Asset Prices and Unemployment Fluctuations: Appendix

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Contents

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A Omitted Proofs and Details

We begin with the omitted proofs and technical details from the paper.

A.1 Competitive Search Equilibrium

Define the elasticity of the job-filling rate with respect to market tightness $\theta_t(z)$ as

$$\eta(\theta_t(z)) \equiv -\frac{d\log(\lambda_f(\theta_t(z)))}{d\log(\theta_t(z))} = -\theta_t(z)\frac{\lambda_f'(\theta_t(z))}{\lambda_f(\theta_t(z))}. \tag{1}$$

Then, as long as $\theta_t(z) > 0$, we have

$$\frac{\lambda_f'(\theta_t(z))}{\lambda_f(\theta_t(z))} = -\frac{\eta(\theta_t(z))}{\theta_t(z)} \text{ and } \frac{\lambda_w'(\theta_t(z))}{\lambda_w(\theta_t(z))} = \frac{1-\eta(\theta_t(z))}{\theta_t(z)}, \tag{2}$$

where the second equality follows from (1), $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$ for $\theta_t(z) > 0$, and

$$\frac{\lambda_w'(\theta_t(z))}{\lambda_w(\theta_t(z))} = \frac{\theta_t(z)\lambda_f'(\theta_t(z))}{\theta_t(z)\lambda_f(\theta_t(z))} + \frac{1}{\theta_t(z)} = -\frac{\eta(\theta_t(z))}{\theta_t(z)} + \frac{1}{\theta_t(z)}.$$

In this expression, the first equality follows from differentiating $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$ and substituting $\lambda_w'(\theta_t(z)) = \theta_t(z)\lambda_f'(\theta_t(z)) + \lambda_f(\theta_t(z))$ and $\lambda_w(\theta_t(z)) = \theta_t(z)\lambda_f(\theta_t(z))$ for $\lambda_w'(\theta_t(z))/\lambda_w(\theta_t(z))$ and the second equality uses (1). Notice also that the second equation in (2) and $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$ imply

$$\lambda_w'(\theta_t(z)) = [1-\eta(\theta_t(z))]\,\lambda_{ft}(\theta_t(z)), \tag{3}$$

which we will use later. It will be convenient for later results to summarize in a compact form the conditions determining the competitive search equilibrium. In the next proposition, we do so and also provide a condition that implies that the deviation market tightness $\tilde{\theta}_t(z)$ equals the symmetric one $\theta_t(z)$.

Proposition 1 (Characterization of Competitive Search Equilibrium). *In a competitive search equilibrium the following hold:*

a. The optimality condition for a firm's wage offer is

$$-\frac{\lambda_f'(\theta_t(z))}{\lambda_f(\theta_t(z))}[Y_t(z) - W_{mt}(z)] = \frac{\lambda_w'(\theta_t(z))}{\lambda_w(\theta_t(z))}[W_{mt}(z) + W_{pt}(z) - U_t(z)] \tag{4}$$

or, equivalently using (2) when $\theta_t(z) > 0$,

$$\eta(\theta_t(z))[Y_t(z) - W_{mt}(z)] = [1-\eta(\theta_t(z))][W_{mt}(z) + W_{pt}(z) - U_t(z)]. \tag{5}$$

b. The free-entry condition is given by

$$\kappa A_t z = \lambda_f(\theta_t(z))[Y_t(z) - W_{mt}(z)]. \tag{6}$$

c. The values of output in a match $Y_t(z)$, the value of unemployment $U_t(z)$, and the post-match *value $W_{pt}(z)$ satisfy*

$$Y_t(z) = A_t z + \phi(1-\sigma)\mathbb{E}_t Q_{t,t+1} Y_{t+1}((1+g_e)z) \tag{7}$$

$$\begin{aligned} U_t(z) &= bA_t z + \phi \mathbb{E}_t Q_{t,t+1} \lambda_w(\theta_{t+1}(z)) \left[W_{mt+1}((1+g_u)z) + W_{pt+1}((1+g_u)z)\right] \\ &\quad + \phi \mathbb{E}_t Q_{t,t+1}[1-\lambda_w(\theta_{t+1}(z))]U_{t+1}((1+g_u)z) \end{aligned} \tag{8}$$

$$W_{pt}(z) = \phi(1-\sigma)\mathbb{E}_t Q_{t,t+1} W_{pt+1}((1+g_e)z) + \phi\sigma \mathbb{E}_t Q_{t,t+1} U_{t+1}((1+g_e)z). \tag{9}$$

d. Suppose that the matching function $m(u_{bt}(z), v_t(z))$ is such that its derivative with respect to the measure of unemployed workers, namely,

$$D(\theta_t(z)) \equiv \frac{\partial m(u_{bt}(z), v_t(z))}{\partial u_{bt}(z)} = \eta_t \lambda_w(\theta_t(z)) \tag{10}$$

is strictly increasing in $\theta_t(z)$ and that match surplus, $Y_t(z) + W_{pt}(z) - U_t(z)$, is different from zero. Then, it is optimal for each firm to choose $\tilde{\theta}_t(z) = \theta_t(z)$ and $\tilde{W}_{mt}(z) = W_{mt}(z)$, where $\theta_t(z)$ and $W_{mt}(z)$ are the common market tightness and offer.

Note that condition (10) in part d is satisfied for many common matching functions, including the Cobb-Douglas matching function that we use.

Proof. For part a, we first derive the firm optimality condition (4). Consider the firm's maximization problem, taking as given that all other firms have chosen $W_{mt}(z)$ and market tightness is $\theta_t(z)$, namely,

$$\begin{aligned} \max_{\tilde{W}_{mt}(z), \tilde{\theta}_t(z)} \quad & -\kappa A_t z + \lambda_f(\tilde{\theta}_t(z))[Y_t(z) - \tilde{W}_{mt}(z)] + [1-\lambda_f(\tilde{\theta}_t(z))]\mathbb{E}_t Q_{t,t+1} V_{t+1}(z) \\ \text{s.t.} \quad \tilde{\mu}_t(z): \quad & \lambda_w(\theta_t(z))[W_{mt}(z) + W_{pt}(z) - U_t(z)] \leq \lambda_w(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)], \end{aligned}$$

where $\tilde{\mu}_t(z)$ is the multiplier on the worker participation constraint. The associated first-order conditions are

$$\tilde{W}_{mt}(z) : \lambda_f(\tilde{\theta}_t(z)) = \tilde{\mu}_t(z)\lambda_w(\tilde{\theta}_t(z)) \tag{11}$$

and

$$\tilde{\theta}_t(z) : \lambda_f'(\tilde{\theta}_t(z))[Y_t(z) - \tilde{W}_{mt}(z) - \mathbb{E}_t Q_{t,t+1} V_{t+1}(z)] = \tilde{\mu}_t(z)\lambda_w'(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)]$$

or, rearranging terms, using the free-entry condition $V_{t+1}(z) = 0$, and (11),

$$-\frac{\lambda_f'(\tilde{\theta}_t(z))}{\lambda_f(\tilde{\theta}_t(z))}[Y_t(z) - \tilde{W}_{mt}(z)] = \frac{\lambda_w'(\tilde{\theta}_t(z))}{\lambda_w(\tilde{\theta}_t(z))}[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)].$$

By assuming that $\tilde{\theta}_t(z) > 0$, multiplying both sides by $\tilde{\theta}_t(z)$, and substituting from (2), we obtain

$$\tilde{\eta}_t(\tilde{\theta}_t(z))[Y_t(z) - \tilde{W}_{mt}(z)] = [1-\tilde{\eta}_t(\tilde{\theta}_t(z))][\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)], \tag{12}$$

which establishes (4) and (5).

For parts b and c, we note that the free-entry condition is in the paper and the equations for $Y_t(z)$, $U_t(z)$, and $W_{pt}(z)$ can all be derived by rearranging the expressions in the paper.

We turn now to part d. We establish this result by using the condition $Y_t(z) + W_{pt}(z) - U_t(z) \neq 0$ to reduce the firm's optimality condition to

$$D(\theta_t(z)) = D(\tilde{\theta}_t(z)), \tag{13}$$

which under the assumption that $D(\cdot)$ is strictly increasing, has a unique solution $\tilde{\theta}_t(z) = \theta_t(z)$. To do so, note that, by the participation constraint, we have

$$\tilde{W}_{mt}(z) = U_t(z) - W_{pt}(z) + \tilde{x}_t(z)[W_{mt}(z) + W_{pt}(z) - U_t(z)] \text{ and } \tilde{x}_t(z) \equiv \frac{\lambda_w(\theta_t(z))}{\lambda_w(\tilde{\theta}_t(z))}. \tag{14}$$

Substituting from (14) into (12) gives

$$\begin{aligned} &\tilde{\eta}_t(\tilde{\theta}_t(z))\{Y_t(z) - U_t(z) + W_{pt}(z) - \tilde{x}_t(z)[W_{mt}(z) + W_{pt}(z) - U_t(z)]\} \\ &\quad = [1 - \tilde{\eta}_t(\tilde{\theta}_t(z))]\tilde{x}_t(z)[W_{mt}(z) + W_{pt}(z) - U_t(z)]. \end{aligned}$$

Rearranging (5) as

$$W_{mt}(z) + W_{pt}(z) - U_t(z) = \eta(\theta_t(z))[Y_t(z) + W_{pt}(z) - W_{mt}(z)]$$

and plugging this into the previous equation yields

$$\begin{aligned} &\tilde{\eta}_t(\tilde{\theta}_t(z))\{Y_t(z) - U_t(z) + W_{pt}(z) - \tilde{x}_t(z)\tilde{\eta}_t(\tilde{\theta}_t(z))[Y_t(z) + W_{pt}(z) - U_t(z)]\} \\ &\quad = [1 - \tilde{\eta}_t(\tilde{\theta}_t(z))]\tilde{x}_t(z)\eta_t(\theta_t(z))[Y_t(z) + W_{pt}(z) - U_t(z)] \end{aligned}$$

or, using that $Y_t(z) + W_{pt}(z) - U_t(z) \neq 0$,

$$\tilde{\eta}_t(\tilde{\theta}_t(z))[1 - \tilde{x}_t(z)\eta_t(\theta_t(z))] = [1 - \tilde{\eta}_t(\tilde{\theta}_t(z))]\tilde{x}_t(z)\eta_t(\theta_t(z)) \text{ or } \tilde{x}_t(z) = \frac{\tilde{\eta}_t(\tilde{\theta}_t(z))}{\eta_t(\theta_t(z))},$$

which using $\tilde{x}_t(z) = \lambda_w(\theta_t(z))/\lambda_w(\tilde{\theta}_t(z))$ and $D(\theta_t(z)) = \eta_t\lambda_w(\theta_t(z))$, can be rewritten as

$$D(\theta_t(z)) = \eta_t(\theta_t(z))\lambda_w(\theta_t(z)) = \tilde{\eta}_t(\tilde{\theta}_t(z))\lambda_w(\tilde{\theta}_t(z)) = D(\tilde{\theta}_t(z)), \tag{15}$$

which implies (13). Since $D(\cdot)$ is monotone, we have that $\tilde{\theta}_t(z) = \theta_t(z)$. Finally, to see that $D(\theta_t(z))$ satisfies (10), we note that since $m_t = v_t\lambda_{ft}$,

$$\begin{aligned} D(\theta_t(z)) &= \frac{\partial\left[v_t(z)\lambda_f(\theta_t(z))\right]}{\partial u_{bt}(z)} = v_t(z)\frac{\partial\lambda_f(\theta_t(z))}{\partial\theta_t(z)}\frac{\partial\theta_t(z)}{\partial u_{bt}(z)} = -\theta_t(z)^2\lambda_f'(\theta_t(z)) \\ &= \left[-\theta_t(z)\frac{\lambda_f'(\theta_t(z))}{\lambda_f(\theta_t(z))}\right]\theta_t(z)\lambda_f(\theta_t(z)) = \eta_t\lambda_w(\theta_t(z)), \end{aligned}$$

where in the third equality we use that $\partial\theta/\partial u_{bt} = \partial(v/u_{bt})/\partial u_{bt} = -v/u_{bt}^2$ and in the fourth equality we used (2) and that $\lambda_w(\theta_t(z)) = \theta_t(z)\lambda_f(\theta_t(z))$. □

A.2 Linearity of Competitive Search Equilibrium

We begin with the statement of Lemma 1 and then provide its proof.

