^t) (1 + \eta [y_i(s^t) - \bar{y}])$  — the object that matters for the pricing and inventory decisions — are now strongly countercyclical. The decline in markups now accounts for 96% of the real effects of monetary policy shocks, much more than in the corresponding economy without convex ordering costs. Once again, the exact source of variability in marginal costs (decreasing returns to scale, adjustment costs, etc.) is not crucial for our results. Rather, any mechanism that prevents distributors from taking advantage of the rigidities in wages by buying inputs cheaply allows the model to account for the variability of inventories in the data and implies an important role for markup variation in accounting for the real effects of monetary shocks.

## B. Alternative Specifications of Monetary Policy

We have assumed earlier that money growth is serially uncorrelated in order to allow the model to account for the drop in nominal interest rates following an expansionary monetary shock. Consider next several alternative specifications of the process for monetary policy.

### *Persistent Money Growth*

We first assume that the money growth rate follows an autoregressive process,

$$g_m(s^t) = \rho_m g_m(s^{t-1}) + \varepsilon_m(s^t),$$

where we set  $\rho_m$ , the serial correlation of money growth rates, equal to 0.61, a number that Kehoe and Midrigan (2010) show best approximates the exogenous component of money growth in the data. Panel 2 of Table A1 shows that when money growth is persistent, our baseline parameterization with decreasing returns and sticky prices produces a slightly countercyclical inventory stock: the elasticity of inventories to sales is equal to -0.17. Net inventory investment remains procyclical, with an elasticity to output of about 0.11<sup>1</sup>.

The reason the inventory stock declines here is that when money growth is persistent, the nominal interest rate increases after an expansionary monetary shock due to a strong expected inflation effect. Higher nominal interest rates make it costlier for firms to hold inventories. Note that this increase in interest rates is not sufficient to overturn the model's predictions for the economies with labor as the only factor or with flexible prices. In both of these economies the inventory stock and inventory investment expand too much relative to the data.

Since nominal interest rates decline after expansionary monetary shocks in the data, we next modify our model to allow it to reproduce this fact even in the presence of serially correlated money growth shocks.<sup>2</sup> We do so by assuming (external) habit persistence in preferences. These preferences

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<sup>1</sup>Note that the *stock* is countercyclical, while the *change* in the stock is procyclical. There is no contradiction here: inventory investment declines in the first few months after the shock (thus persistently reducing the stock), but increases thereafter. The different signs of the two correlations thus reflect differences in the persistence of the stock of inventories and that of inventory investment.

<sup>2</sup>See Canzoneri, Cumby and Diba (2007), who document how the interest rate implications of standard consumption-based Euler equations are grossly at odds with the data.

imply that the nominal interest rate is equal to

$$1 + i(s^t) = \left[ \beta \int \frac{1}{g_m(s^{t+1})} \left( \frac{c(s^{t+1}) - \omega_c c(s^t)}{c(s^t) - \omega_c c(s^{t-1})} \right)^{1-\sigma} \pi(s^{t+1}|s^t) ds^{t+1} \right]^{-1}, \quad (3)$$

where  $\omega_c$  determines the extent of habit persistence.

We choose  $\omega_c = 0.91$  so that the model reproduces the 2 percentage points decline in interest rates in the aftermath of a 1% monetary policy shock. This degree of habit persistence is also consistent with the estimates of habit persistence in Christiano, Eichenbaum and Evans (2005) at the quarterly frequency ( $0.87 = 0.65^{\frac{1}{3}}$  at our monthly frequency).

Note in Panel 3 of Table A1 that our baseline parameterization with decreasing returns and sticky prices now reproduces the slightly procyclical stock of inventories in the data: the elasticity of inventories to sales is equal to 0.18.

### ***Taylor Rule***

We finally assume that monetary policy is described by a Taylor rule. We follow Smets and Wouters (2007) and assume that the monetary authority chooses its instrument so as to ensure that the nominal interest rate evolves according to

$$\begin{aligned} i(s^t) &= c_i + \rho_i i(s^{t-1}) + c_1 \Delta \log P(s^t) + c_2 \log y(s^t) + c_3 \Delta \log y(s^t) + \varepsilon_{it} \\ \varepsilon_{it} &= \rho_\varepsilon \varepsilon_{it-1} + \epsilon_{it} \end{aligned}$$

where  $\Delta \log P(s^t)$  is inflation,  $y(s^t)$  is output and  $\epsilon_{it}$  is a disturbance. As is standard in recent studies, we assume interest rate smoothing, captured by the term  $\rho_i$  on the lagged nominal interest rate, as well as that the nominal interest rate reacts to deviations of inflation, output and the output growth rate from their steady-state level. We use U.S. data to estimate the parameters in this interest rate rule<sup>3</sup> and then study the response of our economy to a monetary expansion given by a negative shock  $\epsilon_{it}$ . With such an interest rate rule, the nominal and real interest rates

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<sup>3</sup>We use data for the post-Volcker period, 1982:01 to 2009:12 on the Fed Funds rate, CPI inflation (excl. food and energy) and industrial production. The estimated coefficients are  $c_i = 0.01$ ,  $\rho_i = 0.981$ ,  $c_1 = 0.711$ ,  $c_2 = 0.052$ ,  $c_3 = 0.071$ ,  $\rho_\varepsilon = 0.458$  and  $\sigma_\varepsilon = 0.0056$ .



persistently decline following a monetary policy expansion, as in the data.

Panel 4 of Table A1 shows that the results for this economy are essentially identical to those in our original experiments with serially uncorrelated money growth shocks. The economy with sticky prices and decreasing returns accounts well for the data on inventories and implies that the drop in markups is responsible for almost 90% of the increase in consumption after a negative interest rate shock. In contrast, economies with flexible prices or with labor as the only factor of production do much more poorly.

### C. Inventory Carrying Costs

We earlier set the rate at which inventories depreciate,  $\delta_z$ , equal to 1.1% per month so that the model can simultaneously account for the inventory-sales ratio of 1.4 in the data and the 5% frequency of stockouts. We next conduct several experiments in which we increase  $\delta_z$  and also allow the depreciation rate to be a convex function of the amount of inventories held.

#### *Higher Inventory Depreciation*

We first increase  $\delta_z$  to 2.5%, a number in the mid-range of those reported by Richardson (1995).<sup>4</sup> With this choice we can no longer simultaneously match the 5% frequency of stockouts and 1.4 inventory-sales ratio in the data. If we keep the standard deviation of demand shocks at 0.63, its original value, the model produces a frequency of stockouts of 9% and an inventory-sales ratio of 1.0. Reproducing the inventory-sales ratio in the data would require a much greater volatility of demand shocks, but that parameterization would produce a much larger frequency of stockouts than observed in the data. Since the standard deviation of demand shocks has a negligible impact on the model's impulse responses to monetary shocks (see the experiment below), we leave the standard deviation of demand shocks at its original value of 0.63 in this experiment.

Panel 5 of Table A1 shows that when the rate of depreciation is equal to 2.5%, our baseline parameterization predicts that the stock of inventories is somewhat less volatile: the elasticity of inventories to sales is equal to 0.10 (0.25 originally), and the elasticity of net inventory investment to output is equal to 0.01. Intuitively, the higher inventory carrying cost reduces the intertemporal substitution motive induced by the decline in nominal interest rates. Panel 5 also shows that the

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<sup>4</sup>Richardson reports annual inventory carrying costs (excluding the “cost of money” which is already accounted for in our model) that range from 19% to 43%, implying monthly carrying costs around 1.5-3.5%.

models with labor as the only factor of production or flexible prices are also characterized by less volatility in inventories and inventory investment, but much more than in the data. The elasticity of inventories to sales is equal to 2.75 in the model with labor only and 1.44 in the model with flexible prices, both much higher than in the data.

Raising the depreciation rate even further, to 3.5%, in the upper range of the numbers reported by Richardson, does not change these counterfactual implications very much. When  $\delta_z = 3.5\%$ , the elasticity of inventories to sales is equal to 0.01 in our baseline parameterization, 2.3 in the model with only labor and 1.38 in the model with flexible prices (these numbers are not reported in the table). Hence, our conclusion that models without variable markups are not capable of accounting for the response of inventories to a monetary shock are robust to allowing for much greater rates of depreciation.

### *Convex Carrying Cost*

We next allow the inventory carrying cost to be a convex, rather than linear, function of the inventory stock. In particular, we now assume that the rate at which inventories of distributor  $i$  depreciate is equal to

$$\delta_{z,i}(s^t) = \delta_0 + \frac{\delta_1}{2}n_i(s^t),$$

where  $\delta_0$  and  $\delta_1$  are parameters. The cost of storing  $n_i(s^t)$  units of inventories is now equal to  $\delta_0 n_i(s^t) + \frac{\delta_1}{2}n_i(s^t)^2$  and is thus a quadratic function of the stock.

