International price dispersion in state-dependent pricing models

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Abstract

Menu-cost models predict a hump-shaped relationship between real and nominal exchange rate volatility. The hump occurs at higher values of nominal exchange rate volatility, the higher trade costs and lower international substitution elasticities are. These predictions accord well with the negative relationship between relative price and nominal exchange rate volatility I document using a data set of prices collected in Eastern Europe in a volatile environment. In contrast, trade costs must be sufficiently high or international substitution elasticities low in order for the model to account for the positive correlation between real and nominal exchange rate volatility in the aggregate data. © 2007 Elsevier B.V. All rights reserved.

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1. Introduction

International relative prices for similar, apparently tradeable goods are volatile and persistent, much more so than relative prices for locations within a country. Engel and Rogers (1996) provide a metric that has often been used to gauge the impact that national borders have on international relative price movements: the border between the United States and Canada increases the volatility of relative prices for two cities it separates by as much as 75,000 miles would if the cities were located within a country.¹

This paper quantifies the sources of these excessive relative price movements. Do they reflect real factors, i.e., transportation costs, trade barriers, combined with low substitutability of goods produced in different countries and other impediments to goods-market arbitrage? Or, rather, do they reflect nominal price stickiness coupled with volatile nominal exchange rates (NERs)?

Both of these explanations have received widespread support in earlier work. International trade costs, broadly defined, are large² and positively correlated with measures of law-of-one-price (LOP) deviations.³ Relative prices across countries, at all levels of aggregation, track NERs closely.⁴ The volatility of NERs across countries is positively correlated with measures of international relative price variability, suggesting that nominal frictions may indeed be partly responsible for the excessively large border effects.⁵

I revisit the question posed above quantitatively, using a model with trade costs and price adjustment (menu) costs. I show that in models in which nominal rigidities arise endogenously, due to fixed costs firms pay to adjust their prices, an increase in NER volatility does not necessarily increase the variability of international relative prices, as is the case in models in which the degree of price stickiness is assumed exogenous.⁶ Rather, an increase in the volatility of the environment has two effects on the volatility of international relative prices. On the one hand, NERs are explicitly used to compute relative prices and higher NER volatility increases relative price variability. On the other hand, as the volatility of NER increases, firms reprice more frequently, thereby more readily correcting relative price fluctuations induced by nominal shocks.

The relative strength of these two effects depends on the size of trade costs, as well as the volatility of NERs in the economy. In environments in which the volatility of NERs is small, prices adjust infrequently: relative prices inherit the properties of NERs and higher NER volatility increases the size of deviations from the LOP. As NER volatility increases, firms find it optimal to pay the menu cost and adjust more readily. More frequent adjustment breaks the tight link between nominal and real exchange rates. Higher NER volatility, through its effect on the frequency of price changes, lowers the variability of relative prices.

¹See also Engel (1993, 1999) and Parsley and Wei (2001a).
²Anderson and van Wincoop (2004) find that total trade costs amount to a 70% ad valorem tax equivalent.
³E.g., Crucini and Shintani (2004), Crucini et al. (2005).
⁵Engel and Rogers (1996, 2001), Parsley and Wei (2001a, b).
⁶E.g., Taylor (1980) or Calvo (1983). See also Devereux and Yetman (2002) who study a related question (the relationship between exchange rate pass-through and inflation) in the context of a time-dependent model in which firms adjust prices at random intervals, in a Calvo fashion, but choose the (ex ante) optimal frequency of price changes given a cost of price adjustment.
The size of trade costs affects both the level as well as the slope of the relationship between relative price and NER volatility. Firms that face smaller trade costs have higher export shares and allow smaller LOP deviations because they find it optimal to reprice more frequently. Relative price fluctuations are costlier for these firms because, given the lower costs of arbitrage, they translate into more volatile movements in quantities. Given that the sign of the relationship between real and NER volatility depends on the frequency of price adjustment, the hump occurs earlier for more tradeable goods, whose producers have a higher incentive to increase the frequency of price adjustment in response to an increase in the volatility of nominal shocks. I use this last prediction of the model to gauge the size of trade costs empirically.

Alvarez et al. (2002) document that high-inflation countries are characterized by less volatility of real exchange rates relative to that of NERs and show that a model with endogenously segmented markets can replicate this behavior. Burstein (2006) shows that the effect of nominal shocks on real activity is smaller in high-inflation environments because firms adjust more frequently in the presence of fixed costs of adjusting prices. He argues that a state-dependent pricing model can also reproduce the lower ratio of real to NER volatility in high-inflation environments. However, my open economy menu-cost model delivers a stronger result. According to the model, real exchange rate volatility itself should eventually decrease with higher NER volatility.

Both aggregate and disaggregated price data are used to test the predictions of the model. The micro-data set includes prices for agricultural products sold in open-air markets, a volatile environment in which prices are changed frequently and where the model predicts, if anything, a negative relationship between relative price and NER volatility. Using data on the prices of 58 products collected biweekly over a three-year period in 12 Eastern European cities, I indeed find, using gravity-type equations, that city pairs and time periods that are subject to larger shocks to their NERs experience smaller deviations from the LOP.