Lemma 1 (Linearity of Competitive Search Equilibrium). *In a competitive search equilibrium, labor market tightness* $\theta_t(z)$*, the job-finding rate* $\lambda_{wt}(\theta_t(z))$*, the job-filling rate* $\lambda_{ft}(\theta_t(z))$*, and the elasticity* $\eta_t(\theta_t(z))$ *are independent of* z*, and values are linear in* z *in that* $W_{mt}(z) = W_{mt}z$*,* $W_{pt}(z) = W_{pt}z$*,* $U_t(z) = U_t z$*,* $\mathcal{W}_t(z) = \mathcal{W}_t z$*, and* $Y_t(z) = Y_t z$*.*

Proof. By Proposition 1, for a given path for the stochastic discount factor, the competitive search equilibrium is characterized by processes $\{\theta_t(z), Y_t(z), U_t(z), W_{pt}(z), W_{mt}(z)\}$ that satisfy (4), (6), (7),(8), and (9).

We guess and then verify that a solution to this system of equations has labor market tightness independent of z, $\theta_t(z) = \theta_t$, and valuations linear in z, $W_{pt}(z) = W_{pt}z$, $U_t(z) = U_t z$, $Y_t(z) = Y_t z$, and $W_{mt}(z) = W_{mt}z$, and so $\mathcal{W}_t(z) = \mathcal{W}_t z$. The verification step replaces the guess in the equations and divides by z both sides of each equation so that

$$\begin{cases} 0 = \dfrac{\lambda'_{ft}(\theta_t)}{\lambda_{ft}(\theta_t)}(Y_t - W_{mt}) + \dfrac{\lambda'_{wt}(\theta_t)}{\lambda_{wt}(\theta_t)}(W_{mt} + W_{pt} - U_t) \\ \kappa A_t = \lambda_{ft}(\theta_t)\,(Y_t - W_{mt}) \\ W_{pt} = \phi(1-\sigma)(1+g_e)\mathbb{E}_t(Q_{t,t+1}W_{pt+1}) + \phi\sigma(1+g_e)\mathbb{E}_t(Q_{t,t+1}U_{t+1}) \\ U_t = bA_t + \phi(1+g_u)\mathbb{E}_t[Q_{t,t+1}\lambda_{wt}(\theta_t)(W_{mt+1} + W_{pt+1})] + \phi(1+g_u)\mathbb{E}_t\{Q_{t,t+1}[1-\lambda_{wt}(\theta_t)]U_{t+1}\} \\ Y_t = A_t + \phi(1-\sigma)(1+g_e)\mathbb{E}_t(Q_{t,t+1}Y_{t+1}) \end{cases}.$$

This system of equations admits a solution independent of z, thereby verifying the guess. Note that the linearity of flow market production, home production, and vacancy costs is key to this result. □

A.3 Laws of Motion for Aggregate Human Capital

To derive these aggregate laws of motion, we first show how to derive the laws of motions for $e_t(z)$ and $u_t(z)$, namely,

$$e_t(z) = \frac{\phi\,(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \lambda_{wt}(\theta_t)u_{bt}(z) \tag{16}$$

and

$$u_t(z) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + [1-\lambda_{wt}(\theta_t)]u_{bt}(z) + (1-\phi)\,\nu(z), \tag{17}$$

and then aggregate them. To see where these laws of motion come from, denote by (z_t, τ_t) the pair of human capital and labor market status $\tau_t \in \{e, u\}$, namely, either employed or unemployed, of a consumer at t with human capital z_{t-1} and market status $\tau_{t-1} \in \{e, u\}$ in $t-1$. Note that

$$(z_t, \tau_t)|\tau_{t-1} = \begin{cases} ((1+g_e)z_{t-1}, e), & \text{with probability } \phi(1-\sigma) \text{ if } \tau_{t-1} = e \\ ((1+g_e)z_{t-1}, u), & \text{with probability } \phi\sigma \text{ if } \tau_{t-1} = e \\ ((1+g_u)z_{t-1}, e), & \text{with probability } \phi\lambda_w(\theta_t) \text{ if } \tau_{t-1} = u \\ ((1+g_u)z_{t-1}, u), & \text{with probability } \phi[1-\lambda_w(\theta_t)] \text{ if } \tau_{t-1} = u \\ (z, u), & \text{with probability } (1-\phi)\nu(z) \text{ for all } \tau_{t-1} \end{cases}$$

with $z > 0$ drawn from the continuous distribution with density $\nu(z)$. Hence,

$$e_t(z) = \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \lambda_w(\theta_t)\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right),$$
$$u_t(z) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + [1-\lambda_w(\theta_t)]\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) + (1-\phi)v(z),$$

where the measure of unemployed workers at the beginning of period t with human capital z is

$$u_{bt}(z) = \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right).$$

From (16) and (17), we can derive the law of motion of the aggregate human capital of employed workers $Z_{et} \equiv \int z e_t(z) dz$ as

$$\begin{aligned}
Z_{et} &= \int z e_{t-1}\left(\frac{z}{1+g_e}\right)\frac{\phi(1-\sigma)}{1+g_e} dz + \int z u_{t-1}\left(\frac{z}{1+g_u}\right)\frac{\phi\lambda_w(\theta_t)}{1+g_u} dz \\
&= \phi(1-\sigma)\int \frac{z}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) dz + \phi\lambda_w(\theta_t)\int \frac{z}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) dz \\
&= \phi(1-\sigma)(1+g_e)\int \frac{z}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) d\left(\frac{z}{1+g_e}\right) \\
&+ \phi\lambda_w(\theta_t)(1+g_u)\int \frac{z}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) d\left(\frac{z}{1+g_u}\right) \\
&= \phi(1-\sigma)(1+g_e)Z_{et-1} + \phi\lambda_w(\theta_t)(1+g_u)Z_{ut-1},
\end{aligned}$$

where the second and fourth equalities follow from simple algebra and the definitions of Z_{et-1} and Z_{ut-1} whereas the third equality follows from the change of variable from $x = z$ to $y = z/(1+g_e)$ in the first integral, which is then multiplied by the Jacobian $dx/dy = 1+g_e$ of this transformation, and from the change of variable from z to $z/(1+g_u)$ in the second integral, which is then multiplied by the Jacobian $dx/dy = 1+g_u$ of this transformation. (Recall that if X and $Y = g(X)$ are two continuous random variables with densities $f_X(x)$ and $f_Y(y)$, then $f_Y(y) = f_X(x)\det(dx/dy')$.)

Similarly, we can derive the law of motion of the aggregate human capital of unemployed workers $Z_{ut} \equiv \int z u_t(z) dz$ as

$$\begin{aligned}
Z_{ut} &= \int z e_{t-1}\left(\frac{z}{1+g_e}\right)\frac{\phi\sigma}{1+g_e} dz + \int z u_{t-1}\left(\frac{z}{1+g_u}\right)\frac{\phi[1-\lambda_w(\theta_t)]}{1+g_u} dz + (1-\phi)\int z\nu(z) dz \\
&= \phi\sigma\int \frac{z}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) dz + \phi[1-\lambda_w(\theta_t)]\int \frac{z}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) dz + (1-\phi)\int z\nu(z) dz \\
&= \phi\sigma(1+g_e)\int \frac{z}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) d\left(\frac{z}{1+g_e}\right) \\
&+ \phi[1-\lambda_w(\theta_t)](1+g_u)\int \frac{z}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) d(\frac{z}{1+g_u}) + (1-\phi)\int z\nu(z) dz \\
&= 1-\phi+\phi\sigma(1+g_e)Z_{et-1} + \phi[1-\lambda_w(\theta_t)](1+g_u)Z_{ut-1},
\end{aligned}$$

where, as before, the second and fourth equalities follow by simple algebra whereas the third equality follows from the change of variable from z to $z/(1+g_e)$ in the first integral and from z to $z/(1+g_u)$ in the second integral, and in the fourth equality we used that the mean of the human capital of newborns, $\int z\nu(z)dz = 1$.

A.4 Efficiency of Competitive Search Equilibrium

We start by characterizing the solution to the planning problem: choose $\{C(s^t), Z_e(s^t), Z_u(s^t), \theta(s^t)\}$ to solve

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)[C(s^t) - X_t(s^t)]^{1-\alpha}/(1-\alpha),$$

subject to the constraints

$$\lambda(s^t) : C(s^t) \leq A(s^t)Z_e(s^t) + bA(s^t)Z_u(s^t) - \phi(1+g_u)\kappa A(s^t)\theta(s^t)Z_{ut-1}(s^{t-1}),$$

$$\lambda(s^t)\mu_e(s^t) : Z_e(s^t) \leq \phi(1+g_e)(1-\sigma)Z_e(s^{t-1}) + \phi(1+g_u)\lambda_w(\theta(s^t))Z_u(s^{t-1}),$$

and

$$\lambda(s^t)\mu_u(s^t) : Z_u(s^t) \leq 1 - \phi + \phi(1+g_e)\sigma Z_e(s^{t-1}) + \phi(1+g_u)[1-\lambda_w(\theta(s^t))]Z_u(s^{t-1})$$

for all $t = 0, \ldots, \infty$ and s^t. The optimality conditions of the planning problem are

$$C_t(s^t) : \lambda(s^t) = \beta^t \pi(s^t) \left(C(s^t) - X(s^t)\right)^{-\alpha}, \tag{18}$$

$$Z_e(s^t) : \lambda(s^t)\mu_e(s^t) = \lambda(s^t)A(s^t) + \phi(1+g_e)\sum_{s^{t+1}} \lambda(s^{t+1}) \left[(1-\sigma)\mu_e(s^{t+1}) + \sigma\mu_u(s^{t+1})\right], \tag{19}$$

$$\begin{aligned} Z_{ut}(s^t) \quad : \quad & \lambda(s^t)\mu_u(s^t) = \lambda(s^t)bA_t(s^t) - \phi(1+g_u)\sum_{s^{t+1}} \lambda(s^{t+1})\kappa A(s^{t+1})\theta(s^{t+1}) \\ & +\phi(1+g_u)\sum_{s^{t+1}} \lambda(s^{t+1})\{\lambda_w(\theta(s^{t+1}))\mu_e(s^{t+1}) + [1-\lambda_w(\theta(s^{t+1}))]\mu_u(s^{t+1})\}, \end{aligned} \tag{20}$$

and

$$\theta(s^t) : \lambda(s^t)\phi(1+g_u)\kappa A(s^t)Z_{ut-1}(s^{t-1}) = \lambda(s^t)\left[\mu_e(s^t) - \mu_u(s^t)\right]\phi(1+g_u)Z_u(s^{t-1})\lambda_w'(\theta(s^t)). \tag{21}$$

To eliminate the multipliers $\lambda(s^t)$ and $\lambda(s^{t+1})$, we use (18) and that $\pi(s^{t+1}|s^t) \equiv \pi(s^{t+1})/\pi(s^t)$ to write

$$\lambda(s^{t+1})/\lambda(s^t) = \frac{\pi(s^{t+1})\beta[C(s^{t+1}) - X(s^{t+1})]^{-\alpha}}{\pi(s^t)[C(s^{t+1}) - X(s^{t+1})]^{-\alpha}} \equiv \pi(s^{t+1}|s^t)Q_{t,t+1}(s^{t+1})$$

and then divide (19), (20), and (21) by $\lambda(s^t)$ and use (11) to obtain

$$\begin{aligned} \mu_e(s^t) &= A(s^t) + \phi(1+g_e)\sum_{s^{t+1}} \frac{\lambda(s^{t+1})}{\lambda(s^t)} \left[(1-\sigma)\mu_e(s^{t+1}) + \sigma\mu_u(s^{t+1})\right] \\ &= A(s^t) + \phi(1+g_e)\sum_{s^{t+1}} \pi(s^{t+1}|s^t)Q_{t,t+1}(s^{t+1}) \left[(1-\sigma)\mu_e(s^{t+1}) + \sigma\mu_u(s^{t+1})\right], \end{aligned}$$

$$\begin{aligned}\mu_u(s^t) &= bA_t(s^t) - \phi(1+g_u)\sum_{s^{t+1}} \pi(s^{t+1}|s^t)Q_{t,t+1}(s^{t+1})\kappa A(s^{t+1})\theta(s^{t+1}) \\ &\quad +\phi(1+g_u)\sum_{s^{t+1}} \pi(s^{t+1}|s^t)Q_{t,t+1}(s^{t+1})\{\lambda_w(\theta(s^{t+1}))\mu_e(s^{t+1}) + [1-\lambda_w(\theta(s^{t+1}))]\mu_u(s^{t+1})\},\end{aligned}$$

and

$$\kappa A(s^t) = \left[\mu_e(s^t) - \mu_u(s^t)\right] \lambda_w'(\theta(s^t)).$$