Consider next our choice of  $\delta_0$  and  $\delta_1$ . For any given  $\delta_0$ , the value of  $\delta_1$  determines the steady-state inventory carrying cost. Since our original experiment has considered one extreme parameterization ( $\delta_0 = 1.1\%$  and  $\delta_1 = 0$ ), we now consider the alternative extreme in which the marginal cost of carrying an additional unit of inventories is equal to 0 when the stock is equal to 0 ( $\delta_0 = 0$ ). We then set the second parameter,  $\delta_1$ , equal to 0.0063, so that the model continues to reproduce the 1.4 inventory-sales ratio and the 5% frequency of stockouts in the data.

Panel 6 of Table A1 shows that all of our results are robust to allowing for a convex cost of carrying inventories. In all models we consider inventories are somewhat less volatile than in our original experiments, since an increase in the inventory stock raises the inventory carrying cost, but the effect is fairly small. The elasticity of the inventory stock to sales is equal to 0.15 in the baseline parameterization with decreasing returns and sticky prices, 3.5 in the model with only labor and

1.38 in the model with flexible prices. We have also studied an economy in which the inventory carrying cost is cubic in the stock, so that  $\delta_{z,i}(s^t) = \frac{\delta_1}{3} n_i(s^t)^2$  and have also found similar results. In particular, the elasticity of inventories to sales is equal to 0.06 in the baseline parameterization, 3.17 in the model with only labor and 1.19 in the model with flexible prices (these numbers are not reported in the table).

#### D. Lower Inventory-Sales Ratio

We next ask whether our results are robust to assuming a lower steady-state inventory-sales ratio. One concern about our original calibration is that the stock of inventories in the data reflects inventories of final goods as well as of intermediate goods. Since in our model firms only hold inventories of final goods, the concern is that our choice of an inventory-sales ratio of 1.4 is too high relative to the data.<sup>5</sup>

Here we ask whether our results are robust to reducing the volatility of demand shocks to  $\sigma_v = 0.356$  so that the model matches an inventory-sales ratio of 0.7, half of that in our original setup. We leave all other parameters of the model unchanged and report the results of these experiments in Panel 7 of Table A1.

We find that the model's implications for the response of inventories to monetary shocks change little. The elasticity of inventories to sales is now equal to 0.37 in the baseline parameterization, only slightly greater than originally. As in the original experiment, the models with labor as the only factor of production and flexible prices predict a much more volatile stock of inventories.

#### E. Productivity Shocks

In the next set of experiments, we study the model's responses to aggregate productivity shocks. In particular, we modify the technology of intermediate goods producers to

$$y(s^t) = a(s^t) \left( l(s^t)^\alpha k(s^{t-1})^{1-\alpha} \right)^\gamma,$$

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<sup>5</sup>Note, however, that the inventory-sales ratio is also equal to 1.4 in the retail sector which only holds inventories of finished goods.

where productivity,  $a(s^t)$ , follows an AR(1) process:

$$\ln a(s^t) = \rho_a \ln a(s^{t-1}) + \varepsilon_a(s^t).$$

We choose the persistence of productivity shocks,  $\rho_a$ , and the standard deviation of innovations,  $\sigma_a$ , so that the model reproduces a quarterly autocorrelation of productivity of 0.95 and a standard deviation of shocks of 0.7%, typical numbers in the RBC literature. Table A2 reports the results of these experiments.<sup>6</sup>

### ***RBC Economy***

Column A of Table A2 reports results for an economy driven solely by productivity shocks, which we refer to as the *RBC economy*. To make our analysis comparable to that of Khan and Thomas (2007), who study the effect of productivity shocks in an  $(S, s)$  inventory model, we also shut down the New Keynesian elements of the model, namely the price and wage rigidities, as well as the capital adjustment costs. The table shows that this economy reproduces well the inventory statistics in the data.<sup>7</sup> The stock of inventories is somewhat more volatile than in the data, but the inventory-sales ratio is nevertheless countercyclical, as in the Khan and Thomas (2007) economy. The model's predictions for the relative volatility of output, sales and inventory investment also line up well with the data.

### ***Productivity Shocks in a New Keynesian Model***

Panel B of Table A2 reports the predictions of a model driven solely by productivity shocks, but in which prices and wages are sticky and investment in capital is subject to adjustment costs. Wage and price rigidities make inventories much more volatile relative to the data: the elasticity of inventories to sales is equal to 2.5, and that of inventory investment to output is equal to 0.62, both much larger relative to the data.

To show why the model with only productivity shocks and sticky prices and wages does so poorly, Figure A1 presents the impulse responses to a productivity shock in this model. An increase

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<sup>6</sup>Since our model is monthly, we set  $\rho_a = 0.984$  and  $\sigma_a = 0.004$ .

<sup>7</sup>The data column now reports the unconditional HP-filtered statistics, not those conditional on monetary shocks as earlier.

in productivity immediately lowers the marginal cost of production, but prices only gradually decline. Hence, a productivity shock is associated with a sharp increase in firm markups, making inventories more valuable. Consequently, production increases much more than sales do and the inventory stock is very volatile.

We conclude that price rigidities worsen the model's predictions for how inventories respond to productivity shocks. While a model with constant markups accounts for the data well, price rigidities imply strongly procyclical markups and a more volatile stock of inventories than that observed in the data.

These counterfactual responses of inventories in sticky price models are due to the fact that price rigidities undermine firms' ability to keep markups from increasing in response to a positive productivity shock. Dupor, Han and Tsai (2009) show, however, that prices are in fact very flexible in response to productivity shocks in the data.<sup>8</sup> Reconciling the models with the data thus requires a framework in which prices can respond to technological disturbances, but not to monetary policy shocks, perhaps due to frictions that segment goods and asset markets and imply a sluggish response of prices to monetary shocks but not to other shocks.<sup>9</sup>

### ***Productivity and Monetary Shocks in a New Keynesian Model***

Column C of Table A2 reports the model's predictions for an economy driven simultaneously by monetary and productivity shocks and in which prices and wages are sticky. When we introduce both types of shocks, the model's ability to account for the inventory data depends on the relative contribution of monetary and productivity shocks to the business cycle. When we feed the model the interest rate rule we have estimated above for the U.S. data, we find, as Table A2 reports, that the model reproduces the behavior of inventories in the data well. If anything, the model now does even better than the model without productivity shocks. The model now matches the low elasticity of inventories to sales in the data, and yet produces a time series for inventory investment that is almost as volatile as in the data. Recall that inventory investment is only about half as volatile as in the data in our baseline model with monetary shocks only.

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<sup>8</sup>Boivin, Giannoni and Mihov (2009) also reach a related conclusion: using a factor-augmented VAR model, they find that prices react quickly to sectoral shocks, but not to monetary shocks.

<sup>9</sup>See the work of Alvarez, Atkeson and Edmond (2009) and Khan and Thomas (2011).

## F. Role of Capital Adjustment Costs

We have assumed in our original setup that intermediate good producers face quadratic capital adjustment costs in order to allow the model to reproduce the relative variability of investment in the data. Such adjustment costs are widely used in New Keynesian models, but there is relatively little micro-evidence for such costs. Here we discuss the consequence of assuming away adjustment costs in our stockout-avoidance model, several alternative approaches to reducing the variability of investment, as well as review the evidence on the nature of capital adjustment costs in Midrigan and Xu (2008).

Table A3 reports the aggregate implications of a model identical to that of our baseline setup with decreasing returns to scale, capital accumulation and sticky prices, but in which there are no capital adjustment costs. For comparison, we also report results from an otherwise identical model in which the volatility of demand shocks is set equal to 0 so that distributors hold no inventories. The latter is a standard New Keynesian model with sticky wages and prices.

Note in Table A3 that the model without capital adjustment costs is grossly at odds with the inventory data: the inventory stock is strongly countercyclical and declines by 0.62% for every 1% increase in sales. Moreover, production is only about 1/3 as volatile as sales, and inventory investment is extremely volatile and countercyclical.

The reason inventory investment is countercyclical here is the sharp increase in the nominal and real interest rates following a monetary expansion: since investment spikes up after a monetary expansion, consumption only gradually increases (the half-life of the consumption response is now 96 months) and interest rates increase due to the household's preference to smooth consumption. The high interest rates make it optimal for distributors to order little and sell out of existing inventories.