I next show that in the aggregate, CPI-based data, the volatility of monthly changes in real exchange rates increases with larger NER volatility even for pairs of countries that have suffered from excessive NER fluctuations. The menu-cost model can account for this lack of a hump in the data, but only in the presence of high trade costs or low substitution elasticities across Home and Foreign varieties of a good. I thus conclude that menu costs, on their own, cannot account for the large border effects documented in work with international relative prices. Large trade frictions or low substitution elasticities (or alternative frictions that reduce a firm’s incentive to respond to NER movements) are necessary ingredients of a sticky price model capable of reproducing the pattern of international relative price movements observed in the data.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 discusses the relationship between relative price variability and NER volatility that the model generates. Section 4 evaluates the model empirically. Section 5 concludes. Appendices that discuss the solution method and the data used in the paper are available online.7

2. The model

The world consists of two countries, Home and Foreign. Each country is inhabited by a continuum of identical consumers and a continuum of firms. Consumers buy goods from Home and Foreign firms and supply labor to firms in their own country. They own local firms and consume their profits. In addition, consumers in both countries have access to a complete set of state-contingent securities denominated in Home’s currency. The Home country is small relative to Foreign so that Foreign variables are unaffected by shocks in the Home economy. This assumption is made in order to maintain computational tractability and to allow the use of non-linear solution techniques.

I next describe the problem of the agents in Home. Foreign agents solve identical problems and asterisks are appended to their variables. Throughout, let \( t \) denote the Home economy. This assumption is made in order to maintain computational tractability and to allow the use of non-linear solution techniques.

2.1. Consumers

A representative consumer has preferences over a continuum of goods indexed by \( z \in (0, 1) \). For each good \( z \), two closely substitutable varieties, one produced in Home and another in Foreign, are available for consumption. Letting \( C(z, i; s') \) denote the consumption of variety \( i \in \{h, f\} \) of good \( z \) in \( s' \), the Home consumer solves

\[
\max_{C(z, i; s'), h(s^{t+1}), n(s')} \sum_{t=0}^{\infty} \beta^t \sum_{s'} U(C(s'), n(s'))
\]

s.t. \[
\int_{0}^{1} (p(z; s') C(z, h; s') + \tau(z) e(s') p^*(z; s') C(z, f; s')) \, dz + \sum_{s^{t+1}} \chi(s^{t+1} | s') b(s^{t+1}) = W(s') n(s') + \Pi(s') + b(s'),
\]

where

\[
C(s') = \left( \int_{0}^{1} C(z; s')^{(\theta-1)/\theta} \, dz \right)^{\theta/(\theta-1)}
\]

is a CES aggregator over the different goods and

\[
C(z; s') = (C(z, h; s')^{(\theta-1)/\theta} + C(z, f; s')^{(\gamma-1)/\gamma})^{\theta/(\theta-1)}
\]

is an aggregator over the two (Home and Foreign) varieties of good \( z \). \( \theta \) and \( \gamma \) are the elasticity of substitution across goods and varieties, respectively. \( p(z; s') \) is the price of the domestically produced variety of \( z \), and \( p^*(z; s') \) is the (Foreign currency) price of the Foreign variety. \( e \) is the exchange rate, denominated in units of Home per Foreign currency. \( \tau(z) \) is an iceberg trade cost the consumer incurs in order to purchase goods from abroad: to consume 1 unit of the Foreign good, \( \tau(z) > 1 \) must be purchased. Goods differ according to their tradeability: \( G(\tau) \), the distribution of trade costs across goods, is calibrated below. \( \chi(s^{t+1} | s') \) is the price of a bond that pays 1 unit of Home currency if state \( s^{t+1} \) is realized tomorrow. \( b(s^{t+1}) \) is the quantity of state-contingent bonds the consumer purchases. Finally, \( \Pi(s') \) are the profits earned by all Home firms, and \( n(s') \) is the...
household’s supply of labor. Implicit in this formulation of the budget constraint is the convention that consumers own only their country’s firms.\(^8\)

The optimal consumption allocations chosen by the consumer that faces the problem above satisfy

\[
C(z,h) = \left( \frac{p(z)}{P(z)} \right)^{-\gamma} \left( \frac{P(z)}{P} \right)^{-\theta} C,
\]

\[
C(z,f) = \left( \frac{\tau(z)e_p^a(z)}{P(z)} \right)^{-\gamma} \left( \frac{P(z)}{P} \right)^{-\theta} C,
\]

where

\[
P(z) = [p(z)^{1-\gamma} + (\tau(z)e_p^a(z))^{1-\gamma}]^{1/(1-\gamma)}
\]

is the price index for good \(z\), and

\[
P = \left( \int_0^1 P(z)^{-\theta} dz \right)^{1/(1-\theta)}
\]

is the aggregate price index in this economy.

The share of good \(z\) imported from abroad depends on the price the Foreign firm charges (in Home currency terms), times the good’s shipping cost, relative to a consumption-weighted average price of the Home and Foreign varieties of this good. This problem is thus a generalization of the standard trade-cost model, exposited, for example, in Sercu et al. (1995), in which Home and Foreign varieties of a good are perfect substitutes \((\gamma \rightarrow \infty)\), and consumers purchase only the cheapest variety of a given good.