Dropping the notation for s^t and the explicit dependence of λ_{wt} and λ_{ft} on θ_t, we have

$$\mu_{et} = A_t + \phi(1+g_e)\mathbb{E}_t Q_{t,t+1}\left[(1-\sigma)\mu_{et+1} + \sigma\mu_{ut+1}\right], \tag{22}$$

$$\mu_{ut} = bA_t - \phi(1+g_u)\mathbb{E}_t\left[Q_{t+1}\kappa A_{t+1}\theta_{t+1}\right] + \phi(1+g_u)\mathbb{E}_t\left[Q_{t,t+1}[\lambda_{wt+1}\mu_{et+1} + (1-\lambda_{wt+1})\mu_{ut+1}]\right], \tag{23}$$

and

$$\kappa A_t = (\mu_{et} - \mu_{ut})\lambda_{wt}' = (1-\eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}), \tag{24}$$

where to obtain the second equality, we used (3). Now, we want to show that (23) reduces to

$$\mu_{ut} = bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}[\eta_{t+1}\lambda_{wt+1}\mu_{et+1} + (1-\eta_{t+1}\lambda_{wt+1})\mu_{ut+1}]. \tag{25}$$

To do so, we substitute $\lambda_{wt+1}' = (1-\eta_{t+1})\lambda_{wt+1}/\theta_{t+1}$ from (2) to rewrite the first equality in the free-entry condition (24) at $t+1$ as

$$\kappa A_{t+1}\theta_{t+1} = (\mu_{et+1} - \mu_{ut+1})(1-\eta_{t+1})\lambda_{wt+1}$$

and substitute this into (23) to obtain

$$\begin{aligned}\mu_{ut} &= bA_t + \phi(1+g_u)\mathbb{E}_t\left[Q_{t,t+1}(\mu_{ut+1} - \mu_{et+1})(1-\eta_{t+1})\lambda_{wt+1}\right] \\ &\quad +\phi(1+g_u)\mathbb{E}_t\left[Q_{t,t+1}\{\lambda_{wt+1}\mu_{et+1} + [1-\lambda_{wt+1}]\mu_{ut+1}\}\right],\end{aligned}$$

which simplifies to (25).

Proposition 1 (Efficiency of Competitive Search Equilibrium). *The competitive search equilibrium allocations solve the planning problem.*

Proof. We have just shown that the optimality conditions of the planning problem are (22), (25), and (24) repeated here

$$\mu_{ut} = bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}\{\eta_{t+1}\lambda_{wt+1}\mu_{et+1} + [1-\eta_{t+1}\lambda_{wt+1}]\mu_{ut+1}\}, \tag{26}$$

$$\mu_{et} = A_t + \phi(1+g_e)\mathbb{E}_t Q_{t,t+1}\left[(1-\sigma)\mu_{et+1} + \sigma\mu_{ut+1}\right], \tag{27}$$

$$\kappa A_t = (1-\eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}), \tag{28}$$

where $Q_{t,t+1} = \beta[S_{t+1}C_{t+1}/(S_tC_t)]^{-\alpha}$. Hence, a solution to the planning problem is completely characterized by these three conditions along with the original constraints to the problem, namely, the resource constraint and the transition laws for Z_{et} and Z_{ut}. The competitive search equilibrium is completely characterized by the equations given in Lemma 1 in the paper namely,

$$W_{pt} = \phi(1-\sigma)(1+g_e)\mathbb{E}_t Q_{t,t+1} W_{pt+1} + \phi\sigma(1+g_e)\mathbb{E}_t Q_{t,t+1} U_{t+1}, \tag{29}$$
$$U_t = bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}\lambda_{wt+1}(W_{mt+1}+W_{pt+1}) + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}[1-\lambda_{wt+1}]U_{t+1}, \tag{30}$$
$$Y_t = A_t + \phi(1-\sigma)(1+g_e)\mathbb{E}_t Q_{t,t+1} Y_{t+1}, \tag{31}$$
$$\kappa A_t = \lambda_{ft}\left(Y_t - W_{mt}\right), \tag{32}$$
$$0 = \frac{\lambda'_{wt}}{\lambda_{wt}}(W_{mt}+W_{pt}-U_t) + \frac{\lambda'_{ft}}{\lambda_{ft}}(Y_t - W_{mt}), \tag{33}$$

along with the the resource constraint and the transition laws for Z_{et} and Z_{ut}. We argue that by suitably redefining the variables $\{W_{mt}, W_{pt}, U_t, Y_t\}$ in the competitive search equilibrium in terms of the variables $\{\mu_{et}, \mu_{ut}\}$ and the allocations in the planning problem, we can show that (29)-(33) are identical to (26)-(28). We conclude that the solution to the planning problem and the allocations in the competitive search equilibrium coincide.

We now claim that if we replace $Y_t + W_{pt}$ with μ_{et} and U_t with μ_{ut}, then equations (29)-(33) reduce to equations (26)-(28) so the allocations in the competitive search equilibrium solve the planning problem. To establish this claim, we first use (2), namely,

$$\frac{\lambda'_{wt}}{\lambda_{wt}} = \frac{1-\eta_t}{\theta_t} \text{ and } \frac{\lambda'_{ft}}{\lambda_{ft}} = -\frac{\eta_t}{\theta_t},$$

to rewrite (33), which after multiplying both sides by $\theta_t > 0$, is

$$(1-\eta_t)(W_{mt}+W_{pt}-U_t) - \eta_t(Y_t - W_{mt}) = 0 \tag{34}$$

so

$$\eta_t(Y_t - W_{mt}) = (1-\eta_t)[Y_t + W_{pt} - U_t - (Y_t - W_{mt})].$$

Adding $(1-\eta_t)(Y_t - W_{mt})$ to both sides gives

$$Y_t - W_{mt} = (1-\eta_t)(Y_t + W_{pt} - U_t). \tag{35}$$

Using this equation to substitute for $Y_t - W_{mt}$ in (34) further gives

$$(1-\eta_t)(W_{mt}+W_{pt}-U_t) = \eta_t(1-\eta_t)(Y_t + W_{pt} - U_t)$$

and dividing by $1-\eta_t$ gives

$$W_{mt}+W_{pt}-U_t = \eta_t(Y_t + W_{pt} - U_t).$$

Adding U_t to both sides then yields

$$W_{mt}+W_{pt} = \eta_t(Y_t + W_{pt}) + (1-\eta_t)U_t. \tag{36}$$

Now use this expression for $W_{mt} + W_{pt}$ to substitute for $W_{mt+1} + W_{pt+1}$ in (30)

$$\begin{aligned} U_t &= bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}\lambda_{wt+1}[\eta_{t+1}(Y_{t+1} + W_{pt+1}) + (1-\eta_{t+1})U_{t+1}] \\ &\quad +\phi(1+g_u)\mathbb{E}_t Q_{t,t+1}[1-\lambda_{wt+1}]U_{t+1} \\ &= bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}\lambda_{wt+1}\eta_{t+1}(Y_{t+1} + W_{pt+1}) \\ &\quad +\phi(1+g_u)\mathbb{E}_t Q_{t,t+1}[\lambda_{wt+1}(1-\eta_{t+1}) + 1 - \lambda_{wt+1}]U_{t+1} \\ &= bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}\lambda_{wt+1}\eta_{t+1}(Y_{t+1} + W_{pt+1}) \\ &\quad +\phi(1+g_u)\mathbb{E}_t Q_{t,t+1}(1-\lambda_{wt+1}\eta_{t+1})U_{t+1} \end{aligned} \tag{37}$$

and so using that $\mu_{et+1} = Y_{t+1} + W_{pt+1}$ and $\mu_{ut+1} = U_{t+1}$ in this last equation gives

$$\mu_{ut} = bA_t + \phi(1+g_u)\mathbb{E}_t[Q_{t,t+1}[\lambda_{wt+1}\eta_{t+1}\mu_{et+1} + (1-\lambda_{wt+1}\eta_{t+1})\mu_{ut+1}]],$$

namely, (26). Proceeding similarly, note that by summing Y_t and W_{pt} from (31) and (29), we obtain

$$Y_t + W_{pt} = A_t + \phi(1+g_e)\mathbb{E}_t[Q_{t,t+1}[(1-\sigma)(Y_{t+1} + W_{pt+1}) + \sigma U_{t+1}]] \tag{38}$$

and so using that $\mu_{et} = Y_t + W_{pt}$ and $\mu_{ut} = U_t$, we obtain

$$\mu_{et} = A_t + \phi(1+g_e)\mathbb{E}_t[Q_{t,t+1}[(1-\sigma)\mu_{et+1} + \sigma\mu_{ut+1}]],$$

namely, (27). Now use (35), $\mu_{et} = Y_t + W_{pt}$, and $\mu_{ut} = U_t$ to rewrite (32) as

$$\kappa A_t = \lambda_{ft}\left(Y_t - W_{mt}\right) = \lambda_{ft}(1-\eta_t)(Y_t + W_{pt} - U_t) = (1-\eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}), \tag{39}$$

namely, (28). Thus, the equations characterizing the competitive search equilibrium allocations coincide with those characterizing the solution to the planning problem. Hence, the allocations in the competitive search equilibrium solve the planning problem. □

Alternative Proof. An alternative way to prove this proposition consists of two steps. In Step 1, we directly calculate the allocations that solve the planning problem. In Step 2, we directly calculate the competitive search equilibrium allocations. By inspection, we will see that these allocations are identical.

Step 1: Directly calculate the conditions that the planning problem satisfies. To do so, we define $\tilde{\mu}_{et} = \mu_{et}/A_t$ and $\tilde{\mu}_{ut} = \mu_{ut}/A_t$ and express the dynamical system for the multipliers (26) and (27) in matrix form as

$$\begin{bmatrix} \tilde{\mu}_{et} \\ \tilde{\mu}_{ut} \end{bmatrix} = \begin{bmatrix} 1 \\ b \end{bmatrix} + \mathbb{E}_t\left\{\Psi(\theta_{t+1})Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix} \tilde{\mu}_{et+1} \\ \tilde{\mu}_{ut+1} \end{bmatrix}\right\}, \tag{40}$$

where

$$\Psi(\theta_{t+1}) \equiv \begin{bmatrix} \phi(1+g_e)(1-\sigma) & \phi(1+g_e)\sigma \\ \phi(1+g_u)\eta\lambda_w(\theta_{t+1}) & \phi(1+g_u)\left[1-\eta\lambda_w(\theta_{t+1})\right] \end{bmatrix}.$$