Finally, notice that this model produces investment responses that are greatly at odds with the data: the standard deviation of investment is 149 times greater than that of consumption, while in the data investment is only about 4 times more volatile than consumption.

Column B of Table A3 shows that these counterfactual implications for investment are not specific to our model with inventories. A standard New Keynesian model without inventories predicts a relative variability of investment to consumption of 141 in the absence of investment adjustment costs, thus very similar to our model with inventories. This highly volatile behavior of investment is driven by intertemporal substitution: since capital depreciates slowly, intermediate

goods producers find it optimal to sharply expand investment so as to take advantage of the temporarily low price of investment goods arising due to price and wage rigidities.

We next discuss several attempts that we have made to reconcile the counterfactual implications for investment of New Keynesian models. For simplicity, our analysis mostly focuses on models without inventories, since this excess volatility of investment characterizes both classes of models.

### *Input-Output Structure*

One approach to reducing the variability of investment is to assume that investment is produced using intermediate goods, rather than final goods. Since intermediate goods' prices are much less sticky, this assumption reduces the variability of investment in the model. For example, an adjustment cost  $\xi$  of only 17.6 is now necessary to match the variability of investment in the data (46.2 earlier). Absent adjustment costs, investment in this model is 19 times more volatile than consumption, much more volatile than in the data, but not as much as originally.

### *Fixed Costs of Investment*

We next ask whether fixed costs of investment in capital can substitute for the quadratic costs in reducing the variability of aggregate investment in New Keynesian models. To pin down the size of the fixed adjustment costs, we use the plant-level dataset for Korean manufacturing plants that Midrigan and Xu (2008) study. Since in the data there is enormous variability in plant-level output and investment, we introduce firm-specific productivity shocks, and so modify the intermediate goods producer's productivity to

$$y_i(s^t) = a_i(s^t) \left( l_i(s^t)^\alpha k_i(s^{t-1})^{1-\alpha} \right)^\gamma, \quad (4)$$

where  $a_i(s^t)$  is the productivity of producer  $i$ , which follows a random walk process:

$$\ln a_i(s^t) = \ln a_i(s^{t-1}) + \varepsilon_i(s^t).$$

We assume a random walk process for productivity following Midrigan and Xu (2010), who show that output at the plant level is highly persistent and that a permanent component of productivity

is necessary to account for the data. Given the random walk in productivity, we render the model stationary by assuming a (small) exogenous exit hazard  $\delta_A$ . Firms that exit sell all capital back to households and are replaced by new firms that start with a constant level of productivity, here normalized to unity.

We assume, as Khan and Thomas (2008) do, that producers face a fixed cost of investing in any given period and that the fixed cost,  $f_i(s^t)$ , is an i.i.d. random variable drawn from a uniform distribution on  $[0, \bar{F}]$ . We denominate this fixed cost in units of labor.

The intermediate good producer now solves

$$\max_{y_i(s^t), x_i(s^t), l(s^t)} \sum_{t=0}^{\infty} \int_{s^t} (1 - \delta_A)^t Q(s^t) \left[ \begin{array}{l} \Omega(s^t) y_i(s^t) - P(s^t) x_i(s^t) \\ -W(s^t) (l_i(s^t) + f_i(s^t) (x_i(s^t) \neq 0)) \end{array} \right] ds^t, \quad (5)$$

subject to the production function in (4) and the law of motion for capital:

$$k_i(s^t) = (1 - \delta) k_i(s^{t-1}) + x_i(s^t). \quad (6)$$

As in Khan and Thomas (2008), fixed costs lead producers to follow generalized  $(S, s)$  rules, invest infrequently and only invest when the benefits from doing so exceed the fixed cost.

We calibrate the size of the fixed cost and the standard deviation of shocks to productivity to ensure that the model captures the volatility of the investment-capital ratio in the data as well as the amount of *inaction* in plant-level investment. We measure inaction as the fraction of plant-year observations whose investment-capital ratio is sufficiently close to zero. In particular, a plant is defined to be inactive if its investment-capital ratio is between zero and one-quarter of the average investment-capital ratio in the data. Since the average investment-capital ratio in the data is equal to 13%, we define a firm as inactive if its investment in that particular year is less than 3.25% ( $= \frac{1}{4} \times 13\%$ ) of its capital stock.

The data column of Table A4 shows that the standard deviation of annual plant-level investment is equal to 0.38, that only 4% of producers sell capital, and that 67% of producers are inactive. The second column of Table A4 shows that a frictionless model without investment adjustment costs cannot account for these patterns in the data: in that model, 36% of producers sell capital in any



given year and only 5% of producers are inactive.<sup>10</sup> In contrast, a model with fixed investment costs accounts for the data very well.

Given that the model with fixed costs of investment accounts for the pattern of inaction in the data well, we now study its implications for aggregate inventory dynamics. We find that the model continues to produce an excessively high variability of investment relative to consumption of 40. This is about one-third as large as in the model with no adjustment costs, but nevertheless ten times greater than in the data. Consistent with what Khan and Thomas (2008) find, fixed costs of investment do not play much role in the aggregate. Even though few producers invest in any given period, those that do react strongly to changes in intertemporal prices.

### *Evidence of Gradual Adjustment*

We next review the arguments of Midrigan and Xu (2008) who show that fixed costs of investment alone are not sufficient to explain the gradual adjustment of the capital stock in response to plant-level shocks. To document the extent of rigidity in the stock of capital of individual producers, note that absent adjustment costs and in the ergodic steady state without aggregate uncertainty, a producer will adjust its capital stock in any given period so as to ensure that the expected marginal product of capital is equal to its user cost:

$$(1 - \alpha) \gamma E_t \left[ \frac{y_{i,t+1}}{k_{i,t}} \right] = \frac{1}{\beta} - 1 + \delta. \quad (7)$$

Since output follows a random walk in our model and is also well approximated by a random walk in the data, (7) implies that

$$\frac{y_{i,t}}{k_{i,t}} = \frac{\frac{1}{\beta} - 1 + \delta}{(1 - \alpha) \gamma}$$

and is thus independent of shocks to productivity. The frictionless model thus implies that the average product of capital is constant over time for any individual producer, and in particular, uncorrelated with changes in output,  $\Delta \log y_{i,t}$ , or with the lagged value of the average product of capital.

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<sup>10</sup>Since the data are sampled at an annual frequency, we time-aggregate our monthly model and construct annual statistics to compare the model and the data.

Consider next the following regression:

$$\log \frac{y_{i,t}}{k_{i,t}} = \alpha_0 + \alpha_1 \Delta \log y_{i,t} + \alpha_2 \log \frac{y_{i,t-1}}{k_{i,t-1}} + \varepsilon_{i,t} \quad (8)$$

in order to evaluate the extent to which the average product of capital covaries with changes in output. When we run this regression on a panel of plants simulated from the model without adjustment costs, we find, as expected, that  $\alpha_1 = 0$  and  $\alpha_2 = 0$  (see the last few rows of Table A4), since capital increases one-for-one with output. In contrast, in the data these elasticities are much closer to unity (so that capital reacts very little to changes in output), even when we only focus on those producers that we classify as active in any given year. For example, the estimate of  $\alpha_1$  is equal to 0.96 when we include all 392,000 observations in our sample of Korean manufacturing plants, and equal to 0.95 when we restrict the regression to only those 147,000 observations that are not classified as inactive. Similarly, the average product of capital is very persistent: the autocorrelation coefficient  $\alpha_2$  is equal to about 0.99. Given the large number of observations in the data, all these elasticities are very precisely estimated with standard errors less than 0.001.

We next compute these elasticities in the models with fixed and convex adjustment costs. Note that the model with fixed costs produces some autocorrelation in the average product of capital ( $\alpha_2 = 0.55$ ), but much less than in the data. Moreover, the fixed cost model predicts essentially no relationship between changes in output and the average product of capital ( $\alpha_1 = -0.04$ ). Since producers that do adjust offset the effect of productivity shocks on the average product of capital, the model with fixed costs only predicts little relationship between changes in output and the average product of capital.

Table A4 also shows that the model with convex adjustment costs that are large enough to account for the relative volatility of aggregate investment to consumption in the data does much better along this dimension. It predicts a much more persistent average product of capital at the producer level (an autocorrelation of  $\alpha_2 = 0.71$ ) as well as a much greater sensitivity of the average product of capital to output:  $\alpha_1 = 0.81$ . In fact, much greater adjustment costs (about 10 times greater) are necessary for the model with convex adjustment costs to fully match the elasticities in the data.