Finally, the optimal supply of labor and demand for state-contingent securities is governed by

\[
n(s^t) : \frac{W(s^t)}{P(s^t)} = -\frac{U_n(s^t)}{U_c(s^t)},
\]

\[
b(s^{t+1}) : \frac{P(s^{t+1})}{P(s^t)} \xi(s^{t+1} | s^t) = \beta \pi(s^{t+1} | s^t) \frac{U_c(s^{t+1})}{U_c(s^t)},
\]

where \(U_n, U_c\) are the derivatives of the utility function with respect to its arguments.

Foreign consumers face a similar problem. In the exercises, it is assumed that the Foreign economy is large compared to the Home economy. Formally, there is a unit mass of Home agents and that Foreign’s population is \(N\) times larger, where \(N \rightarrow \infty\). Moreover, Foreign consumers place little weight on their consumption of Home varieties of goods

\[
C^a(z; s^t) = (\omega^a C^a(z, h; s^t)^{(\gamma-1)/\gamma} + C^a(z, f; s^t)^{(\gamma-1)/\gamma})^{\gamma/(\gamma-1)},
\]

and \(\omega^a \rightarrow 0\). Without this assumption, Home firms would de facto sell only abroad independent of the size of their trade costs, and this would preclude us from studying the

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\(^8\)This, as well as the restriction that state-contingent bonds are denominated in Home’s currency, are notational conventions rather than assumptions. Given the availability of a complete set of state-contingent Arrow–Debreu securities, any additional asset in this economy is redundant (it does not affect equilibrium allocations), because it can be constructed using a combination of the state-contingent bonds available for trade. Chari et al. (2002) employ a similar convention.
role of tradeability in the model. Foreign consumers’ demand for the Home-produced variety of a given good is

$$C^*(z, h; s') = \omega^* \left( \frac{\tau(z) p(z; s')}{e(s')} P^*(z; s') \right)^{-\gamma} C^*(z; s'),$$

(12)

where

$$P^*(z; s') = \left( p^*(z; s')^{1-\gamma} + \omega^* \left( \frac{\tau(z) p(z; s')}{e(s')} \right)^{1-\gamma} \right)^{1/(1-\gamma)}$$

(13)

is the aggregate price of good $z$ from the perspective of the Foreign agent. Note here the consequence of the assumption that $\omega^* \rightarrow 0$: Foreign prices and quantities are independent of disturbances originating in Home. Nevertheless, because Foreign is large relative to Home, total demand for Home goods is non-zero.

The NER in this model is determined from the Euler equation (10) that governs the Home consumer’s demand for state-contingent securities. Together with its Foreign counterpart, this equation implies the following relationship between the ratio of the marginal utilities of consumption in Home and Foreign and the real exchange rate:

$$U_c(C^*(s'), n(s')) = A \frac{P^*(s')}{e(s') P^*(s')},$$

(14)

where $A$ is a constant that depends on initial conditions. Its value is normalized to ensure that $e = 1$ in the steady state. Even though households trade a complete set of state-contingent securities, the Home and Foreign marginal utilities fail to move in tandem to the extent to which the real exchange rate deviates away from unity. This is the case whenever transport costs are positive (so that consumption is skewed toward domestically produced goods; otherwise, given the assumption of producer currency pricing, the real exchange rate is equal to unity) and prices are irresponsive to nominal shocks.

2.2. Firms

Firms set a unique price $p(z; s')$ or $p^*(z; s')$ denominated in their own country’s currency. Both countries’ consumers buy at this price. Customers from abroad pay an additional (iceberg) trade cost $\tau(z) - 1$: in order to consume 1 unit of the good, $\tau(z)$ must be purchased.

Labor is the sole variable input in production: the firm’s technology is $y(z) = l(z)\xi$, where $l(z)$ is the quantity of labor the firm uses for production. Firms also face menu costs of adjusting prices: the firm must hire $\xi(z)$ additional units of labor every time it resets its price. $\xi$ differs across firms; more on this below. A typical firm in Home solves

$$\max_{p(z; s')} \sum_{i=0}^{\infty} \sum_{s'} Q(s') \Pi(z; s'),$$

(15)

where $Q(s') = b' \pi(s') U_c(C(s'), n(s')) / U_c(C(s^0), n(s^0))$ is the rate at which real profits at $s'$ are discounted in period 0, and $\Pi(z; s')$ are the firm’s profits in state $s'$,

$$\Pi(z; s') = \frac{p(z; s')}{P(s')} D(z; s') - \frac{W(s')}{P(s')} D(z; s')^{1/\gamma} - \xi(z) \frac{\tau(z)}{P(s')} \mathcal{I}_{p(z; s') \neq p(z; s'-1)},$$

(16)
where \( \xi(z) W(s')/P(s') \) is the additional cost the firm must pay every time it reprices, and \( D(z; s') \) is total demand:

\[
D(z; s') = C(z, h; s') + N\tau(z)C^*(z, h; s').
\]

Note that per capita demand from Foreign consumers is \( \tau(z)C^*(z, h) \) because, given a desired level of consumption \( C^*(z, h) \), the consumer must purchase a total of \( \tau(z)C^*(z, h) \) units, of which a fraction melts in transit. Parameter values in the Foreign economy are chosen to ensure that per capita consumption and aggregate price levels are equal in both countries at the steady state: \( C = C^* \), \( P = P^* \), and that \( \omega^* N = 1 \). These assumptions imply that, at the steady state, a Home firm that faces no trade costs treats the Home and Foreign markets symmetrically because its export share is close to \( \frac{1}{2} \).