Then, we solve out this system for a formula for $\tilde{\mu}_{et} - \tilde{\mu}_{ut}$ in terms of allocations as follows,

$$
\begin{aligned}
\begin{bmatrix} \tilde{\mu}_{et} \\ \tilde{\mu}_{ut} \end{bmatrix} &= \begin{bmatrix} 1 \\ b \end{bmatrix} + \mathbb{E}_t \left[\Psi(\theta_{t+1}) Q_{t,t+1} \frac{A_{t+1}}{A_t} \left(\begin{bmatrix} 1 \\ b \end{bmatrix} + \mathbb{E}_{t+1} \left\{ \Psi(\theta_{t+2}) Q_{t+1,t+2} \frac{A_{t+2}}{A_{t+1}} \begin{bmatrix} \tilde{\mu}_{et+2} \\ \tilde{\mu}_{ut+2} \end{bmatrix} \right\} \right) \right] \\
&= \begin{bmatrix} 1 \\ b \end{bmatrix} \left\{ 1 + \mathbb{E}_t \left[\Psi(\theta_{t+1}) Q_{t,t+1} \frac{A_{t+1}}{A_t} + \Psi(\theta_{t+1}) \Psi(\theta_{t+2}) Q_{t,t+2} \frac{A_{t+2}}{A_t} \right] \right\} \\
&\quad + \mathbb{E}_t \left\{ \Psi(\theta_{t+1}) \Psi(\theta_{t+2}) Q_{t,t+2} \frac{A_{t+2}}{A_t} \begin{bmatrix} \tilde{\mu}_{et+2} \\ \tilde{\mu}_{ut+2} \end{bmatrix} \right\} \\
&= \begin{bmatrix} 1 \\ b \end{bmatrix} \left(1 + \mathbb{E}_t \left\{ \sum_{s=t+1}^{T} \Psi(\theta_{t+1}) \cdots \Psi(\theta_s) Q_{t,s} \frac{A_s}{A_t} + \Psi(\theta_{t+1}) \Psi(\theta_{t+2}) \cdots \Psi(\theta_T) Q_{t,T} \frac{A_T}{A_t} \begin{bmatrix} \tilde{\mu}_{eT} \\ \tilde{\mu}_{uT} \end{bmatrix} \right\} \right) \\
&= \begin{bmatrix} 1 \\ b \end{bmatrix} \left\{ 1 + \mathbb{E}_t \left[\sum_{s=t+1}^{\infty} \Psi(\theta_{t+1}) \cdots \Psi(\theta_s) Q_{t,s} \frac{A_s}{A_t} \right] \right\}, \qquad (41)
\end{aligned}
$$

where in the second line we used the law of iterated expectations, $\mathbb{E}_t \mathbb{E}_{t+1} x_{t+2} = \mathbb{E}_t x_{t+2}$, and that $Q_{t,t+2} = Q_{t,t+1} Q_{t+1,t+2}$, and in the last line we used the limiting condition

$$
\lim_{T \to \infty} \mathbb{E}_t \left\{ \Psi(\theta_{t+1}) \Psi(\theta_{t+2}) \cdots \Psi(\theta_T) Q_{t,T} \frac{A_T}{A_t} \begin{bmatrix} \tilde{\mu}_{eT} \\ \tilde{\mu}_{uT} \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
$$

Now, to obtain a formula for $\tilde{\mu}_{et} - \tilde{\mu}_{ut}$, we premultiply both sides of (41) by the vector $\begin{bmatrix} 1 & -1 \end{bmatrix}$ to obtain

$$
\tilde{\mu}_{et} - \tilde{\mu}_{ut} = \left\{ 1 - b + \mathbb{E}_t \sum_{s=t+1}^{\infty} \left[\Psi(\theta_{t+1}) \cdots \Psi(\theta_s) \right] Q_{t,s} \frac{A_s}{A_t} (1-b) \right\}, \qquad (42)
$$

which, when substituted into (105), gives that the stochastic process for job-finding rates satisfies

$$
\kappa = (1-\eta) \lambda_{ft}(\theta_t) \left\{ 1 - b + \mathbb{E}_t \sum_{r=t+1}^{\infty} \left[\Psi(\theta_{t+1}) \cdots \Psi(\theta_r) \right] Q_{t,r} \frac{A_r}{A_t} (1-b) \right\}, \qquad (43)
$$

where

$$
Q_{t,r} = \beta^{r-t} \left(\frac{S_r}{S_t} \frac{C_r}{C_t} \right)^{-\alpha} \qquad (44)
$$

and $s_t = \log(S_t)$ follows

$$
s_{t+1} = (1-\rho_s) s + \rho_s s_t + \lambda_a(s_t) \sigma_a \varepsilon_{at+1}, \qquad (45)
$$

C_t follows

$$
C_t = A_t Z_{et} + b A_t Z_{ut} - \kappa A_t \phi \theta_t (1+g_u) Z_{ut-1}, \qquad (46)
$$

and Z_{et} and Z_{ut} follow

$$
Z_{et} = \phi (1-\sigma)(1+g_e) Z_{et-1} + \phi \lambda_{wt}(\theta_t)(1+g_u) Z_{ut-1}, \qquad (47)
$$

and

$$
Z_{ut} = \phi \sigma (1+g_e) Z_{et-1} + \phi (1 - \lambda_{wt}(\theta_t))(1+g_u) Z_{ut-1} + 1 - \phi. \qquad (48)
$$

Here conditions (43)-(48) completely characterize the solution to the planning problem for any initial conditions $Z_{e,-1}$ and $Z_{u,-1}$.

Step 2. Directly calculate the conditions that the competitive search equilibrium satisfies. We can write the dynamical system (38) and (37) for the values $\tilde{Y}_t + \tilde{W}_{pt} \equiv (Y_t + W_{pt})/A_t$ and $\tilde{U}_t \equiv U_t/A_t$ in matrix form as

$$\begin{bmatrix} \tilde{Y}_t + \tilde{W}_{pt} \\ \tilde{U}_t \end{bmatrix} = \begin{bmatrix} 1 \\ b \end{bmatrix} + \mathbb{E}_t \left\{ \Psi(\theta_{t+1}) Q_{t,t+1} \frac{A_{t+1}}{A_t} \begin{bmatrix} \tilde{Y}_{t+1} + \tilde{W}_{pt+1} \\ \tilde{U}_{t+1} \end{bmatrix} \right\}$$

and follow the *identical* manipulations from (40) to (42) to arrive at

$$\tilde{Y}_t + \tilde{W}_{pt} - \tilde{U}_t = \left\{ 1 - b + \mathbb{E}_t \sum_{s=t+1}^{\infty} \left[\Psi(\theta_{t+1}) \cdots \Psi(\theta_s) \right] Q_{t,s} \frac{A_s}{A_t} (1-b) \right\}. \tag{49}$$

Now substituting the answer for $\tilde{Y}_t + \tilde{W}_{pt} - \tilde{U}_t$ in (49) into the free-entry condition (39) expressed as

$$\kappa = \lambda_{ft}(1-\eta_t)(\tilde{Y}_t + \tilde{W}_{pt} - \tilde{U}_t),$$

along with the resource constraints and laws of motion for Z_{et} and Z_{ut}, yields the identical equations (43)-(48), which completely characterize the solution to the planning problem. Hence, the allocations coincide. $\square$

A.5 Constant Job-Finding Rate Under CRRA

Proposition 2 (Constant Job-Finding Rate and Unemployment Under CRRA). *Starting from the steady-state values of the total human capital of employed and unemployed workers, Z_e and Z_u, with preferences of the form $E_0 \sum_{t=0}^{\infty} \beta^t C_t^{1-\alpha}/(1-\alpha)$, both the job-finding rate and unemployment are constant.*

Proof. Here we show that with CRRA utility and random-walk productivity, job-finding rates are constant in our competitive search equilibrium, where

$$\tilde{\mu}_{ut} = \tilde{\mu}_u, \tilde{\mu}_{et} = \tilde{\mu}_e, \theta_t = \theta, \tilde{C}_t = \tilde{C}, Z_{et} = Z_e, \text{ and } Z_{ut} = Z_u \tag{50}$$

and variables with $\tilde{}$ are scaled by productivity. We do so by showing that the equations characterizing the solution to the planning problem, namely, (26)-(28) along with that problem's constraints admit a solution of the form just described. To this purpose, consider first the difference equation for the value of employment, (27). Now substitute $\mathbb{E}_t(Q_{t,t+1}) = \beta \mathbb{E}_t \left(C_{t+1}/C_t\right)^{-\alpha}$ to obtain

$$\mu_{et} = A_t + \phi(1+g_e)\beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left[(1-\sigma)\mu_{et+1} + \sigma \mu_{ut+1} \right] \right\}, \tag{51}$$

which after dividing both sides by A_t gives

$$\frac{\mu_{et}}{A_t} = 1 + \phi(1+g_e)\beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \frac{A_{t+1}}{A_t} \right)^{-\alpha} \frac{A_{t+1}}{A_t} \left[(1-\sigma) \frac{\mu_{et+1}}{A_{t+1}} + \sigma \frac{\mu_{ut+1}}{A_{t+1}} \right] \right\}.$$

Using $\tilde{\mu}_{et} = \mu_{et}/A_t$, $\tilde{\mu}_{et} = \mu_{et}/A_t$, and $\tilde{C}_t = C_t/A_t$, we further obtain

$$\tilde{\mu}_{et} = 1 + \phi(1+g_e)\beta\mathbb{E}_t\left\{\left(\frac{A_{t+1}}{A_t}\right)^{1-\alpha}\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\alpha}\left[(1-\sigma)\tilde{\mu}_{et+1} + \sigma\tilde{\mu}_{ut+1}\right]\right\}$$

or, equivalently,

$$\tilde{\mu}_{et} = 1 + \phi(1+g_e)\beta\mathbb{E}_t\left\{e^{(1-\alpha)(g_a+\varepsilon_{at+1})}\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\alpha}\left[(1-\sigma)\tilde{\mu}_{et+1} + \sigma\tilde{\mu}_{ut+1}\right]\right\}, \tag{52}$$

where in the last step we used that $\log(A_{t+1}) = g_a + \log(A_t) + \varepsilon_{at+1}$ implies

$$\left(\frac{A_{t+1}}{A_t}\right)^{1-\alpha} = e^{(1-\alpha)(g_a+\varepsilon_{at+1})}. \tag{53}$$

At our conjectured solution, (50) implies that (52) becomes

$$\tilde{\mu}_e = 1 + \phi(1+g_e)\delta\left[(1-\sigma)\tilde{\mu}_e + \sigma\tilde{\mu}_u\right] \tag{54}$$

with $\delta \equiv \beta e^{(1-\alpha)g_a+(1-\alpha)^2\sigma_a^2/2}$, since ε_{at+1} distributed as $N(0,\sigma_a^2)$ implies that

$$\mathbb{E}_t e^{(1-\alpha)(g_a+\varepsilon_{at+1})} = e^{(1-\alpha)g_a+(1-\alpha)^2\sigma_a^2/2}.$$

Proceeding in a similar fashion with (26) gives

$$\tilde{\mu}_{ut} = b + \phi(1+g_u)\mathbb{E}_t\left(\left(\frac{A_{t+1}}{A_t}\right)^{1-\alpha}\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\alpha}\left\{\lambda_w(\theta_{t+1})\eta_{t+1}\tilde{\mu}_{et+1} + [1-\eta_{t+1}\lambda_w(\theta_{t+1})]\tilde{\mu}_{ut+1}\right\}\right),$$

which using (53) simplifies to

$$\tilde{\mu}_{ut} = b + \phi(1+g_u)\beta\mathbb{E}_t\left(e^{(1-\alpha)(g_a+\varepsilon_{at+1})}\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\alpha}\left\{\lambda_w(\theta_{t+1})\eta(\theta_{t+1})\tilde{\mu}_{et+1} + [1-\eta(\theta_{t+1})\lambda_w(\theta_{t+1})]\tilde{\mu}_{ut+1}\right\}\right). \tag{55}$$

At our conjectured solution, (50) implies that (55) becomes

$$\tilde{\mu}_u = b + \phi(1+g_u)\delta\left[\lambda_w(\theta)\eta(\theta)\tilde{\mu}_e + (1-\eta(\theta)\lambda_w(\theta))\tilde{\mu}_u\right]. \tag{56}$$

Also, evaluated at this conjectured solution, the resource constraint and the transition equations for Z_{et} and Z_{ut} are

$$\tilde{C} = Z_{et} + bZ_u - \kappa\phi(1+g_u)Z_u, \tag{57}$$
$$Z_e = \phi(1+g_e)(1-\sigma)Z_e + \phi(1+g_u)\lambda_w Z_u, \tag{58}$$
$$Z_u = 1 - \phi + \phi(1+g_e)\sigma Z_e + \phi(1+g_u)(1-\lambda_w)Z_u. \tag{59}$$

Hence, the system of equations for the economy admits a solution given by (54), (56), (57), (58), and

(59), in which all variables are constant and the initial conditions for Z_{et} and Z_{ut} are equal to the posited constants Z_e and Z_u. □

A.6 Job-Findings Rates with Human Capital Depreciation

In most of the paper, we have assumed that $g_u = 0$ for algebraic simplicity. Here we state and prove Proposition 3 for the general case when g_u is nonzero. Recall that to develop intuition for the solution to the dynamical system (22) and (23), we considered an approximation to it in which we ignore the variation in future job-finding rates, $\lambda_w(\theta_s) = \lambda_w(\theta)$ for $s > t$, for a given θ. The formulas we derive for any choice of θ.