Given that convex adjustment costs are arguably ad hoc, we next discuss several mechanisms

that may give rise to this inertia in the producer's stock of capital observed in the micro data. First, as Midrigan and Xu (2010) and Wang and Wen (forthcoming) point out, the firm's stock of capital may be insufficiently responsive to shocks because of the presence of financial frictions. In fact, Wang and Wen show how Kiyotaki-Moore (1997)-type financial frictions can give rise to investment dynamics that at the aggregate level are observationally equivalent to those produced by models with convex adjustment costs, and yet at the same time generates lumpiness in plant-level investment. Second, models with time-to-build frictions as in Kydland and Prescott (1982) can also give rise to a sluggish adjustment of the capital stock to shocks.

Finally, we note that an important reason why investment is extremely volatile in response to shocks in standard models without adjustment costs is the assumption, implicitly embedded in the law of motion for capital in (6), that current and old vintages of capital goods are perfect substitutes. Consider next the consequence of relaxing this assumption and rather assuming that current and old vintages of capital goods are imperfectly substitutable. In particular, we modify the law of motion for capital to

$$k_i(s^t) = \left[ (1 - \delta) (k_i(s^{t-1}))^{\frac{\omega-1}{\omega}} + \delta^{\frac{1}{\omega}} (x_i(s^t))^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (9)$$

It is straightforward to show that (9) reduces to (6) as the elasticity of substitution between old vintages of capital and current investment,  $\omega$ , goes to infinity. As the elasticity  $\omega$  decreases, current investment becomes less substitutable with old capital, making it difficult for producers to expand their capital stock by increasing investment. To see this, notice that the Euler equation for capital accumulation becomes:

$$\left( \frac{x_i(s^t)}{\delta k_i(s^{t-1})} \right)^{\frac{1}{\omega}} = \beta \int_{s^{t+1}} \frac{Q(s^{t+1})P(s^{t+1})}{Q(s^t)P(s^t)} \left[ \frac{R_i(s^{t+1})}{P(s^{t+1})} + (1 - \delta) \left( \frac{x_i(s^{t+1})}{\delta k_i(s^t)} \right)^{\frac{1}{\omega}} \right] ds^{t+1} \quad (10)$$

and is similar to that in the convex adjustment cost model:

$$1 + \xi \left( \frac{x_i(s^t)}{k_i(s^{t-1})} - \delta \right) = \beta \int_{s^{t+1}} \frac{Q(s^{t+1})P(s^{t+1})}{Q(s^t)P(s^t)} \left[ \frac{R_i(s^{t+1})}{P(s^{t+1})} + (1 - \delta) + \xi \left( \frac{x_i(s^{t+1})}{k_i(s^t)} - \delta \right) + \frac{\xi}{2} \left( \frac{x_i(s^{t+1})}{k_i(s^t)} - \delta \right)^2 \right] ds^{t+1} \quad (11)$$

We calibrate the elasticity between old and new varieties,  $\omega$ , to match the relative standard deviation of investment to consumption of 4. Table A4 shows that this model's implications for the micro-investment dynamics are identical to those of the convex adjustment cost model. The last column of Table A3 shows that this model's implications for the behavior of inventories are virtually identical to those of our model with convex adjustment costs.

### G. Inventories at Two Stages of Production

Our analysis has focused so far exclusively on finished goods inventories held by distributors. In the data, however, inventories are held at all stages of production. We next report on the fraction of inventories held at different stages of production, and modify our model to allow for inventories of both finished goods as well as materials.

Table A5 breaks down inventories in the U.S. data by sector and stage of production. Notice that inventories of intermediate inputs (raw materials and work-in-progress) amount to about two-thirds of all inventories in the manufacturing sector. However, since the wholesale and retail sectors hold large stocks of finished good inventories, the share of inventories of intermediate inputs in the total stock of inventories in the U.S. manufacturing and trade sectors is equal to only 23.2%. This relatively small share of inventories of intermediate inputs validates our focus on inventories of finished goods in the original model.<sup>11</sup>

We next modify our model to introduce inventories of finished goods as well as intermediate inputs. We make several modifications to our original model in order to allow it to better capture the input-output structure of the data. We assume that intermediate goods now have three uses: as an input into the production of other intermediate goods producers (materials), as investment goods, and as goods sold to distributors. As earlier, distributors hold inventories of goods due to a

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<sup>11</sup>These numbers are fairly similar to those in Ramey and West (1999), who report a similar breakdown using the 1995 data.

stockout-avoidance motive. We assume that intermediate goods producers must order new inputs of materials one period in advance of production. Hence, as in cash-in-advance models of money, intermediate goods producers hold inventories of materials from one period to another.

The technology with which intermediate goods producers operate is

$$y(s^t) = \left( l(s^t)^{\alpha(1-\eta)} k(s^{t-1})^{(1-\alpha)(1-\eta)} d(s^t)^\eta \right)^\gamma \quad (12)$$

where  $\eta$  governs the share of materials,  $d(s^t)$ , used in production. Given  $\Omega(s^t)$ , the price of the intermediate good, the producer solves:

$$\max_{x(s^t), l(s^t), x_d(s^t)} \sum_{t=0}^{\infty} \int_{s^t} Q(s^t) \begin{bmatrix} \Omega(s^t) y(s^t) - \Omega(s^t) (x(s^t) + \phi(s^t)) - \\ \Omega(s^t) [x_d(s^t) + \phi_d(s^t)] - W(s^t) l(s^t) \end{bmatrix}, \quad (13)$$

where  $x_d(s^t)$  are purchases of materials and  $\phi_d(s^t)$  is a quadratic adjustment cost. The producer's stock of materials,  $n_d(s^t)$ , evolves according to

$$n_d(s^t) = (1 - \delta_d) (n_d(s^{t-1}) - d(s^t) + x_d(s^t)), \quad (14)$$

where  $\delta_d$  is the rate at which the stock of materials depreciates.

We introduce a motive for holding inventories of materials by assuming a one-period delay between when materials are purchased and when they can be used in production. This implies that the producer can only produce using the stock of materials currently available,  $n_d(s^{t-1})$ :

$$d(s^t) \leq n_d(s^{t-1}), \quad (15)$$

hence the analogy with cash-in-advance constraints in the monetary literature. As in that literature, equation (15) binds here for the size of the monetary shocks we consider.

Note also that we now assume that investment is also produced using inputs of intermediate goods, rather than final goods as earlier. This assumption modifies the resource constraint for final goods to

$$c(s^t) = q(s^t) \tag{16}$$

and that for intermediate inputs to

$$y(s^t) = x(s^t) + \phi(s^t) + x_d(s^t) + \phi_d(s^t) + \int y_i(s^t) di. \tag{17}$$

Final goods are thus only used for consumption, while intermediate goods are used for investment, as materials, and sold directly to distributors.

Table A6 reports on the business cycle properties of this economy. We set  $\eta$ , the parameter governing the share of materials in production, equal to 0.4, in order to match the 0.23 share of intermediate goods' inventories in the data. We consider two versions of the model. In the first one, we set the adjustment cost parameter,  $\phi_d$ , equal to 0. In the second, we calibrate  $\phi_d$  so as to reproduce the variability of the stock of materials inventories in the data.

Note in Table A6 that the models both with and without adjustment costs for materials do a good job at reproducing the variability of the overall stock of inventories: the elasticity of inventories to sales is equal to 0.22 and 0.32, respectively, close to the 0.34 in the data. Importantly, countercyclical markups continue to account for the bulk of the real effects of monetary shocks in this model, suggesting that our results are robust to introducing inventories at multiple stages of production.

## H. Lower Share of Inventories in the Production of Final Goods

Our Benchmark model assumes that final goods are produced solely using inputs of intermediate goods that can be stored as inventories. This feature, commonly used in the New Keynesian literature, contrasts with the assumption made in the work of Wen (2011) and Khan and Thomas (2007), in which the share of goods held in inventory in the production of final goods is equal to 0.7 and 0.5, respectively. Here we show that our results are robust to reducing the share of storable goods in the production of final goods. In particular, we now assume that the technology for producing final goods is

$$c(s^t) + x(s^t) + \phi(s^t) = a(s^t) = q(s^t)^\varphi \left[ l_F(s^t)^\alpha k_F(s^t)^{1-\alpha} \right]^{1-\varphi},$$

where  $q(s^t)$  is, as earlier, a CES aggregator over varieties of storable intermediate inputs:

$$q(s^t) = \left( \int_0^1 v_i(s^t)^{\frac{1}{\theta}} q_i(s^t)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

while  $k_F$  and  $l_F$  are the amounts of capital and labor used in the final goods sector. We set  $\varphi$ , the share of storable goods in the production of final goods, equal to 1/2, and report, in Panel 8 of Table A1, results from our experiments.