### 2.3. Equilibrium

Money enters via a quantity theory money demand equation that assumes unitary velocity: \( P(s')C(s') = M(s') \), and similarly for the Foreign economy. The Foreign money supply is assumed constant. This, as well as the assumption that Foreign preferences are heavily skewed toward the consumption of Foreign goods (\( \omega^* \rightarrow 0 \)), implies that Foreign aggregate variables are constant at their steady-state values. The only source of uncertainty in the economy is shocks to the growth rate of the Home money supply:

\[
g(s') = \log(M(s')/M(s'-1)).
\]

The equilibrium is a sequence of prices \( p(z; s'), e(s'), W(s'), \chi(s'+1|s') \) and allocations \( n(s'), C(z, i, s'), b(s'+1) \) that satisfy firm and household maximization and market-clearing and resource constraints.

The algorithm used to characterize the equilibrium of this economy recursively, including the non-linear solution method employed, and the use of a Krusell and Smith (1997)-type approach to deal with the infinite-dimensional state space is discussed in detail in a technical appendix available online.

### 3. The relationship between RPV and NER volatility

I next discuss the model’s key implication: the relationship between relative price and NER volatility. I first discuss the parameterization of the model and then conduct numerical experiments.

#### 3.1. Parametrization and calibration

The utility function is

\[
U(C, n) = \frac{1}{1 - \kappa} C^{1-\kappa} - \psi n.
\]

The length of the period is one month, and the discount factor is equal to \( \beta = 0.997 \). The value of \( \psi \) is chosen to ensure that households work 30% of their discretionary time in the steady state. I follow Chari et al. (2002) and set \( \kappa = 5 \) so that the model produces sufficiently large movements in real exchange rates. The elasticity of substitution across goods is set equal to \( \theta = 3 \) to highlight the fact that goods are imperfect substitutes,
a lower bound of the range of elasticities used in closed-economy models. On the other hand, varieties of a good are assumed closely substitutable, and I set $\gamma = 15$, somewhat higher than the estimates reported by Hummels (2001) using disaggregated international trade data. The high elasticity of substitution between Home and Foreign goods is chosen because my goal is to quantify the ability of the model to generate relative price fluctuations for seemingly identical goods. Indeed, the extent of international law-of-one-price deviations in the data is puzzling especially because goods for which relative prices fluctuate so much are arguably closely substitutable. This assumption is relaxed below.

The production function $y = \ell^a$ reflects the fact that other factors of production are fixed in the short run. The share of labor is set to $\alpha = 2/3$.

To calibrate the distribution of trade costs, I use the US Imports and Exports Data compiled by Robert Feenstra. This data set contains information on the total value of exports by manufacturing firms at the SIC four-digit level for 1958–1994. I supplement this data set with the NBER Productivity Database, which contains, among others, data on total shipments for these industries. Using the two sets of data, I calculate export shares for 396 industries for 1993. Based on these shares, I compute (weighted by the revenue share of each industry) second (11.26%), third (1.43), and fourth (4.74) moments of the distribution of export shares that I ask the model to match. I do not use the US data to calibrate the mean export share given that the United States is a relatively closed economy and also because the disaggregated data are available only for the manufacturing sector. Instead, I use the OECD STAN Indicators Database for 1993, which provides data on export shares for most of the OECD countries. The average (across OECD economies) export share is 11.87%. I assume a parametric functional form for the distribution of trade costs $G(\tau)$: the beta distribution, for its flexibility. The two parameters of the distribution are chosen so that the model delivers moments of the export shares distribution close to those in the data. The calibrated distribution is $G(\tau) = \text{beta}(a_1, a_2)$ with $a_1 = 3.4$ and $a_2 = 13$. Trade costs range in the model from 2% to 48% (in a 50-node quadrature-based discretization of this distribution), with a mean of 20% ($\tau = 1.2$). This large dispersion of trade costs across varieties is consistent with results from earlier work that has provided direct measurements of the size of transport costs. A notable example is Hummels (2001), who has assembled an extensive data set of freight rates for two-digit SITC industries for the United States and several other countries. He finds large dispersion in freight rates, across both commodities, and transportation modes: average freight rates range from 3.5% (Office Machines) to 28.6% (Coal, Coke) for the United States, and are as large as 50% for other countries.

Menu costs of price adjustment $\xi(z)$ are assumed to differ across firms: firms pay 1.2% of their steady-state revenues each time they adjust prices. Firms that sell more abroad face larger menu costs because of their larger market shares. This assumption is made in order to ensure that the result that more tradeable firms adjust more frequently is not simply due to the fact that these firms have smaller menu costs relative to their revenues. The choice of the size of the menu cost implies that firms adjust every seven months on average for monetary shocks of a size similar to that in the US data, an average price duration in the neighborhood of that documented by Bils and Klenow (2004) and

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9See Obstfeld and Rogoff (2000) for a brief survey.
10I discovered several errors for the 1994 data set downloaded from the UC Davis website and went back one year in time to avoid them.
Nakamura and Steinsson (2006) for a large data set of BLS-collected consumer prices in the US economy. These menu costs are also consistent with the evidence presented by Zbaracki et al. (2004), who study the price adjustment practices of a large US manufacturing firm.