Proposition 3 (Job-Finding Rate). *The job-finding rate approximately satisfies*

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right)\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)\frac{P_{nt}}{A_t}\right], \tag{60}$$

where χ is a constant,

$$\delta_{\ell,s} = \phi(1+g_u+\lambda)/2 \pm \phi[(1+g_u-\lambda)^2 + 4\eta\lambda_w(1+g_u)(g_e-g_u)]^{1/2}/2 \tag{61}$$

with $\lambda \equiv (1-\sigma)(1+g_e) - \eta\lambda_w(1+g_u)$, and

$$c_\ell = \frac{(\phi\lambda - \delta_s)(1-b) + \phi(g_e-g_u)b}{\delta_\ell - \delta_s} \text{ and } c_s = 1 - b - c_\ell. \tag{62}$$

Proof. Consider the system

$$\begin{bmatrix}\mu_{et}\\ \mu_{ut}\end{bmatrix} = \sum_{n=0}^{\infty}\Psi^n\begin{bmatrix}1\\ b\end{bmatrix}\mathbb{E}_t Q_{t,t+n}A_{t+n}, \tag{63}$$

where Ψ is the transition matrix given by

$$\Psi = \begin{bmatrix}\phi(1+g_e)(1-\sigma) & \phi(1+g_e)\sigma\\ \phi(1+g_u)\eta\lambda_w & \phi(1+g_u)(1-\eta\lambda_w)\end{bmatrix}.$$

Letting V be the matrix of eigenvectors, we can decompose Ψ^n as

$$\Psi^n = V\begin{bmatrix}\delta_\ell^n & 0\\ 0 & \delta_s^n\end{bmatrix}V^{-1}, \quad \text{with} \quad \Psi V = V\begin{bmatrix}\delta_\ell & 0\\ 0 & \delta_s\end{bmatrix},$$

where the eigenvalues of Ψ are given by

$$\delta_\ell = \frac{tr(\Psi)}{2} + \frac{1}{2}\sqrt{tr(\Psi)^2 - 4det(\Psi)} \text{ and } \delta_s = \frac{tr(\Psi)}{2} - \frac{1}{2}\sqrt{tr(\Psi)^2 - 4det(\Psi)},$$

with

$$tr(\Psi) = \phi(1+g_e)(1-\sigma) + \phi(1+g_u)(1-\eta\lambda_w)$$

and

$$det(\Psi) = \phi^2(1+g_e)(1+g_u)(1-\sigma-\eta\lambda_w).$$

Solving this out explicitly, we obtain

$$\delta_{\ell,s} = \frac{\phi(1+g_u+\lambda)}{2} \pm \frac{\phi}{2}\sqrt{(1+g_u-\lambda)^2+4\eta\lambda_w(1+g_u)(g_e-g_u)} \tag{64}$$

$$= \begin{cases} \phi(1+g_u) + \frac{\phi}{2}\left[\sqrt{(1+g_u-\lambda)^2+4\eta\lambda_w(1+g_u)(g_e-g_u)} - \sqrt{(1+g_u-\lambda)^2}\right] \\ \phi\lambda - \frac{\phi}{2}\left[\sqrt{(1+g_u-\lambda)^2+4\eta\lambda_w(1+g_u)(g_e-g_u)} - \sqrt{(1+g_u-\lambda)^2}\right] \end{cases}, \tag{65}$$

where in (65) we used

$$\frac{\phi(1+g_u+\lambda)}{2} = \phi(1+g_u) - \frac{\phi}{2}(1+g_u-\lambda) \text{ and } \frac{\phi(1+g_u+\lambda)}{2} = \phi\lambda + \frac{\phi}{2}(1+g_u-\lambda).$$

Now, note that

$$\begin{bmatrix}1 & -1\end{bmatrix} V \begin{bmatrix}\delta_\ell^n & 0 \\ 0 & \delta_s^n\end{bmatrix} V^{-1} \begin{bmatrix}1 \\ b\end{bmatrix} = \delta_\ell^n c_\ell + \delta_s^n c_s, \tag{66}$$

that is, the left-side of (66) has the form of a sum of the roots δ_ℓ^n and δ_s^n multiplied by some unknown constants c_ℓ and c_s. To derive the constants c_ℓ and c_s, we evaluate (66) for the first two periods $n = 0$ and $n = 1$ to obtain two equations in two unknowns, namely, for $n = 0$,

$$\begin{bmatrix}1 & -1\end{bmatrix} \begin{bmatrix}1 \\ b\end{bmatrix} = 1 - b = c_\ell + c_s \tag{67}$$

and for $n = 1$,

$$\begin{aligned} \begin{bmatrix}1 & -1\end{bmatrix} \Psi \begin{bmatrix}1 \\ b\end{bmatrix} &= \phi[(1+g_e)(1-\sigma) - \eta\lambda_w(1+g_u)](1-b) + \phi(g_e-g_u)b \\ &= \phi\lambda(1-b) + \phi(g_e-g_u)b = c_\ell\delta_\ell + c_s\delta_s. \end{aligned} \tag{68}$$

Solving these two equations for c_ℓ and c_s gives (62). (Observe that by using (67) and (68), we have avoided having to explicitly solve for the eigenvectors in V associated with the roots δ_ℓ and δ_s.) $\square$

To solve for the constant χ, note that $m(u_t, v_t) = Bu_t^\eta v_t^{1-\eta}$ implies that $\lambda_{ft}^{1-\eta} = B\lambda_{wt}^{-\eta}$ since

$$\lambda_{ft}^{1-\eta} = \left(\frac{Bu_t^\eta v_t^{1-\eta}}{v_t}\right)^{1-\eta} = B^{1-\eta}\left[\left(\frac{u_t}{v_t}\right)^\eta\right]^{1-\eta}$$

and

$$B\lambda_{wt}^{-\eta} = B\left(\frac{Bu_t^\eta v_t^{1-\eta}}{u_t}\right)^{-\eta} = B^{1-\eta}\left[\left(\frac{u_t}{v_t}\right)^{\eta-1}\right]^{-\eta} = B^{1-\eta}\left[\left(\frac{u_t}{v_t}\right)^{\eta}\right]^{1-\eta}.$$

Next, note that $\lambda_{ft}^{1-\eta} = B\lambda_{wt}^{-\eta}$ implies that $\lambda_{ft} = B^{\frac{1}{1-\eta}}\lambda_{wt}^{-\frac{\eta}{1-\eta}}$, which we substitute into (28) to obtain

$$\kappa A_t = (1-\eta)\left(B^{\frac{1}{1-\eta}}\lambda_{wt}^{-\frac{\eta}{1-\eta}}\right)(\mu_{et} - \mu_{ut}),$$

so

$$\lambda_{wt}^{\frac{\eta}{1-\eta}} = \left(\frac{1-\eta}{\kappa}\right) B^{\frac{1}{1-\eta}} \left(\frac{\mu_{et} - \mu_{ut}}{A_t}\right).$$

Raising both sides to the $(1-\eta)/\eta$ power, we obtain

$$\lambda_{wt} = \left[\left(\frac{1-\eta}{\kappa}\right) B^{\frac{1}{1-\eta}}\right]^{\frac{1-\eta}{\eta}} \left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right)^{\frac{1-\eta}{\eta}}$$

and taking logs

$$\log(\lambda_{wt}) = \frac{1-\eta}{\eta}\log\left[\left(\frac{1-\eta}{\kappa}\right) B^{\frac{1}{1-\eta}}\right] + \frac{1-\eta}{\eta}\log\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right),$$

so

$$\chi = \frac{1-\eta}{\eta}\log\left[\left(\frac{1-\eta}{\kappa}\right) B^{\frac{1}{1-\eta}}\right].$$

This concludes the proof. □

A.7 Price of Productivity Claims

We apply the risk-adjusted affine approximation around the risky steady state described by Lopez et al. (2017) to the pricing equation for claims to productivity in n periods, which can be written recursively as

$$\frac{P_{nt}}{A_t} = \mathbb{E}\left(Q_{t,t+1}\frac{A_{t+1}}{A_t}\frac{P_{n-1,t+1}}{A_{t+1}}\right) \tag{69}$$

with $P_{0t} = A_t$ or, in logs,

$$\log\left(\frac{P_{nt}}{A_t}\right) = \log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1} + \Delta a_{t+1} + \log(P_{n-1,t+1}/A_{t+1})\right)\right]\right\}, \tag{70}$$

where $q_{t,t+1} = \log(Q_{t,t+1})$ and $\Delta a_{t+1} = \log(A_{t+1}) - \log(A_t)$. Now, defining $\hat{s}_t = s_t - s$, use the approximation

$$\log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n\hat{s}_t \tag{71}$$

to rewrite both sides of (70) to obtain

$$a_n + b_n\hat{s}_t = \log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1} + \Delta a_{t+1} + a_{n-1} + b_{n-1}\hat{s}_{t+1}\right)\right]\right\}. \tag{72}$$

We next use (72) to derive a recursion for a_n and b_n to establish Lemma 2 in the paper. (Note that here this approximation does not depend on the point θ about which it is taken, so the use of the risky steady state versus a deterministic steady state does not matter. In contrast, the sufficient statistic result in Proposition 4 will depend on the point θ.)

Lemma 2 (Price of Productivity Claims). *The price of a claim to productivity in n periods approximately satisfies*

$$\log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n(s_t - s), \tag{73}$$

where $a_0 = b_0 = 0$,

$$a_n = \log(\beta) + (1-\alpha)g_a + a_{n-1} + \left[1 - b_{n-1} - (\alpha - b_{n-1})/S\right]^2\sigma_a^2/2, \tag{74}$$

and b_n satisfies

$$b_n = \alpha(1-\rho_s) + \rho_s b_{n-1} + \left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\sigma_a^2. \tag{75}$$

Under the assumption that $\alpha > 1$ and $1 - \rho_s + \left(1 - \frac{\alpha}{S}\right)\frac{\sigma_a^2}{S} > 0$, *the coefficients b_n grow monotonically with n and converge to α.*

Proof. The idea of the proof is to write out the terms on the right side of (72), evaluate them, and then match up the undetermined coefficients of the constants and the terms in $\hat{s}_t$ on both sides of (72). Doing so will give the recursive formulas for a_n and b_n in (74) and (75).

Now, the pricing kernel for our baseline preferences in log form is

$$q_{t,t+1} = \log(\beta) - \alpha\Delta c_{t+1} - \alpha\Delta s_{t+1}, \tag{76}$$

with $\Delta a_{t+1} = g_a + \sigma_a\varepsilon_{at+1}$, and the law of motion for $\hat{s}_t$ is

$$\hat{s}_{t+1} = \rho_s\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1} \tag{77}$$

so

$$\Delta\hat{s}_{t+1} = (\rho_s - 1)\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}. \tag{78}$$

The approximation that $\Delta c_{t+1} = \Delta a_{t+1}$, (78), and $\Delta s_{t+1} = \Delta\hat{s}_{t+1}$ imply that we can write the argument inside the expectation in (72) as

$$\begin{aligned}
&q_{t,t+1} + \Delta a_{t+1} + a_{n-1} + b_{n-1}\hat{s}_{t+1} \\
&= [\log(\beta) - \alpha\Delta a_{t+1} - \alpha\Delta s_{t+1}] + \Delta a_{t+1} + a_{n-1} + b_{n-1}\hat{s}_{t+1} \\
&= \{\log(\beta) - \alpha\Delta a_{t+1} - \alpha[(\rho_s - 1)\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}]\} + \Delta a_{t+1} + a_{n-1} + b_{n-1}[\rho_s\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}] \\
&= \log(\beta) + (1-\alpha)(g_a + \sigma_a\varepsilon_{at+1}) + [b_{n-1}\rho_s - \alpha(\rho_s - 1)]\,\hat{s}_t + a_{n-1} + (b_{n-1} - \alpha)\lambda_a(s_t)\sigma_a\varepsilon_{at+1} \\
&= \log(\beta) + (1-\alpha)g_a + [b_{n-1}\rho_s - \alpha(\rho_s - 1)]\hat{s}_t + a_{n-1} + \{1 - \alpha[1 + \lambda_a(s_t)] + b_{n-1}\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1}.
\end{aligned} \tag{79}$$

Next, we evaluate the right side of (72) using the equality in (79). Note first that, except for the last term, all of the variables in (79) are known at t so that

$$\begin{aligned}
&\log[\mathbb{E}_t(\exp\{\log(\beta) + (1-\alpha)g_a + [b_{n-1}\rho_s - \alpha(\rho_s - 1)]\,\hat{s}_t + a_{n-1}\})] \\
&\quad = \log(\beta) + (1-\alpha)g_a + [b_{n-1}\rho_s - \alpha(\rho_s - 1)]\,\hat{s}_t + a_{n-1}.
\end{aligned} \tag{80}$$