The table shows that the models' implications regarding the response of inventories and inventory investment to monetary policy shocks changes little in this alternative model. As earlier, the model with sticky prices and decreasing returns to labor predicts inventory responses consistent with the data. The elasticity of inventories to sales is now equal to 0.49, slightly greater than in the Benchmark model (0.25) and in the data (0.34). In contrast, versions of the model with flexible prices or constant returns to labor which imply much less variation in markups, predict that inventories are much more volatile than in the data.

## 2. Data Appendix

We next check the robustness of the inventory facts reported in the data section in the text with respect to alternative detrending methods, stage of fabrication and the “speed of adjustment” approach to measuring the persistence of the inventory-sales ratio. We also provide details on how we measured the responses of consumption and interest rates to monetary policy shocks.

### A. Alternative Detrending Methods

Since our sample ends in a deep recession, we now ask whether our inventory facts are robust with respect to the detrending method we use. Table A7 compares the inventory moments for the HP (14400)-filtered time series for manufacturing and trade with the same moments derived using a sample that ends in 2005, before the recent recession, and also using the Baxter and King (1999) bandpass filter. For the latter, we restrict the band to frequencies between 12 and 96 months, and use a lead-lag length of the filter of 24 months.

The facts reported in the paper are virtually unchanged when we use the shorter sample. The difference between the HP and BP only shows up in the inventory investment statistics. The BP filter implies a much lower volatility of inventory investment: 0.12 as opposed to 0.23 for the

HP filter. Moreover, the correlation of inventory investment with output is a bit higher under the BP filter and equal to 0.71 (0.55 for the HP filter). This discrepancy arises because the BP filter eliminates the high-frequency variation in the time series. Note, however, that the *elasticity* of inventory investment with respect to output is not very different for the two filters. This elasticity is equal to 0.09 for the BP-filtered data and 0.13 for the HP-filtered data. We thus conclude that most of the inventory facts we report in the paper are not very sensitive to the detrending method.

## B. Persistence of the Inventory-Sales Ratio: “Speed of Adjustment”

Here we ask whether our results about the persistence of the inventory-sales ratio are robust to using an alternative “speed of adjustment” metric that has been popular in recent work.<sup>12</sup> Namely, we use the approach described in Ramey and West (1999) to estimate a target level of the inventory stock:  $I_t^* = \theta S_t$ . The speed of adjustment of  $I_t$  to  $I_t^*$  is then measured by the serial correlation of the gap between  $I_t$  and  $I_t^*$ .

We find that the speed of adjustment is very low in the data. The autocorrelation of  $I_t - I_t^*$  is equal to 0.97 in the manufacturing and trade sector and 0.89 in the retail sector. Our baseline model predicts a somewhat greater speed of adjustment: the autocorrelation of the gap is equal to 0.78 in our baseline model. As discussed above, adding a realistic interest rate rule and productivity shocks raises the persistence of the inventory-sales ratio in the model. The speed of adjustment decreases as well: the autocorrelation of  $I_t - I_t^*$  is equal to 0.92 in that model. Hence, our facts regarding the persistence of the inventory-sales ratio are robust to using this alternative “speed of adjustment” metric.

## C. Dynamics of the Inventory-Sales Ratio by Stage of Fabrication

As we have discussed above, around 77% of all manufacturing and trade inventories are finished goods inventories, and 23% are raw materials and work-in-progress inventories. We next report elasticities of the inventory-sales ratio with respect to sales for inventories at all stages of production, for the period from 1967:01 to 1996:12. Sales are total manufacturing sales. Each time series is log HP-detrended.

The elasticity of the inventory-sales ratio of the entire stock of manufacturing inventories is

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<sup>12</sup>See Khan and Thomas (2007) for a description.



equal to -1.01, slightly greater in absolute value than the -0.66 for manufacturing and trade sectors. The elasticities of the inventory-sales ratio for stocks at different stage of fabrication are very similar: -1.01 for raw materials, -0.87 for work-in-progress and -1.16 for finished good inventories. Hence, inventories at different stages of fabrication exhibit very similar dynamics as the aggregate stock of inventories.

#### **D. Responses of Consumption and Interest Rates to Monetary Policy Shocks**

To document the response of consumption to a monetary shock in the data, we use the methodology outlined in Christiano, Eichenbaum and Evans (1999) (hereafter, CEE). We estimate monetary shocks using a monthly VAR for the period 1965:1 to 1995:6. We include the following variables: total non-farm employment, the PCE deflator, a smoothed index of sensitive commodity prices, the federal funds rate, non-borrowed reserves, total reserves and the M1 or M2 money stocks. As CEE do, we define monetary shocks as innovations to the federal funds rate. We assume a Cholesky ordering such that shocks to the federal funds rate do not have a contemporaneous effect on employment, the price deflator and commodity prices, only on the monetary variables. We also estimate VARs for a longer sample period than that used in CEE from 1965:1 to 2004:4.

Table A8 reports the response of consumption (defined as in the model as the ratio of the money stock to the price level) and interest rates to an expansionary monetary policy shock. We normalize the size of the shock so that the maximum response of the money stock is 1%, as in the model.

The maximum consumption response to the 1% monetary shock is about 1% for all specifications we have considered. The average consumption response in the first two years after the shock is equal to 0.55 (0.75) percentage points, and the half-life of the response is equal to 16 months (22 months) in the specifications with M1 (M2) as the money stock.

Table A8 also shows that the federal funds rate declines by about 2 to 2.5% after the monetary shock and is on average 30 to 60 basis points below its initial value in the first two years after the shock.

### **3. Computations**

We now describe the methods we have used to solve the equilibrium in our model economies. We separately describe how we have solved the wage-setter's problem and then the problem of

distributors. Since intermediate producers are all identical, no computational issues arise here: these agents' decision rules are described by the optimality conditions for capital and labor discussed in the main text and are straightforward to compute.

To solve the wage-setter's problem and the problem of distributors, we have employed first- and second-order perturbations methods, as well as global projection methods in conjunction with a shooting method to compute transitions after one time shocks. Since the first- and second-order perturbation methods produce nearly identical results, we contrast the perturbation methods with the global projection methods.

First-order perturbation methods provide an accurate approximation as long as the firm's pricing and inventory decisions are approximately linear, as well as if the second- and higher-order moments of the distributions of inventories, prices, and wages have a negligible impact on the aggregates. We next argue that this is indeed the case in our economy without fixed ordering costs. Since the economy with fixed ordering costs is highly non-linear due to non-convexities, we only use global projection methods to solve that problem.

### A. Wage Problem

The aggregate wage in this economy evolves according to

$$W_t = \left[ \lambda_w (W_{t-1})^{1-\vartheta} + (1 - \lambda_w) (W_t^R)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}, \quad (18)$$

where  $W_t^R$  solves

$$(W_t^R)^{1+\vartheta\chi} = \psi \frac{\vartheta}{\vartheta - 1} \frac{E_t \sum_{k=0}^{\infty} (\beta\lambda_w)^k [W_{t+k}^\vartheta l_{t+k}]^{1+\chi}}{E_t \sum_{k=0}^{\infty} (\beta\lambda_w)^k \frac{c_{t+k}^{-\sigma}}{P_{t+k}} W_{t+k}^\vartheta l_{t+k}} \quad (19)$$

To solve this problem recursively, let

$$H_t^1 = E_t \sum_{k=0}^{\infty} (\beta\lambda_w)^k [W_{t+k}^\vartheta l_{t+k}]^{1+\chi} \quad (20)$$

and

$$H_t^2 = E_t \sum_{k=0}^{\infty} (\beta \lambda_w)^k \frac{c_{t+k}^{-\sigma}}{P_{t+k}} W_{t+k}^\vartheta l_{t+k} \quad (21)$$

denote the numerator and denominator in (19). Clearly, these objects solve

$$H_t^1 = \beta \lambda_w E_t H_{t+1}^1 + [W_t^\vartheta l_t]^{1+\chi} \quad (22)$$

and

$$H_t^2 = \beta \lambda_w E_t H_{t+1}^2 + \frac{c_t^{-\sigma}}{P_t} W_t^\vartheta l_t \quad (23)$$

and are functions of the aggregate state of the economy. Here, the aggregate state consists of the distribution of past wages across unions, the distribution of prices across distributors, as well as the aggregate stock of capital and inventories. Since the distributor's problem is linear in its existing stock, as inspection of the distributor's problem makes it clear, the only moment of the distribution of inventories that affects aggregate variables is the aggregate inventory stock.