The model is used to study how goods prices respond to movements in exchange rates. I thus require the model to generate NER fluctuations consistent with those observed in the data. The process for the growth rate of the money supply is chosen to ensure that NERs follow, in equilibrium, a random walk process:

\[
\log e(s') = \log e(s'^{-1}) + \varepsilon(s'),
\]

where \(\varepsilon \sim N(0, \sigma^2)\). I vary \(\sigma\) in simulations to study how the variability of relative prices depends on the volatility of changes in NERs. To be clear, even though the NER follows in equilibrium a random walk subject to random disturbances, it does not constitute an exogenous variable in the model. Rather, the growth rate of the money supply is calibrated in order to generate NER fluctuations as in the data.11

3.2. Results

Fig. 1 plots a firm’s value of leaving its price unchanged: \(V_n\), and that of adjusting its price and paying the menu cost, \(V_a\), as a function of the log deviation of the firm’s price from its optimum when all other variables are at their steady-state levels. The figure plots these functions for two firms: one that sells tradeable goods (\(\tau = 1.02\)), and one whose good is virtually untradeable, at the upper end of the distribution of trade costs in the model: \(\tau = 1.48\). The two functions are normalized so that the maximum of the value of not changing its price is 1.

If the firm adjusts, it resets the price to its optimal level: the firm’s value of adjustment is therefore independent of \(p^{-1}\). The best a firm can do by not adjusting is if its past price coincides with today’s optimum: the firm’s value of inaction in this case exceeds the value of adjustment by the size of the menu cost. The intersection of the two value functions determines the region of inaction—the \((S, s)\) bands. Notice here that this inaction region is narrower for the more tradeable good. For firms that have low trade costs, even small relative price movements are costly because they generate large changes in quantities: consumers in both markets can easily switch expenditure toward the cheaper variety (Home or Foreign) of a particular good. In contrast, firms that sell goods that are less tradeable face less competition from the Foreign producer of the same variety, and face less elastic demand, given the assumption that \(\theta < \gamma\).

To see this, consider, for example, the elasticity of the Home consumers’ demand for a Home firm’s good:

\[
-\frac{\partial \log D^H(z)}{\partial \log p(z)} = [\gamma(1 - s) + \theta s],
\]

where

\[
s = \frac{p(z)^{1-\gamma}}{p(z)^{1-\gamma} + (\tau(z)e^{p^*}(z))^{1-\gamma}} = \frac{1}{1 + \tau(z)^{1-\gamma}}.
\]

11\(\log(M(s')/M(s'^{-1})) = (1 - k) \log(C(s')/C(s'^{-1})) + \varepsilon(s')\).
is the Home firm’s market share for good $z$ in the Home market.\(^{12}\) (The last equality assumes a steady state in which the LOP holds and $p(z) = ep^*(z)$.) This last expression illustrates the role that trade costs play in the model: when $\tau$ is close to 1, the firm’s market share in a particular market is $\frac{1}{2}$ and the demand elasticity is high: $(\gamma + \theta)/2$. As the transport cost increases, the firm’s share in the Home market is 1 and in the Foreign market is 0, so the total demand elasticity is $\theta$ because the firm only faces competition from producers of other (much less substitutable) goods. This in turn implies that the inaction region is wider for the high transport cost firm: it tolerates prices as far as 4% away from its optimum. A low trade-cost firm adjusts more readily: every time its price deviates from the optimum by 1.5% in absolute value.

Given that the process for the nominal wage rate follows a random walk (the choice of preferences implies $W \sim P/U(C) \sim \epsilon$), an important component of the firm’s marginal costs is unforecastable based on current information. Moreover, even though $a$, say, monetary expansion increases aggregate consumption and thus the marginal cost of production, firms that do adjust in times of a monetary expansion set prices that are higher than those of other firms in the economy who have not yet responded to the nominal shock. Substitution by consumers toward the cheaper goods thus reduces these firms’ total sales and hence their marginal costs. These two opposite effects cancel each other out for my choice of parameter values. As a result, adjusting firms in this economy respond approximately one-for-one to a nominal shock, and the LOP holds for firms that have just reset prices. Relative price variability is thus solely a function of the frequency of price adjustment and the volatility of NERs in this environment.

\(^{12}\)Atkeson and Burstein (2006) study the properties of a two-country model with a similar market structure and the role of time-varying market shares and therefore markups in shaping international relative price movements.
It is helpful, at this point, to relate the predictions of this model to those of the standard trade-cost model that assumes that goods are perfectly substitutable and arbitrage eliminates LOP deviations whenever they are too large. A key prediction of the standard trade-cost model is that the relative price of the good can deviate away from unity, but is bounded by the size of the transport cost. As discussed above, models with imperfect competition and menu costs share similar features with the simple trade-cost model.\(^{13}\) The firm allows its price to deviate away from its optimum, as long as the deviation is within the \(S_s\) bounds, and charges a price that (approximately) ensures that the LOP holds otherwise. Although the inaction bands are functions of the trade cost, elasticities of substitution, as well as the volatility of the environment, the similarity with the standard trade-cost model is evident. In particular, more tradeable goods command narrower inaction regions in the menu-cost model, and allow smaller relative price fluctuations.