For the last term in (79), we use that the conditional expectation of a log-normal random variable with mean 0 and variance σ^2 is $\exp \sigma^2/2$ so that

$$
\begin{aligned}
&\log[\mathbb{E}_t(\exp\{1-\alpha[1+\lambda_a(s_t)]+b_{n-1}\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1})] \\
&\quad = \frac{\sigma_a^2}{2}\{1-\alpha[1+\lambda_a(s_t)]+b_{n-1}\lambda_a(s_t)\}^2 \\
&\quad \cong \frac{\sigma_a^2}{2}\{1-\alpha[1+\lambda_a(s)]+b_{n-1}\lambda_a(s)\}^2+(\sigma_a^2\,(b_{n-1}-\alpha)\,\lambda_a'(s)\{1-\alpha[1+\lambda_a(s)]+b_{n-1}\lambda_a(s)\})\hat{s}_t \\
&\quad = \left[1-\frac{\alpha}{S}+b_{n-1}\left(\frac{1}{S}-1\right)\right]^2\frac{\sigma_a^2}{2}+\sigma_a^2\frac{(\alpha-b_{n-1})}{S}\left[1-\frac{\alpha}{S}+b_{n-1}\left(\frac{1}{S}-1\right)\right]\hat{s}_t \\
&\quad = \left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)^2\frac{\sigma_a^2}{2}+\sigma_a^2\left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)\left(\frac{\alpha-b_{n-1}}{S}\right)\hat{s}_t, \qquad (81)
\end{aligned}
$$

where in the third line we have performed a first-order approximation around $s_t = s$ and in the fourth line we have used that in a steady state,

$$\lambda_a(s_t) = \frac{1}{S}\left[1-2\left(s_t-s\right)\right]^{1/2}-1 \qquad (82)$$

satisfies

$$1+\lambda_a(s) = 1/S \text{ and } \lambda_a'(s) = -1/S. \qquad (83)$$

Adding (80) and (81) and grouping together the constants and the terms in $\hat{s}_t$, we have that

$$
\begin{aligned}
&\log\{\mathbb{E}_t[\exp\left(q_{t,t+1}+\Delta a_{t+1}+a_{n-1}+b_{n-1}\hat{s}_{t+1}\right)]\} \\
&\quad = \log(\beta)+(1-\alpha)g_a+a_{n-1}+\left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)^2\frac{\sigma_a^2}{2} \\
&\qquad + \left[\alpha(1-\rho_s)+\rho_s b_{n-1}+\left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)\left(\frac{\alpha-b_{n-1}}{S}\right)\sigma_a^2\right]\hat{s}_t. \qquad (84)
\end{aligned}
$$

Now we are ready to use the recursion in (72), namely,

$$a_n+b_n\hat{s}_t = \log\{\mathbb{E}_t[\exp\left(q_{t,t+1}+\Delta a_{t+1}+a_{n-1}+b_{n-1}\hat{s}_{t+1}\right)]\}.$$

Matching up the constants and the coefficients of $\hat{s}_t$ on both sides of this equation using (84) gives

$$a_n = \log(\beta)+(1-\alpha)g_a+a_{n-1}+\left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)^2\frac{\sigma_a^2}{2}$$

and

$$b_n = \alpha(1-\rho_s)+\rho_s b_{n-1}+\left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)\left(\frac{\alpha-b_{n-1}}{S}\right)\sigma_a^2,$$

which are the formulas in (74) and (75) above.

Finally, to establish the claim that the assumptions that $\alpha > 1$ and $1-\rho_s+\left(1-\frac{\alpha}{S}\right)\frac{\sigma_a^2}{S} > 0$ imply that the coefficients b_n grow monotonically from 0 to α, we rearrange the formula for b_n as

$$b_n - b_{n-1} = (\alpha - b_{n-1})\phi(b_{n-1}), \qquad (85)$$

where

$$\phi(b_{n-1}) \equiv 1-\rho_s+\left(1-\frac{\alpha}{S}\right)\frac{\sigma_a^2}{S}+b_{n-1}\left(\frac{1-S}{S}\right)\frac{\sigma_a^2}{S} \tag{86}$$

so $\phi(b_{n-1})$ is a strictly increasing function on $(0,\alpha)$. We first note that if b_n converges to b^*, then from (85), b^* solves the quadratic equation $(\alpha-b^*)\phi(b^*)=0$, for which the relevant root is the positive one with $b^*=\alpha$. We now show that if $\alpha>1$ and $1-\rho_s+\left(1-\frac{\alpha}{S}\right)\frac{\sigma_a^2}{S}>0$, then b_n converges monotonically to α from 0. Note first that these two conditions imply that $\phi(b_{n-1})>0$ for all $b_{n-1}\geq 0$ because $S\leq 1$ and

$$\phi(\alpha)=1-\rho_s+(1-\alpha)\frac{\sigma_a^2}{S}\in(0,1). \tag{87}$$

Clearly, $\phi(\alpha)\geq 0$ because $S\leq 1$ implies $\phi(\alpha)=1-\rho_s+(1-\alpha)\frac{\sigma_a^2}{S}\geq 1-\rho_s+\left(1-\frac{\alpha}{S}\right)\frac{\sigma_a^2}{S}>0$, and $\phi(\alpha)\leq 1$ because $\alpha>1$ implies that the second term in (87) is negative.

Next, we claim that if $b_{n-1}\in[0,\alpha]$, then $b_n\geq b_{n-1}$ and $b_n\in[0,\alpha]$. To prove this claim, note that since $\phi(\cdot)$ is an increasing function and $b_{n-1}\in[0,\alpha]$, then $\phi(b_{n-1})\leq\phi(\alpha)$. It then follows that

$$b_n=b_{n-1}+(\alpha-b_{n-1})\phi(b_{n-1})\leq b_{n-1}+(\alpha-b_{n-1})\phi(\alpha)=[1-\phi(\alpha)]b_{n-1}+\phi(\alpha)\alpha\leq\alpha, \tag{88}$$

where the last inequality follows because $[1-\phi(\alpha)]b_{n-1}+\phi(\alpha)\alpha$ is a convex combination of b_{n-1} and α. Moreover, $b_n\geq b_{n-1}$ since $(\alpha-b_{n-1})\phi(b_{n-1})\geq 0$. Hence, we have established that $b_n\geq b_{n-1}$ and $b_n\in[0,\alpha]$.

From this claim, it follows that since $b_0=0\in[0,\alpha]$, then $b_n\geq b_{n-1}$, so the series increases monotonically, and $b_n\in[0,\alpha]$ for all n. Since b_n is bounded above by α, the series converges. Since b_n is always nonnegative, the series converges to the relevant stationary point of (85), namely, $b^*=\alpha$. □

A.8 Sufficient Statistic for Job-Finding Rate Volatility

In what follows, we define the risky steady state as in Coeurdacier et al. (2011) and Lopez et al. (2017) as the limit point of the deterministic system in which all shocks are zero but in which agents expect shocks to be realized according to their true distribution and agents' approximate decision rules are computed using a first-order approximation around this point. Mechanically, the risky steady state for market tightness θ_t is simply the mean value of $\log(\theta_t)$ under our log-linear approximation $\log(\theta_t)=\log(\theta)+\psi_\theta\hat{s}_t$ and thus is $\log(\theta)$.

Proposition 4 (Sufficient Statistic for Job-Finding Rate Volatility). *Under the approximation in Lemma 2, the response of the job-finding rate to a change in s_t evaluated at a risky steady state is given by*

$$\frac{d\log(\lambda_{wt})}{ds_t}=\left(\frac{1-\eta}{\eta}\right)\sum_{n=0}^{\infty}\omega_n b_n \quad\text{with}\quad \omega_n=\frac{e^{a_n}(c_\ell\delta_\ell^n+c_s\delta_s^n)}{\sum_{n=0}^{\infty}e^{a_n}(c_\ell\delta_\ell^n+c_s\delta_s^n)}, \tag{89}$$

*where a_n and b_n are given in Lemma 2 and the standard deviation of the job-finding rate $\sigma(\lambda_{wt})$*satisfies

$$\sigma(\lambda_{wt})=\frac{d\log(\lambda_{wt})}{ds_t}\sigma(s_t). \tag{90}$$

Proof. Substitute $P_{nt}/A_t = e^{a_n+b_n\hat{s}_t}$ from Lemma 2 into (60) in Proposition 3 to obtain

$$\log(\lambda_{wt}) = \chi + \frac{1-\eta}{\eta}\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n+b_n(s_t-s)}\right] \tag{91}$$

$$\cong \chi + \frac{1-\eta}{\eta}\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n}\right] + \frac{1-\eta}{\eta}\left[\sum_{n=0}^{\infty}\frac{(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n}}{\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n}}b_n\right](s_t - s)$$

$$= const + \frac{1-\eta}{\eta}\left(\sum_{n=0}^{\infty}\omega_n b_n\right)(s_t - s). \tag{92}$$

Denoting the right-side of (11) by $f(s_t)$, in (92) we took a first-order Taylor expansion of this right side around s using $f(s_t) \cong f(s) + f'(s)(s_t - s)$. Differentiating this last expression, (89) and (90) are immediate. $\square$

Here we expand on the risky steady state of the log-linear model. The proposition just established holds for any point θ at which we take our first-order expansion. But it is logically most consistent to take it around the risky steady state of the log-linear model, as we will explain below. We start by showing how that state is calculated.

A. Computing the risky steady state. Since we will take a first-order expansion around θ_t, it is convenient to begin by rewriting the free-entry condition

$$\kappa A_t = (1-\eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}) \tag{93}$$

in terms of θ_t rather than λ_{ft}. Using $\lambda_f = B\theta^{-\eta}$, we obtain

$$\theta^\eta = \frac{B(1-\eta_t)}{\kappa}\left(\frac{\mu_{et} - \mu_{ut}}{A_t}\right),$$

so that we can write (93) as

$$\log(\theta_t) = \frac{1}{\eta}\log\left[\frac{B(1-\eta_t)}{\kappa}\right] + \frac{1}{\eta}\log\left(\frac{\mu_{et} - \mu_{ut}}{A_t}\right). \tag{94}$$

Hence, using (94), we can write (60) in terms of θ_t as

$$\log(\theta_t) = \frac{1}{\eta}\log\left[\frac{B(1-\eta)}{\kappa}\right] + \frac{1}{\eta}\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)\frac{P_{nt}}{A_t}\right]. \tag{95}$$

We now take a first-order approximation to the log of the variables θ_t and P_{nt}/A_t around the risky steady state θ. This first-order approximation for these variables has the same log-linear form in the demeaned state $\hat{s}_t$. Hence,

$$\log(\theta_t) = \log(\theta) + \psi_\theta\hat{s}_t \quad \text{and} \quad \log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n\hat{s}_t. \tag{96}$$

To find this solution, substitute in (60) the expressions in (96) for all of these variables to obtain

$$\log(\theta) + \psi_\theta \hat{s}_t = \frac{1}{\eta} \log\left[\frac{B(1-\eta)}{\kappa}\right] + \frac{1}{\eta} \log\left[\sum_{n=0}^{\infty} (c_\ell \delta_\ell^n + c_s \delta_s^n) e^{a_n + b_n \hat{s}_t}\right]. \tag{97}$$

Now, to find the risky steady state, namely, $\log(\theta)$, set $\hat{s}_t = 0$ in (97) to obtain

$$\log(\theta) = \frac{1}{\eta} \log\left[\frac{B(1-\eta)}{\kappa}\right] + \frac{1}{\eta} \log\left\{\sum_{n=0}^{\infty} \left[c_\ell(\theta)\delta_\ell(\theta)^n + c_s(\theta)\delta_s(\theta)^n\right] e^{a_n}\right\}, \tag{98}$$

where we have made explicit that the roots $\delta_\ell(\theta)$ and $\delta_s(\theta)$ and the constants $c_\ell(\theta)$ and $c_s(\theta)$ depend on θ so that the risky steady state is the fixed point of (98). To solve for the slope term ψ_θ, we take a first-order approximation of the general form $f(s_t) = f(s) + f'(s)(s_t - s)$ around θ at $\hat{s}_t = 0$ to obtain

$$\log(\theta) + \psi_\theta \hat{s}_t = \frac{1}{\eta} \log\left[\frac{B(1-\eta)}{\kappa}\right] + \frac{1}{\eta} \log\left[\sum_{n=0}^{\infty} (c_\ell \delta_\ell^n + c_s \delta_s^n) e^{a_n}\right] + \frac{1}{\eta} \sum_{n=0}^{\infty} \frac{(c_\ell \delta_\ell^n + c_s \delta_s^n) e^{a_n} b_n}{\sum_{n=0}^{\infty} (c_\ell \delta_\ell^n + c_s \delta_s^n) e^{a_n}} \hat{s}_t.$$