Clearly, up to a first-order approximation the only moments of the distribution of wages and prices that affect the aggregate variables are the first moments. We would like to ask, however, whether a first-order approximation is accurate. To this end, we follow Krusell and Smith (1998) and i) guess that aggregate variables are a function of the first moments of these distributions only, in addition to capital and the aggregate inventory stock; ii) solve the system (19)-(23) using non-linear projection methods and the guess in i); and iii) update the guess in i) by simulating the wage-setters' decision rules over time and regressing simulated paths for aggregate variables on the value of the aggregate state variables in each period. As Krusell and Smith (1998) have done, we use the  $R^2$  in these regressions to evaluate the extent to which the higher-order moments of the distributions matter for the aggregates.<sup>13</sup>

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<sup>13</sup>To keep the notation simple, we wrote the equations above as if the nominal variables were stationary. In our model they are not, since the stock of money follows a random walk process. In our computations we have detrended all nominal variables by the money stock to induce stationarity. For example, (18) reduces to

$$w_t^{1-\vartheta} = \left( \frac{w_{t-1}}{g_t} \right)^{1-\vartheta} + (w_t^R)^{1-\vartheta},$$

while (22) reduces to

$$h_t^1 = \beta \lambda_w E_t h_{t+1}^1 g_{t+1}^{\vartheta(1+\chi)} + [w_t^\vartheta l_t]^{1+\chi},$$

The Calvo assumption on the constant hazard of wage adjustment conveniently implies that the law of motion for the aggregate wage in (18) is a function of only the past aggregate wage. The distribution of wages thus only matters for computing moments of the distribution of labor supplied by households. For example, total hours supplied by the household are equal to

$$L_t = \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{-\vartheta} l_t = \left[ \lambda_w \int_0^1 \left( \frac{W_{j,t-1}}{W_t} \right)^{-\vartheta} di + (1 - \lambda_w) \frac{W_t^R}{W_t} \right] l_t = \Theta_t l_t, \quad (24)$$

which states that wage dispersion across unions drives a wedge,  $\Theta_t$ , between the amount of labor employed by firms,  $l_t$ , and the total amount of hours supplied by households,  $L_t$ . The greater the dispersion in wages, the greater the inefficiency, and thus the greater the disutility from work associated with the same amount of labor used in production.

Figure A2 presents the decision rules for reset wages,  $W^R(W_{t-1}, P_{t-1}, K_{t-1}, N_{t-1})$ , in each of the four state variables (expressed as percentage deviations from the steady state).<sup>14</sup> As one would expect, the reset wage is increasing in the past aggregate wage (a form of strategic complementarity here since low wages by the union's competitors imply greater hours supplied by this particular union, increasing the disutility from work), decreasing in the past aggregate price (since a greater aggregate price level decreases aggregate consumption and therefore reduces hours supplied by the union and thus the disutility from work), etc. The stock of inventories and capital has a much more muted effect on reset wages (a 1% increase in the inventory stock lowers the reset wages by only 0.01%, and a 1% increase in the capital stock raises the reset wage by only 0.04%), suggesting that the dynamics of wages and therefore the labor supply decisions are less impacted by the latter.

Most importantly, note that there are no important non-linearities in these decision rules: the non-linear rules (approximated using 5-th order cubic splines) are very similar to the linear decision rules derived using perturbation methods. Given the standard deviations of monetary shocks that we use (0.23%), wages and prices never exceed the bounds on the state space we impose here, so the linear decision rules are highly accurate.

Figure A3 illustrates the role of the higher-order moments of the distribution of wages for

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where lower case nominal variables denote variables appropriately detrended by the money supply and  $g_t$  is the growth of the money supply from  $t - 1$  to  $t$ :  $g_t = M_t/M_{t-1}$ .

<sup>14</sup>We set the value of all other states in each figure equal to their steady-state values.

the aggregate dynamics of the economy. The left panel shows a time-series simulation of the wedge,  $\Theta_t$ , arising due to the dispersion in wages across unions. As discussed above, higher-order moments of the distribution only affect the dynamics of this wedge. Note first that  $\Theta_t$  is very close to 1 (0 in logs) on average: its average is equal to 1.000068. Moreover, the wedge does not fluctuate much in response to monetary policy shocks: its highest value in these simulations is 0.012% greater than the average. These fluctuations are orders of magnitude smaller than the fluctuations in the amount of labor supplied by households induced by monetary shocks (see the right panel of Figure A3) and thus play a trivial role in the aggregate. When we regress  $L_t$ , computed based on the non-linear decision rules using (24), on the four states,  $W_{t-1}, P_{t-1}, K_{t-1}, N_{t-1}$ , we find an  $R^2$  equal to 0.999993, suggesting that variation in the higher-order moments of the distribution of  $W_{j,t-1}$  plays essentially no role in this economy.

## B. Distributor's Problem

We next discuss how we have verified the accuracy with which first-order perturbations solve the distributor's problem. We have done so by solving the model's responses to a monetary shock using non-linear methods. We have first solved the distributor's problem in the ergodic steady state without aggregate uncertainty using projection methods, subjected the economy to a one time, unanticipated increase in the money supply, and solved for the economy's transition using a shooting method. We then compared the transitions implied by the first-order approximation with those implied by the non-linear methods. We discuss first how we have solved for the decision rules in the ergodic steady state, how we computed the transitions, and then compare the two solution methods.

We discuss the most general formulation of the problem, allowing for fixed costs of ordering. We simply set these equal to 0 when solving our baseline stockout-avoidance model.

At the beginning of period  $t$ , the state of an individual distributor  $i$  is characterized by its price in the preceding period,  $P_{i,t-1}$  and its inventory stock  $n_{i,t-1}$ . It is convenient to normalize all nominal prices and wages by the current money supply. Specifically, let  $p_{i,t} = P_{i,t}/M_t$  and  $\omega_t = \Omega_t/M_t$  and use similar notation for other prices. Let  $\bar{p}_t = \frac{P_t}{M_t}$  denote the normalized aggregate price level. With this normalization, we can write the state of an individual firm  $i$  in  $s^t$  as  $[p, n]$ .

Let

$$q(v, p, z) = \min \left( v \left( \frac{p}{\bar{p}} \right)^{-\theta}, q, z \right)$$

be the amount a distributor with a stock of available goods  $z$  that charges a price  $p$  sells when its demand shock is equal to  $v$ .

We can write the firm's problem in the ergodic steady state recursively using the following system of functional equations:

$$V^{a,a}(p, n) = \max_{p', z} \int_v [p'q(v, p', z) - \omega(z - n) + \beta V(p', (1 - \delta_z)(z - q(v, p', z)))] dF(v) \quad (25)$$

$$V^{a,n}(p, n) = \max_{p'} \int_v [p'q(v, p', n) + \beta V(p', (1 - \delta_z)(n - q(v, p', n)))] dF(v) \quad (26)$$

$$V^{n,a}(p, n) = \max_z \int_v [p'q(v, p, z) - \omega(z - n) + \beta V(p, (1 - \delta_z)(z - q(v, p, z)))] dF(v) \quad (27)$$

$$V^{n,n}(p, n) = \int_v [p'q(v, p, n) + \beta V(p', (1 - \delta_z)(n - q(v, p, n)))] dF(v), \quad (28)$$

where  $V^{a,a}$  is the value of resetting the price and inventory stock,  $V^{a,n}$  is the value of resetting only the price but selling out of the existing stock,  $V^{n,a}$  is the value of leaving the price unchanged but ordering new inventories and  $V^{n,n}$  is the value of leaving the price unchanged and not ordering. All of these values are expressed gross of the fixed costs associated with ordering. The continuation value is given by

$$V(p, n) = (1 - \lambda_p) \left[ \int_0^{\bar{\kappa}} \max(V^{a,a}(p, n) - w\kappa, V^{a,n}) dG(\kappa) \right] + \lambda_p \left[ \int_0^{\bar{\kappa}} \max(V^{n,a}(p, n) - w\kappa, V^{n,n}) dG(\kappa) \right], \quad (29)$$

where  $\kappa$  is the fixed ordering cost drawn from a distribution  $G$ .

We approximate the value functions using cubic splines and solve the system (25)-(29) using projection methods and Gaussian quadrature to approximate the distribution of demand shocks. We then use these decision rules to compute the ergodic distribution and hence the steady state of this economy.