Fig. 2 examines the relationship between the standard deviation of first-differenced log relative prices and NER volatility that the model predicts. The left panel plots this relationship for several values of the trade cost. The right panel plots the frequency of price adjustment as a function of NER volatility.\(^{14}\) As the figure indicates, the model predicts a hump-shaped relationship between relative price and NER volatility. When the volatility of NERs is sufficiently low, firms adjust infrequently and relative prices inherit the

\[^{13}\text{Although time-dependent models with CRS production functions predict no relationship between the degree of tradeability and the extent of relative price fluctuations, this is not the only model of imperfect competition that predicts larger LOP deviations for more tradeable goods. See, e.g., Betts and Kehoe (2001).}\]

\[^{14}\text{These curves are computed from simulations of the model in which I vary the volatility of shocks to the growth rate of the money supply in order to vary the volatility of NERs. The plots are Chebyshev interpolants on model-simulated data.}\]
properties of the NER. An increase in NER volatility then increases the volatility of relative prices. Notice, however, that the positive relationship between NER volatility and relative price variability holds only locally, in environments in which firms change prices sufficiently infrequently. As the frequency of price changes increases, the tight link between real and nominal international relative prices breaks down, and higher NER variability, through its effect on the frequency of price changes, decreases the variability of relative prices. In the calibration of the model, whenever NERs are sufficiently large to induce firms to adjust prices every two to three months, higher NER variability decreases the volatility of relative prices.

Note also that the hump occurs for lower NER volatility the more tradeable goods are. As the right panel of Fig. 2 indicates, the difference arises because the frequency of price adjustment is less sensitive to changes in the volatility of the environment the more tradeable goods are. As a result, larger NER fluctuations are necessary in order to induce high trade-cost firms to adjust price sufficiently frequently so as to break the link between real and nominal international relative prices.

![Persistence of relative prices](image1)

![Real exchange rate](image2)

![Persistence of relative prices](image3)

![Unconditional volatility of relative price](image4)

Fig. 3. Other measures of LOP/PPP deviations.
I next turn to other measures of LOP deviations. Fig. 3 plots the persistence of the relative price series, as measured by its AR(1) coefficient, $\hat{\rho}$, as well as the unconditional volatility of relative prices. The figure plots these statistics for a firm with the median transport cost, $\tau = 1.2$, as a function of the volatility of NERs in each simulation (upper panels) and, also in the $\tau$ space, for an economy in which $\sigma = 0.01$.

Notice in the upper-left panel of the figure that the persistence of relative prices declines with higher NER volatility: as $\sigma$ varies from 0% to 6%, the serial correlation of the relative price varies from 1 to 0. Similarly, as the lower-left panel shows, the persistence of relative prices depends on the good’s trade costs. Similar results hold for other measures of LOP deviations. In particular, CPI-based real exchange rates behave similarly to the relative price of the good with the median trade cost with a hump at $\sigma = 2.5\%$. Moreover, unconditional measures of relative price variability are also hump-shaped in the volatility of NERs.

4. The data

I next study the relationship between relative price variability and NER volatility in the data. I first turn to a micro-data set of prices and then to CPI-based relative price series in order to test the model’s predictions.

4.1. Evidence from micro-price data

The first empirical exercise tests the predictions of the model using a micro-data set of prices. The model predicts a negative relationship between relative price variability and NER volatility in environments in which firms adjust prices sufficiently frequently and/or goods in different markets are highly tradeable or closely substitutable. The data set used here is collected in precisely such an environment: I work with prices for homogeneous agricultural products sold in open-air markets, an environment in which prices change frequently and are highly volatile. The data were collected in 12 cities in six Eastern European countries by CAMIB, an NGO aimed at providing informational support to food exporters in the region. The sample covers the years 2000–2002. Prices are collected on average once every two weeks (usually during weekends) in all years except for 2000, when prices are sampled weekly. A total of 83 time periods are available.\footnote{A data appendix available online describes the data in more detail and provides several robustness checks.}

Data on goods in four product categories (meat, fruit, vegetables, as well as a small number of other agricultural products such as oil, honey, etc.) are available. The agency hires representatives in all 12 cities who sample prices several times during the day, from a number of vendors of a given product. An average price, across all vendors surveyed, is then reported. To ensure homogeneity of goods sampled, the agency treats the most and least expensive varieties of a good as separate items and records their prices separately. I follow this convention as well and treat high- and low-quality varieties of a given product as separate goods in the sample.

Not all prices are sampled in all periods: some goods drop out of the sample occasionally, especially during winter. I choose to work only with those goods for which a panel of at least six cities is available in at least 75% of the time periods in the sample. A total of 58 goods make this cutoff.
Let $q_{t g}^i$ be the log-relative price of good $g$ at time $t$ for city pair $i$. I focus on the time-series properties of $q$ in order to characterize relative LOP deviations. In particular, I calculate the time-series standard deviation of changes and levels of the relative price series in order to gauge the extent to which relative prices fluctuate over time. Relative prices are very volatile in the data, with an average standard deviation (across goods/city pairs) in excess of 26% (changes) and 37% (levels). In contrast, the volatility of NERs is high, but much lower than that of relative prices. The average across city pairs separated by national borders is 4.2%. This is, then, the type of environment in which the model predicts, if anything, a negative relationship between NER volatility and the variability of relative prices. To test this prediction, I resort to a cross-sectional gravity-type equation relating the time-series volatility of changes (or levels) of $q_{t g}^i$ for each of the 66 city pairs in the data set to the volatility of NERs, as well as a border dummy and the distance between locations.