Then, matching the coefficients of $\hat{s}_t$ on both sides gives

$$\psi_\theta = \frac{1}{\eta} \sum_{n=0}^{\infty} \frac{\left[c_\ell(\theta)\delta_\ell(\theta)^n + c_s(\theta)\delta_s(\theta)^n\right] e^{a_n} b_n}{\sum_{n=0}^{\infty} \left[c_\ell(\theta)\delta_\ell(\theta)^n + c_s(\theta)\delta_s(\theta)^n\right] e^{a_n}}. \tag{99}$$

B. Why the risky steady state is the only point consistent with this approximation. Imagine that we picked an arbitrary θ, say θ_1, and consider log-linear rules of the form (96) but with the log-linear rule for θ_t given by $\log(\theta_t) = \log(\theta_1) + \psi_{\theta_1} \hat{s}_t$, where ψ_{θ_1} is the expression in (99) evaluated at $c_\ell(\theta_1)$, $c_s(\theta_1)$, $\delta_\ell(\theta_1)$, and $\delta_s(\theta_1)$. If for some arbitrary θ_1, we substitute these values for $c_\ell(\theta_1)$, $c_s(\theta_1)$, $\delta_\ell(\theta_1)$, and $\delta_s(\theta_1)$ into the approximation to obtain the right-side of (97), and take the means of both sides, we obtain

$$\log(\theta_2) = \frac{1}{\eta} \log\left[\frac{B(1-\eta)}{\kappa}\right] + \frac{1}{\eta} \log\left\{\sum_{n=0}^{\infty} \left[c_\ell(\theta_1)\delta_\ell(\theta_1)^n + c_s(\theta_1)\delta_s(\theta_1)^n\right] e^{a_n}\right\} \tag{100}$$

for some $\theta_2 \neq \theta_1$. That is, if we take the approximation to the right side of (100) at any point $\tilde{\theta}$ other than the risky steady state, the process for $\log(\theta_t)$ will not have a mean that is consistent with the point θ about which we took the approximation. In this sense, the risky steady state has a special property: it is the only point θ that is consistent with this approximation in the sense just described. Hence, in this type of approximation, we need to solve for the point θ, about which we are approximating, endogenously as part of the approximation, by solving the fixed point problem in (98). This is the sense in which our approximation differs from many of the standard ones.

A.9 Extensions of Propositions 3 and 4

In the approximation given in Proposition 3 in the paper, we assumed that future λ_{wt+s} were constant when we derived the dynamical system governing the multipliers (μ_{et}, μ_{ut}). Here we do not make such an approximation and state an extension of Proposition 3, which features more terms corresponding to those from the time-varying λ_{wt+s}, which are prices to two new strips involving θ_{t+n} as well as A_{t+n}. Define then $P^{\theta}_{nt} = \mathbb{E}_t(Q_{t,t+n}A_{t+n}\theta_{t+n})$ to be the prices of claims to assets that pay $A_{t+n}\theta_{t+n}$ and $P^{\theta\eta}_{nt} = \mathbb{E}_t(Q_{t,t+n}A_{t+n}\theta^{1-\eta}\theta^{\eta}_{t+n})$ to be the prices of claims to assets that pay $A_{t+n}\theta^{1-\eta}\theta^{\eta}_{t+n}$ at $t+n$, where θ is the market tightness in the risky steady state and we have used the Cobb-Douglas form of the matching function.

Proposition 3' (Extension of Proposition 3). *The job-finding rate satisfies*

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right)\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)\frac{P_{nt}}{A_t} + (b_\ell\delta_\ell^n + b_s\delta_s^n)\left(\frac{P^{\theta}_{n+1t}}{A_t} - \frac{P^{\theta\eta}_{n+1t}}{A_t}\right)\right], \tag{101}$$

where for $\tau \equiv \phi(1+g_e)\sigma - \phi(1+g_u)[1-\eta\lambda_w(\theta)]$,

$$b_s = -\phi(1+g_u)\frac{\kappa\eta}{1-\eta}\frac{\tau+\delta_\ell}{\delta_\ell-\delta_s}, b_\ell = \phi(1+g_u)\frac{\kappa\eta}{1-\eta}\frac{\tau+\delta_s}{\delta_\ell-\delta_s} \tag{102}$$

and the expressions for c_ℓ, c_s, δ_ℓ, *and* δ_s *are as before.*

Notice that, in contrast to Proposition 3, this formula is *exact* since it involves no approximations.

Proof. Defining the scaled multipliers $\tilde{\mu}_{et} \equiv \mu_{et}/A_t$ and $\tilde{\mu}_{ut} \equiv \mu_{ut}/A_t$, we can write the first-order conditions for the planning problem as

$$\tilde{\mu}_{et} = 1 + \phi(1+g_e)\mathbb{E}_t\left\{Q_{t,t+1}\frac{A_{t+1}}{A_t}\left[(1-\sigma)\tilde{\mu}_{et+1} + \sigma\tilde{\mu}_{ut+1}\right]\right\}, \tag{103}$$

$$\tilde{\mu}_{ut} = b + \phi(1+g_u)\mathbb{E}_t\left(Q_{t,t+1}\frac{A_{t+1}}{A_t}\left\{\eta\lambda_w(\theta_{t+1})\tilde{\mu}_{et+1} + \left[1-\eta\lambda_w(\theta_{t+1})\right]\tilde{\mu}_{ut+1}\right\}\right), \tag{104}$$

and

$$\kappa = (1-\eta)\lambda_{ft}(\theta_t)(\tilde{\mu}_{et} - \tilde{\mu}_{ut}). \tag{105}$$

Thus, we can express (103) and (104) as a nonlinear matrix difference equation,

$$\begin{bmatrix}\tilde{\mu}_{et}\\ \tilde{\mu}_{ut}\end{bmatrix} = \begin{bmatrix}1\\ b\end{bmatrix} + \mathbb{E}_t\left\{\Psi(\theta_{t+1})Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix}\tilde{\mu}_{et+1}\\ \tilde{\mu}_{ut+1}\end{bmatrix}\right\}, \tag{106}$$

where the transition matrix $\Psi(\theta_{t+1})$ is defined as

$$\Psi(\theta_{t+1}) = \begin{bmatrix}\phi(1+g_e)(1-\sigma) & \phi(1+g_e)\sigma\\ \phi(1+g_u)\eta\lambda_w(\theta_{t+1}) & \phi(1+g_u)\left[1-\eta\lambda_w(\theta_{t+1})\right]\end{bmatrix}.$$

To understand what terms we dropped in Proposition 3 in the paper, define

$$\Psi(\theta) \equiv \begin{bmatrix}\phi(1+g_e)(1-\sigma) & \phi(1+g_e)\sigma\\ \phi(1+g_u)\eta\lambda_w(\theta) & \phi(1+g_u)\left[1-\eta\lambda_w(\theta)\right]\end{bmatrix}$$

for any (feasible) constant θ and rewrite (106) as

$$\begin{bmatrix}\tilde{\mu}_{et}\\ \tilde{\mu}_{ut}\end{bmatrix} = \begin{bmatrix}1\\ b\end{bmatrix} + \Psi(\theta)\mathbb{E}_t\left\{Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix}\tilde{\mu}_{et+1}\\ \tilde{\mu}_{ut+1}\end{bmatrix}\right\} + \mathbb{E}_t\left\{[\Psi(\theta_{t+1}) - \Psi(\theta)]Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix}\tilde{\mu}_{et+1}\\ \tilde{\mu}_{ut+1}\end{bmatrix}\right\}, \tag{107}$$

where we used the trivial identity

$$\Psi(\theta_{t+1}) = \Psi(\theta) + [\Psi(\theta_{t+1}) - \Psi(\theta)]. \tag{108}$$

We show that when we solve (107), we end up with the formula in (101). Because of the identity in (108), we obtain an identical value for λ_{wt} on the left side of (101) regardless of the value θ we select. Mechanically, as we move from, say, θ_1 to θ_2, the roots $\delta_s(\theta_i)$ and $\delta_\ell(\theta_i)$ adjust to offset the changes in the strip prices $P^{\theta}_{nt} = \mathbb{E}_t(Q_{t,t+n}A_{t+n}\theta_{t+n})$ and $P^{\theta^\eta}_{nt} = \theta^{1-\eta}\mathbb{E}_t(Q_{t,t+n}A_{t+n}\theta^{\eta}_{t+n})$.

In the paper, we omitted the third term in (107) of our approximation because setting future $\lambda_{wt+s}(\theta_{t+s})$ to the constant value $\lambda_w(\theta)$ is equivalent to setting $\Psi(\theta_{t+s}) = \Psi(\theta)$. Now, to solve (107), let us simplify the third term on the right side of (107). Noting that

$$\Psi(\theta_{t+1}) - \Psi(\theta) = \phi(1+g_u)\eta\left[\lambda_w(\theta_{t+1}) - \lambda_w(\theta)\right]\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$$

and that $Q_{t,t+1}$ and A_{t+1}/A_t are scalars, we can write the third term on the right side of (107) as

$$\begin{aligned}
&\mathbb{E}_t\left\{[\Psi(\theta_{t+1}) - \Psi(\theta)]Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix}\tilde{\mu}_{et+1}\\ \tilde{\mu}_{ut+1}\end{bmatrix}\right\}\\
&= \phi(1+g_u)\eta\mathbb{E}_t\left\{\left[\lambda_w(\theta_{t+1}) - \lambda_w(\theta)\right]Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix}\tilde{\mu}_{et+1}\\ \tilde{\mu}_{ut+1}\end{bmatrix}\right\}\\
&= \phi(1+g_u)\eta\mathbb{E}_t\left\{Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix} 0 \\ \left[\lambda_w(\theta_{t+1}) - \lambda_w(\theta)\right]\left[\tilde{\mu}_{et+1} - \tilde{\mu}_{ut+1}\right]\end{bmatrix}\right\} && (109)\\
&= \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\mathbb{E}_t\left\{Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix} 0 \\ \theta_{t+1} - \theta^{\eta}_{t+1}\theta^{1-\eta}\end{bmatrix}\right\}, && (110)
\end{aligned}$$

where in (110) we used that our matching function implies $\lambda_w = B\theta^{1-\eta}$ and $\lambda_f = B\theta^{-\eta}$, so that

$$\begin{aligned}
\lambda_w(\theta_{t+1}) - \lambda_w(\theta) &= B\theta^{1-\eta}_{t+1} - B\theta^{1-\eta} = B\theta^{-\eta}_{t+1}(\theta_{t+1} - \theta^{\eta}_{t+1}\theta^{1-\eta}) = \lambda_f(\theta_{t+1})(\theta_{t+1} - \theta^{\eta}_{t+1}\theta^{1-\eta})\\
&= \frac{\kappa}{(1-\eta)(\tilde{\mu}_{et+1} - \tilde{\mu}_{ut+1})}(\theta_{t+1} - \theta^{\eta}_{t+1}\theta^{1-\eta}), && (111)
\end{aligned}$$

which implies that

$$\left[\lambda_w(\theta_{t+1}) - \lambda_w(\theta)\right](\tilde{\mu}_{et+1} - \tilde{\mu}_{ut+1}) = \frac{\kappa}{1-\eta}(\theta_{t+1} - \theta^{\eta}_{t+1}\theta^{1-\eta}), \tag{112}$$

and in (110) we used (112). Substituting for the third term in (107) using (110) gives