Consider next how we compute the transition after a one-time shock to the money supply. Given the shock, we conjecture that the system converges to the steady state in  $T$  periods, as well as conjecture a path for aggregate prices and quantities along the transition  $Q_t^{(0)}, q_t^{(0)}, \bar{p}_t^{(0)}, \omega_t^{(0)}, w_t^{(0)}$ ,  $t = 1, \dots, T$ . We then solve for the firm’s value functions and decision rules using backward induction. Given the steady-state values in period  $T$ , we compute the value functions and decision rules for period  $T - 1$  using the guess for aggregate variables in that period. We then use the  $T - 1$  value functions to compute the value functions in period  $T - 2$  and continue iterating until  $t = 1$ . Given these value functions at all dates, we compute the firms’ decision rules for each period along the transition, and aggregate those in order to compute new paths for aggregate prices and quantities,  $Q_t^{(i)}, q_t^{(i)}, \bar{p}_t^{(i)}, \omega_t^{(i)}, w_t^{(i)}$  for iterations  $i = 1, \dots, I$ , until these objects converge.

Figure A4 compares the impulse responses to a monetary shock of the aggregate stock of inventories and the aggregate price level in our baseline economy. We report two sets of responses: those computed using the first-order perturbations, as well as those computed using the non-linear method described above. Clearly, the two are nearly identical. We have also computed impulse responses to monetary shocks starting away from the ergodic steady state (for example for an economy that was in the steady state one period ago but had experienced another shock in the previous period). We found that these responses are very close to those that start in the ergodic steady state, suggesting that the economy’s responses to monetary shocks are not affected much by higher-order moments of the joint distribution of distributors’ inventories and prices, consistent with what Khan and Thomas (2007) have found for an  $(S, s)$  inventory model.<sup>15</sup>

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<sup>15</sup>An earlier version of this paper, Kryvtsov and Midrigan (2010), also used the Krusell-Smith (1998) approach to compute simpler versions of our economy without capital or sticky prices. We have found in those cases that the first moments of the distributions approximate aggregate dynamics extremely well (the  $R^2$  were in excess of 0.99). The full-blown baseline model we study here has, however, a very large number of continuous state variables (6) relative to what tensor-based projection methods can handle, and this precludes us from using the Krusell-Smith approach here.

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Table A1: Robustness Extensions

		markup contribution	elast. $I_t$ to $S_t$	elast. $\Delta I_t$ to $Y_t$
	Data	-	0.34	0.12
	Baseline	0.89	0.25	0.09
1	Original model			
	<i>No firm DRS</i>	0.76	0.49	0.16
	<i>Increasing returns</i>	0.49	1.19	0.32
	<i>Variable K utiliz.</i>	0.55	1.57	0.46
	<i>Flexible Wages</i>	3.22	-1.20	-0.68
	<i>Convex Order Costs</i>	0.96	0.35	0.13
2	Persistent money growth			
	<i>Baseline</i>	0.88	-0.17	0.11
	<i>Labor only</i>	0.37	1.98	0.97
	<i>Flexible prices</i>	0.05	0.87	0.34
3	Persistent money growth and habit			
	<i>Baseline</i>	0.49	0.18	0.06
	<i>Labor only</i>	0.11	2.88	0.98
	<i>Flexible prices</i>	0.02	1.66	0.32
4	Taylor rule			
	<i>Baseline</i>	0.88	0.27	0.11
	<i>Labor only</i>	0.35	4.48	0.79
	<i>Flexible prices</i>	0.09	1.83	0.39
5	High depreciation			
	<i>Baseline</i>	0.84	0.10	0.01
	<i>Labor only</i>	0.34	2.75	0.68
	<i>Flexible prices</i>	0.06	1.44	0.25
6	Convex depreciation			
	<i>Baseline</i>	0.90	0.15	0.08
	<i>Labor only</i>	0.37	3.5	0.84
	<i>Flexible prices</i>	0.07	1.38	0.32
7	Lower inventory- sales ratio			
	<i>Baseline</i>	0.88	0.37	0.06
	<i>Labor only</i>	0.33	3.73	0.66
	<i>Flexible prices</i>	0.05	1.74	0.22
8	Lower share inventories in final goods			
	<i>Baseline</i>	0.71	0.49	0.15
	<i>Labor only</i>	0.33	4.36	0.87
	<i>Flexible prices</i>	0.08	1.85	0.41

Table A2: Economy with Productivity Shocks

	Data	A. RBC	B. Sticky prices/wages and technology shocks	C. Sticky prices/wages and both shocks
$\rho(IS_t, S_t)$	-0.82	-0.92	0.54	-0.60
$\sigma(IS_t) / \sigma(S_t)$	1.03	0.39	2.78	1.12
elast. $IS_t$ to $S_t$	-0.84	-0.36	1.50	-0.67
elast. $I_t$ to $S_t$	0.16	0.64	2.50	0.33
$\rho(IS_t, IS_{t-1})$	0.87	0.83	0.96	0.85
$\sigma(Y_t) / \sigma(S_t)$	1.12	1.13	1.11	1.12
$\rho(Y_t, \Delta I_t)$	0.55	0.56	0.61	0.50
$\sigma(\Delta I_t) / \sigma(Y_t)$	0.23	0.25	1.01	0.27
elast. $\Delta I_t$ to $Y_t$	0.13	0.14	0.62	0.14

Note: All variables HP-filtered with smoothing parameter 14400.

Table A3: Business Cycle Predictions of Economies Without Adjustment Costs

	Data	A. Inventories, No Adjustment Costs	B. No Inventories, No Adjustment Costs	C. Inventories. Imperfectly Substitutable Capital Vintages
<i>Impulse response of consumption to monetary shock</i>				
average response	0.55	0.45	0.71	0.45
maximum response	1.01	0.48	0.79	0.90
half-life, months	16.20	96.0	48.3	10.5
markup contribution		1.01	1.40	0.89
$\rho(IS_t, S_t)$	-0.71	-0.87	-	-0.91
$\sigma(IS_t) / \sigma(S_t)$	0.93	1.86	-	0.82
elast. $IS_t$ to $S_t$	-0.66	-1.62	-	-0.75
elast. $I_t$ to $S_t$	0.34	-0.62	-	0.25
$\rho(IS_t, IS_{t-1})$	0.88	0.65	-	0.74
$\sigma(Y_t) / \sigma(S_t)$	1.11	0.35	-	1.10
$\rho(Y_t, \Delta I_t)$	0.63	-0.46	-	0.88
$\sigma(\Delta I_t) / \sigma(Y_t)$	0.20	2.18	-	0.11
elast. $\Delta I_t$ to $Y_t$	0.13	-1.01	-	0.10
$\sigma(x_t) / \sigma(c_t)$	4	149	141	4

Note: All variables HP-filtered with smoothing parameter 14400.

Average output response computed for first 24 months after shock.

Table A4: Economies with Fixed Cost of Investing

	Data	No adj. cost	Fixed cost	Convex adj. cost	Imperfectly substitutable capital vintages	
<i>Parameters</i>						
$\sigma_i$	s.d. productivity shocks		0.096	0.065	0.178	0.178
$F$	upper bound on fixed cost		-	0.0236	-	-
$\xi$	quadratic adjustm. cost		-	-	46.2	-
$\omega$	elast. subst. capital vintages		-	-	-	2.14
<i>Microeconomic implications</i>						
	s.d. ( $x_i/k_i$ )	0.38	0.38	0.38	0.38	0.38
	fraction $x_i/k_i < 0$	0.04	0.36	0.06	0	0
	fraction $x_i/k_i$ in $(0, \text{avg}(x_i/k_i)/4)$	0.67	0.05	0.67	0.26	0.26
	elast. $y_i/k_i$ to $\Delta y_i$	0.96 (0.95)	0	-0.04	0.84	0.84
	elast $y_i/k_i$ to lagged $y_i/k_i$	0.99 (0.98)	0	0.55	0.71	0.71
<i>Aggregate implications</i>						
	$\sigma(x_t) / \sigma(c_t)$	4	141	40	4	4

Note: The fixed investment cost is expressed as fraction of the producer's mean investment (conditional on ordering) in the ergodic steady state. The elasticities of the average product of capital to output and the lagged product of capital in the data are reported for all observations, as well as only for observations without inaction (parentheses).

Table A5: Inventories by Stage of Fabrication, U.S. NIPA

	Billions of chained 2000 dollars	% of total inventory stock in manufacturing & trade
Manufacturing	0.461	36.2
raw materials	0.157	12.3
work-in-progress	0.138	10.8
finished goods	0.165	13.0
Wholesale trade	0.353	27.8
Retail trade	0.457	35.9
Manufacturing and trade	1.272	100.0
Private nonfarm	1.503	118.2

Note: Real manufacturing and trade inventories, seasonally adjusted, end of period, 2008:Q2.