Given that prices are sometimes sampled at irregular intervals, it is assumed that changes in relative prices are serially uncorrelated in order to compute the biweekly changes in relative prices. Although this is a strong assumption, results based on first differences in relative prices are reported for comparability with earlier work. The first estimated equation is

$$vol(\Delta q \text{ or } q)_{gi} = \gamma_0 + \gamma_1 \text{Border}_i + \gamma_2 \log(\text{dist})_i + \gamma_3 \text{std}(\Delta e)_i + \mu_g + \epsilon_{gi},$$  

(22)

where $vol(\Delta q \text{ or } q)_{gi}$ is the time-series volatility of changes or levels of relative prices. I use two measures of relative price variability, the standard deviation as well as the interquartile range, to ensure robustness against outliers. City and type of good (meat, fruit, vegetables) dummies are included in all regressions.

This regression pools together all 58 goods in the sample and assumes that $\mu_g$, the good specific effect, is a random variable uncorrelated with other regressors. Table 1 presents the results. Note that the volatility of NERs always enters with a negative sign with a coefficient that is, in most cases, significantly different from zero at conventional significance levels. The data set used provides support for the state-dependent sticky price

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Notes:
1. Coefficient estimates and standard errors (in parentheses) of a random effects model reported.
2. The different columns use different measures of relative price variability as a dependent variable.
3. Coefficient estimate and standard errors on log-distance and border multiplied by 1000.
4. Regressions include meat/fruit/vegetable as well as city dummies.
model: larger NER volatility is associated with less relative price variability in an environment in which costs of changing prices are low, idiosyncratic shocks volatile, and prices adjust frequently.\footnote{I have performed several checks to ensure the robustness of these results. A negative relationship between RPV and NER volatility is present when I estimate the gravity equation above separately for different good-categories, for goods of high/low quality, by eliminating subsets of cities from the sample, and by allowing for a quadratic NER volatility term. An iteratively weighted robust regression that penalizes outliers produces qualitatively similar results.} This result differs from that documented in a large number of studies that, since Mussa’s (1986) seminal contribution, have documented a pervasive positive relationship between the volatility of real and NERs and suggests that this relationship, just like the menu-cost model predicts, holds locally rather than globally.

I next use an alternative measure of deviations from the LOP, by calculating the volatility of $\Delta q_t$, across goods, rather than time. This is a typical measure of relative price variability used in order to test the effect inflation has on relative price variability in closed-economy studies. Parsley and Wei (2001a) use a similar statistic in a context similar to ours. The reason movements in exchange rates generate relative price variability across goods in sticky price models is staggered price adjustment: firms that adjust in different periods have different relative prices. In the model, firm pricing is not synchronized because of differences in trade costs. For this measure of LOP deviations, the model predicts a similar hump-shaped relationship between price dispersion and NER volatility: when NER volatility is sufficiently high, more volatile NERs induce more frequent price adjustment, firms are more likely to synchronize their price changes and the cross-sectional volatility of relative prices is smaller.

Let $d_{it}^g$ be again the relative price in period $t$ for good $g$ for city pair $i$. When working with levels of relative prices, $d_{it}^g$ is detrended by its time-series mean because some cities are more expensive on average than others: $Q_{it}^g = q_{it}^g - q_{gt}^g$. To compute the volatility of changes in relative prices, I work with $\Delta Q_{it}^g = \Delta q_{it}^g - \Delta q_{gt}^g$ to filter the data of good-specific trend growth in the relative price for a given city pair. To calculate the volatility of NERs in any given period, I use daily exchange rate data and calculate the standard deviation of changes in daily NERs in periods between price collections.\footnote{Here it is again assumed that $\Delta Q_{it}^g$ is serially uncorrelated in order to compare observations in different periods given the irregular intervals between price collections.}

I next estimate the following system of time-series regressions, pooled over all city pairs:

$$vol(\Delta Q \text{ or } Q)_{it} = \beta_0 + \beta_1 \text{Border}_i + \beta_2 \log(\text{dist})_i + \beta_3 \text{std}(\Delta e)_{it} + \mu_i + e_{it}, \quad (23)$$

where $i$ is an index over city pairs and $t$ over time. Once again, the measures of volatility are the interquartile range and the standard deviation, and it is assumed that time-specific effects are uncorrelated with NERs. City fixed effects are included as well. As Table 2 suggests, the results of this exercise are consistent with those in the cross-sectional regressions. Periods and city pairs for which NERs are less volatile suffer from less price dispersion, consistent with the predictions of the model.

4.2. Evidence from macro-data

In the second exercise, I turn to trade-weighted (effective) real and NER data for all 77 countries for which data are available from the International Financial Statistics (IFS) for
years 1980–1998. A broad set of countries is used because identifying the non-linear relationship between real and NER volatility that the model predicts requires a broader sample of countries, some of which have experienced large NER movements. The measure of volatility is the interquartile range because it is less sensitive to episodes of large nominal (and associated real) devaluations on which the model is silent, although using the standard deviation generates qualitatively similar conclusions.