Table A6: Economy with Two Types of Inventories

	Data	Model	
		A. No materials adj cost	B. With materials adj cost
average response	0.55	0.42	0.47
maximum response	1.01	0.90	0.91
half-life, months	16.20	7.6	9.3
markup contribution		1.00	0.81
$\rho(I/S_t, S_t)$	-0.71	-1.00	-0.93
$\sigma(IS_t) / \sigma(S_t)$	0.93	0.79	0.73
elast. $I/S_t$ to $S_t$	-0.66	-0.79	-0.68
elast. $I_t$ to $S_t$	0.34	0.22	0.32
$\rho(IS_t, IS_{t-1})$	0.88	0.80	0.74
$\sigma(Y_t) / \sigma(S_t)$	1.11	1.03	1.10
$\rho(Y_t, \Delta I_t)$	0.63	1.00	0.96
$\sigma(\Delta I_t) / \sigma(Y_t)$	0.20	0.29	0.19
elast. $\Delta I_t$ to $Y_t$	0.12	0.29	0.18

Table A7: Alternative Detrending Methods

	A. Unconditional			B. Conditional on monetary shocks		
	HP filter	HP filter, prior to 2005	BP filter	HP filter	HP filter, prior to 2005	BP filter
$\rho(IS_t, S_t)$	-0.82	-0.82	-0.79	-0.71	-0.71	-0.65
$\sigma(IS_t) / \sigma(S_t)$	1.03	1.03	0.96	0.93	0.93	0.89
elast. $IS_t$ w.r.t. $S_t$	-0.84	-0.84	-0.76	-0.66	-0.66	-0.58
elast. $I_t$ w.r.t. $S_t$	0.16	0.16	0.24	0.34	0.34	0.42
$\rho(IS_t, IS_{t-1})$	0.87	0.87	0.98	0.88	0.88	0.93
$\rho(Y_t, S_t)$	0.98	0.98	1.00	0.98	0.98	0.99
$\sigma(Y_t) / \sigma(S_t)$	1.12	1.12	1.09	1.11	1.11	1.09
$\rho(Y_t, \Delta I_t)$	0.55	0.54	0.71	0.63	0.64	0.73
$\sigma(\Delta I_t) / \sigma(Y_t)$	0.23	0.23	0.12	0.20	0.20	0.13

Notes: U.S. NIPA, manufacturing and trade, Jan. 1967 - Dec. 2009.

All series are real, at monthly frequency.  $IS_t$ ,  $\Delta I_t$ ,  $S_t$ ,  $Y_t$  denote real inventory-sales ratio, inventory investment, and final sales, respectively.

Detrending methods: HP with smoothing parameter 14400, and bandpass filter with frequencies between 12 and 96 months, using 24 lags.

The column labeled "Unconditional" reports statistics for detrended data.

The column labeled "Conditional on monetary shocks" reports statistics computed using detrended data projected on current and 36 lags of Christiano, Eichenbaum, and Evans (1999) measures of monetary policy shocks estimated using a VAR for 1960:01-2000:12.



Table A8: Consumption and Interest Rate Responses to CEE Expansionary Monetary Shock

sample money variable	CEE M1	CEE M2	Updated M1	Updated M2
<i>Consumption</i>				
Average response	0.55	0.75	0.74	0.78
Peak response	1.01	1.02	0.99	1.03
Peak period	4	3	5	6
Half-life (from IRF)	16.2	21.8	22.2	26.6
<i>Interest Rate</i>				
Average response	-0.34	-0.43	-0.38	-0.57
Peak response	-2.44	-2.77	-1.96	-2.66
Peak period	1	1	1	1
Half-life (from IRF)	3.8	3.7	3.9	3.9

Notes: Monetary shocks are estimated by a VAR for monthly U.S. data for the period from 1965:M1 to 1995:M6 (CEE sample) and 1965:M1 to 2004:M4 (updated sample). See the text for the description of the VAR. The shock is normalized so that maximal response in money stock is 1% within 6 months after the shock. Consumption is defined as real money balances, i.e., the ratio of money stock and PCE deflator. All responses are in percentage points. We report the average response for the first 24 months after the shock.

Figure A1: Impulse Response to Productivity Shock. Sticky prices & wages.

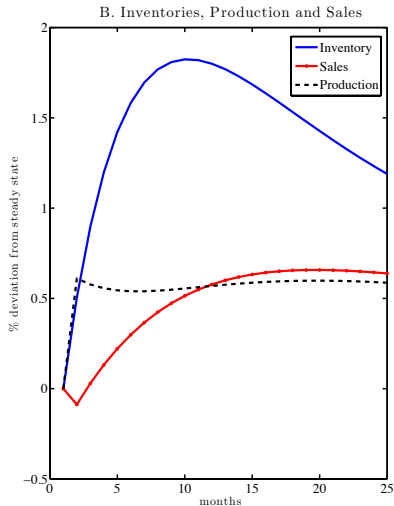
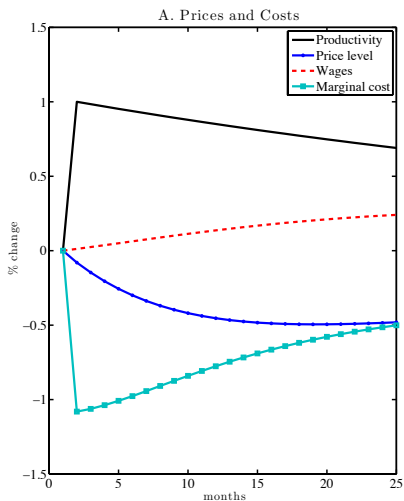


Figure A2: Reset Wages. Linear vs. 5th-order spline approximation.

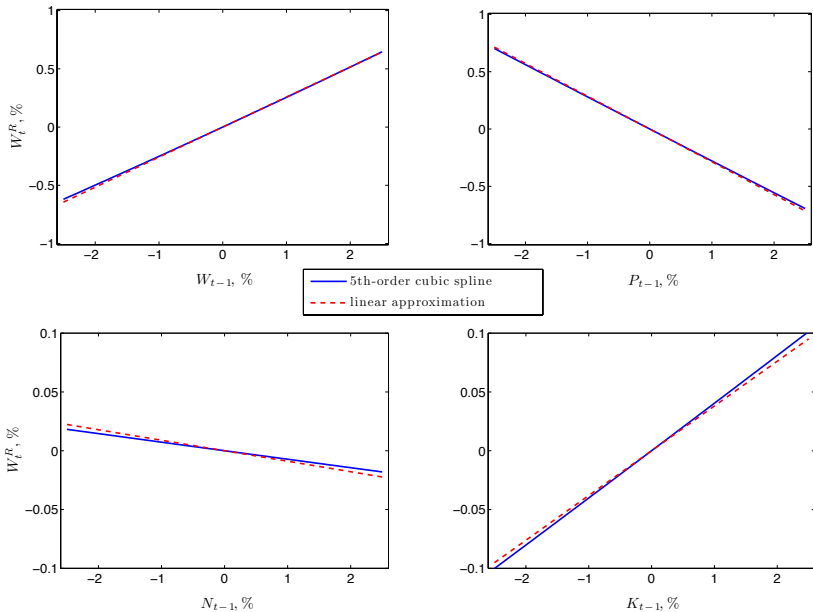


Figure A3: Role of higher-order wage moments.

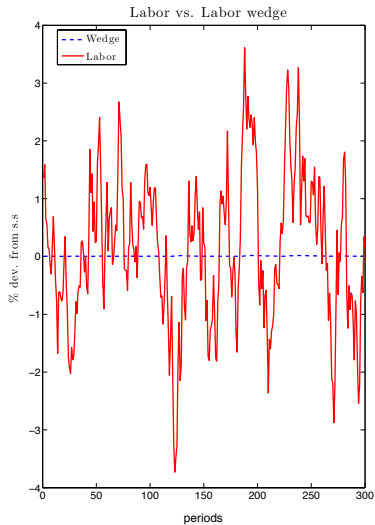
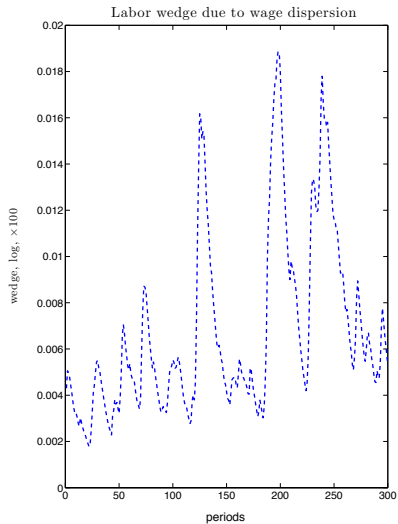


Figure A4: Linear vs. non-linear solution of distributor's problem