Fig. 4 presents scatterplots of the interquartile range of changes in real exchange rates vs. that of changes in NERs, as well as the AR(1) coefficient of the real exchange rate series against the volatility of changes in NERs. The solid lines in this figure are smoothed scatterplots obtained from locally weighted regressions in which observations are weighted using Cleveland’s (1979) tricube function and employ a bandwidth length of 0.5. As the figure indicates, there is little evidence of a hump in the relationship between real and NER variability in the aggregate price data. Moreover, although a weak negative relationship between the persistence of real exchange rates and the volatility of NERs is apparent in the figure, even countries subject to excessively large movements in NERs are characterized by very persistent real exchange rate movements, in contrast to what the model predicts.

How can one reconcile the lack of a hump in the data with the predictions of the menu-cost model? Recall that the reason higher NER volatility eventually decreases the volatility of relative prices in the model is the fact that firms, especially those with higher export shares, find it optimal to increase the frequency of price adjustment in response to more volatile NER movements. Work by Gopinath and Rigobon (2006) suggests, however, that the duration of price spells of at-the-dock import prices varies little with the importer’s NER volatility: an increase in volatility up to 20% lowers the duration of price spells by only one month. A second prediction of the model that receives little support in the data is that of a positive relationship between the firm’s export share and the frequency of price adjustment. The correlation between the frequency of price adjustment across different consumption goods from Bils and Klenow (2004) with the export share of four-digit

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Notes: 1. Coefficient estimates and standard errors (in parentheses) of a random effects model reported. 2. The different columns use different measures of relative price variability as a dependent variable. 3. Coefficient estimate and standard errors on log-distance and border multiplied by 1000. 4. Regressions include city dummies.
industries (available from Robert Feenstra’s data set) for 95 consumption categories for which a close match between the two data sets is possible is low and, in fact, negative (−0.16).18

The reason firms with higher export shares adjust prices more frequently in the model is the assumption that Home and Foreign varieties of a good are closely substitutable, with an elasticity of substitution, \( \gamma \), equal to 15. This is a natural assumption to start with, given my interest in LOP violations for seemingly identical goods. This assumption, however, generates several predictions that are at odds with the data. I therefore ask next whether lowering this elasticity allows the model to account for the data by lowering the elasticity of substitution across varieties of a good \( z \) to \( \gamma = 5 \). In doing so, I also recalibrate the distribution of trade costs to allow the model to match the export share moments in the data. Given that Home and Foreign goods are now much less substitutable, matching the low export shares observed in the data requires much larger trade costs: the ad valorem trade cost in the model now ranges from 0% to 129%, with a median of 85%.

As Fig. 5 illustrates, this parametrization of the model can indeed account for the lack of a hump in the relative price–NER volatility relationship: even for goods that face no trade costs, the hump now occurs for values of NER volatility in excess of 5%. Now that Home and Foreign varieties of a good are much less substitutable, firms’ profits are less sensitive to NER-induced relative price movements, and the frequency of price adjustment increases less with increases in the volatility of NERs. Moreover, as the right panel of Fig. 5 indicates, differences in the size of the trade cost across goods induce small differences in

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18The concordance between the two data sets is available from the author upon request.
the frequency of price adjustment. Finally, Fig. 6 superimposes the relationship between real and NER volatility in the aggregate data, together with the predictions of the two parametrizations of the menu-cost model considered. The model with lower elasticities of substitution, although incapable of accounting for all the volatility of real exchange rates in the data, is qualitatively consistent with the lack of a hump-shaped relationship between real and NER volatility I have documented.
5. Conclusion

Recent research\textsuperscript{19} has emphasized the importance of aggregation and substitution biases in generating the observed correlation between nominal and real exchange rates in the data. Because aggregate real exchange rates are constructed using consumption-based price indices, they may reflect substitution by consumers toward cheaper, non-tradeable varieties of goods whose prices do not immediately respond to exchange rate fluctuations. Moreover, some of the properties of the real exchange rate series are heavily influenced by the behavior of its most volatile components. Crucini et al. (2005) provide support for this idea. They use a large data set of actual transaction prices to construct real exchange rate series immune to substitution biases and find that (at five-year horizons) real and NERs are weakly (in fact, negatively) correlated in a sample of European countries. Crucini and Shintani (2004) show, using disaggregated relative price data, that the degree of a good’s tradeability affects the behavior of its relative prices: relative prices of tradeable goods are more transitory (half-lives less than two years) than the relative prices of non-tradeable goods (half-lives in excess of four years).

Unless aggregation bias is indeed important in generating the type of real exchange rate movements observed in the data—a question whose resolution is beyond the scope of this paper—a sticky price model can rationalize the lack of a hump-shaped relationship between real and NER volatility only if arbitraging international relative price fluctuations is costly, because of high costs of international trade, low elasticities of substitution, or additional frictions I have abstracted from in the quantitative analysis. Frictions that prevent goods-market arbitrage are therefore an important ingredient of a micro-founded sticky price model capable of replicating the relationship between nominal and real exchange rate volatility in the data.

References


\textsuperscript{19} Burstein et al. (2005), Campa and Goldberg (2005), Imbs et al. (2005), and Taylor (2001).