$$\begin{aligned}
\begin{bmatrix}\tilde{\mu}_{et}\\ \tilde{\mu}_{ut}\end{bmatrix} &= \begin{bmatrix}1\\ b\end{bmatrix} + \Psi(\theta)\mathbb{E}_t\left\{Q_{t,t+1}\frac{A_{t+1}}{A_t}\begin{bmatrix}\tilde{\mu}_{et+1}\\ \tilde{\mu}_{ut+1}\end{bmatrix}\right\} + \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\mathbb{E}_t\left[Q_{t,t+1}\frac{A_{t+1}}{A_t}(\theta_{t+1}-\theta_{t+1}^{\eta}\theta^{1-\eta})\right]\begin{bmatrix}0\\ 1\end{bmatrix}\\
&= \sum_{n=0}^{\infty}\Psi(\theta)^n\begin{bmatrix}1\\ b\end{bmatrix}\mathbb{E}_t\left(Q_{t,t+n}\frac{A_{t+n}}{A_t}\right)\\
&\quad + \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\sum_{n=0}^{\infty}\Psi(\theta)^n\begin{bmatrix}0\\ 1\end{bmatrix}\mathbb{E}_t\left[Q_{t,t+n+1}\frac{A_{t+n+1}}{A_t}(\theta_{t+n+1}-\theta^{1-\eta}\theta_{t+n+1}^{\eta})\right]\\
&= \sum_{n=0}^{\infty}\Psi(\theta)^n\begin{bmatrix}1\\ b\end{bmatrix}\tilde{P}_{nt} + \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\sum_{n=0}^{\infty}\Psi(\theta)^n\begin{bmatrix}0\\ 1\end{bmatrix}\left(\tilde{P}_{nt+1}^{\theta}-\tilde{P}_{nt+1}^{\theta\eta}\right), \qquad (113)
\end{aligned}$$

where in the second line we solved the difference equation forward as we did earlier and in the third line we substituted for the scaled strip prices, defined as

$$\tilde{P}_{nt} = \mathbb{E}_t\left(Q_{t,t+n}\frac{A_{t+n}}{A_t}\right),\ \tilde{P}_{nt}^{\theta} = \mathbb{E}_t\left(Q_{t,t+n}\frac{A_{t+n}}{A_t}\theta_{t+n}\right),\ \text{and}\ \tilde{P}_{nt}^{\theta\eta} = \theta^{1-\eta}\mathbb{E}_t\left(Q_{t,t+n}\frac{A_{t+n}}{A_t}\theta_{t+n}^{\eta}\right).$$

Thus, to evaluate $\tilde{\mu}_{et}-\tilde{\mu}_{ut}$, we premultiply both sides of (113) by $\begin{bmatrix}1 & -1\end{bmatrix}$ to obtain

$$\tilde{\mu}_{et}-\tilde{\mu}_{ut} = \sum_{n=0}^{\infty}\begin{bmatrix}1 & -1\end{bmatrix}\Psi(\theta)^n\begin{bmatrix}1\\ b\end{bmatrix}\tilde{P}_{nt}+\phi(1+g_u)\frac{\kappa\eta}{1-\eta}\sum_{n=0}^{\infty}\begin{bmatrix}1 & -1\end{bmatrix}\Psi(\theta)^n\begin{bmatrix}0\\ 1\end{bmatrix}\left(\tilde{P}_{nt+1}^{\theta}-\tilde{P}_{nt+1}^{\theta\eta}\right). \qquad (114)$$

To evaluate (114), we need to evaluate the terms

$$\begin{bmatrix}1 & -1\end{bmatrix}\Psi(\theta)^n\begin{bmatrix}1\\ b\end{bmatrix} = c_\ell\delta_\ell^n + c_s\delta_s^n \text{ and } \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\begin{bmatrix}1 & -1\end{bmatrix}\Psi(\theta)^n\begin{bmatrix}0\\ 1\end{bmatrix} = b_\ell\delta_\ell^n + b_s\delta_s^n, \qquad (115)$$

that is, the left side of the left equation in (115) has the form of a sum of the roots δ_ℓ^n and δ_s^n multiplied by some unknown constants c_ℓ and c_s, and the left side of the right equation has a similar form but with the unknown constants b_ℓ and b_s.

We proceed as before. We decompose $\Psi(\theta)$ into eigenvectors V and eigenvalues as

$$\Psi(\theta)^n = V\begin{bmatrix}\delta_\ell^n & 0\\ 0 & \delta_s^n\end{bmatrix}V^{-1} \quad \text{with} \quad \Psi(\theta)V = V\begin{bmatrix}\delta_\ell & 0\\ 0 & \delta_s\end{bmatrix}.$$

Hence, the constants c_ℓ and c_s are the same as before. To solve for the constants b_ℓ and b_s, we evaluate

$$\phi(1+g_u)\frac{\kappa\eta}{1-\eta}\begin{bmatrix}1 & -1\end{bmatrix}\Psi(\theta)^n\begin{bmatrix}0\\ 1\end{bmatrix} = b_\ell\delta_\ell^n + b_s\delta_s^n \qquad (116)$$

at $n=0$ to obtain one equation in b_ℓ and b_s,

$$\frac{\phi(1+g_u)\kappa\eta}{1-\eta}\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = -\frac{\phi(1+g_u)\kappa\eta}{1-\eta} = b_\ell + b_s,$$

namely,

$$b_\ell + b_s = -\omega, \tag{117a}$$

where $\omega \equiv \phi(1+g_u)\kappa\eta/(1-\eta)$. Evaluating (116) at $n = 1$ gives the second equation in b_ℓ and b_s,

$$\begin{aligned}
b_\ell\delta_\ell + b_s\delta_s &= \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\begin{bmatrix}1 & -1\end{bmatrix}\Psi(\theta)\begin{bmatrix}0\\1\end{bmatrix}\\
&= \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}\phi(1+g_e)(1-\sigma) & \phi(1+g_e)\sigma\\ \eta\lambda_w(\theta)\phi(1+g_u) & \phi(1+g_u)(1-\eta\lambda_w(\theta))\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\\
&= \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}\phi(1+g_e)\sigma\\ \phi(1+g_u)(1-\eta\lambda_w(\theta))\end{bmatrix}\\
&= \frac{\phi(1+g_u)\kappa\eta}{1-\eta}\left[\phi(1+g_e)\sigma - \phi(1+g_u)(1-\eta\lambda_w(\theta))\right],
\end{aligned}$$

namely,

$$b_\ell\delta_\ell + b_s\delta_s = \omega\tau, \tag{118}$$

where $\tau \equiv \phi(1+g_e)\sigma - \phi(1+g_u)[1-\eta\lambda_w(\theta)]$. We can solve the two equations (117a) and (118) in the two unknowns b_ℓ and b_s to obtain

$$b_s = -\omega\frac{\tau+\delta_\ell}{\delta_\ell-\delta_s} \text{ and } b_\ell = \omega\frac{\tau+\delta_s}{\delta_\ell-\delta_s}, \tag{119}$$

which establishes (102). Hence, using (115) in (114), we obtain

$$\tilde{\mu}_{et} - \tilde{\mu}_{ut} = \left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)\frac{P_{nt}}{A_t} + (b_\ell\delta_\ell^n + b_s\delta_s^n)\left(\frac{P^\theta_{n+1t}}{A_t} - \frac{P^{\theta\eta}_{n+1t}}{A_t}\right)\right],$$

which, when substituted into the free-entry condition

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right)\log\left(\tilde{\mu}_{et} - \tilde{\mu}_{ut}\right), \tag{120}$$

gives (101). □

We turn now to an extension of Proposition 4 that takes into account the extra terms in (101).

Proposition 4' (Extension of Proposition 4). *Under affine approximations to the strips P_{nt}, P^θ_{nt} and $P^{\theta\eta}_{nt}$ at the risky steady state, the response of the job-finding rate with respect to a change in s_t evaluated at a risky steady-state is given by*

$$\frac{d\log(\lambda_{wt})}{ds_t} = \frac{1-\eta}{\eta}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\frac{(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n}b_n + (b_\ell\delta_\ell^n + b_s\delta_s^n)\,\theta\left[e^{d_{n+1}}(e_{n+1}+\psi_\theta) - e^{f_{n+1}}(g_{n+1}+\eta\psi_\theta)\right]}{\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n} + (b_\ell\delta_\ell^n + b_s\delta_s^n)\,\theta\left(e^{d_{n+1}} - e^{f_{n+1}}\right)},$$

where a_n and b_n are given in Lemma 2, b_ℓ, b_s are given in Proposition 3', d_n, e_n, f_n, g_n are derived *below, and the standard deviation of the job-finding rate $\sigma(\lambda_{wt})$ satisfies*

$$\sigma(\lambda_{wt}) = \frac{d\log(\lambda_{wt})}{ds_t}\sigma(s_t).$$

Proof. Since we will take a first-order expansion around θ_t, it is convenient to begin by rewriting the free-entry condition

$$\kappa A_t = (1-\eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}) \tag{121}$$

in terms of θ_t rather than λ_{ft}. Using $\lambda_f = B\theta^{-\eta}$, we obtain

$$\theta^\eta = \frac{B(1-\eta_t)}{\kappa}\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right)$$

so that we can write (121) as

$$\log(\theta) = \frac{1}{\eta}\log\left[\frac{B(1-\eta_t)}{\kappa}\right] + \frac{1}{\eta}\log\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right). \tag{122}$$

Hence, using (122), we can write (101) in terms of θ_t as

$$\begin{aligned}\log(\theta_t) &= \frac{1}{\eta}\log\left[\frac{B(1-\eta)}{\kappa}\right] \\ &+ \frac{1}{\eta}\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)\frac{P_{nt}}{A_t} + (b_\ell\delta_\ell^n + b_s\delta_s^n)\left(e^{\log(\theta_t)}\frac{P_{n+1t}^{\theta}}{A_t\theta_t} - \theta^{1-\eta}e^{\eta\log(\theta_t)}\frac{P_{n+1t}^{\theta\eta}}{A_t\theta^{1-\eta}\theta_t^\eta}\right)\right].\end{aligned} \tag{123}$$

We wish to take a first-order approximation to the log of the variables θ_t, P_{nt}/A_t, $P_{n+1t}^{\theta}/A_t\theta_t$, and $P_{n+1t}^{\theta\eta}/(A_t\theta^{1-\eta}\theta_t^\eta)$ around the risky steady state θ. This first-order approximation for each of these variables has the same log-linear form in the demeaned state $\hat{s}_t$

$$\begin{aligned}\log(\theta_t) = \log(\theta) + \psi_\theta\hat{s}_t,\ \log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n\hat{s}_t,\ \log\left(\frac{P_{nt}^{\theta}}{A_t\theta_t}\right) = d_n + e_n\hat{s}_t, \\ \text{and}\ \log\left(\frac{P_{nt}^{\theta\eta}}{A_t\theta^{1-\eta}\theta_t^\eta}\right) = f_n + g_n\hat{s}_t.\end{aligned} \tag{124}$$

To find this solution, substitute in (123) the expressions in (124) for all of these variables to obtain

$$\begin{aligned}\log(\theta) + \psi_\theta\hat{s}_t &= \frac{1}{\eta}\log\left[\frac{B(1-\eta)}{\kappa}\right] + \frac{1}{\eta}\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n+b_n\hat{s}_t}\right] \\ &+ \frac{1}{\eta}\log\left[\sum_{n=0}^{\infty}(b_\ell\delta_\ell^n + b_s\delta_s^n)\left(e^{\log(\theta)+\psi_\theta\hat{s}_t}e^{d_n+e_n\hat{s}_t} - \theta^{1-\eta}e^{\eta[\log(\theta)+\psi_\theta\hat{s}_t]}e^{f_n+g_n\hat{s}_t}\right)\right],\end{aligned}$$

then take a first-order approximation of the general form $f(s_t) = f(s) + f'(s)(s_t - s)$ around θ at $\hat{s}_t = 0$ to obtain

$$\begin{aligned}\log(\theta) + \psi_\theta\hat{s}_t &= \frac{1}{\eta}\log\left[\frac{B(1-\eta)}{\kappa}\right] + \frac{1}{\eta}\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n} + (b_\ell\delta_\ell^n + b_s\delta_s^n)\,\theta\left(e^{d_{n+1}} - e^{f_{n+1}}\right)\right] \\ &+ \frac{1}{\eta}\sum_{n=0}^{\infty}\frac{(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n}b_n + (b_\ell\delta_\ell^n + b_s\delta_s^n)\,\theta\left[e^{d_{n+1}}(e_{n+1}+\psi_\theta) - e^{f_{n+1}}(g_{n+1}+\eta\psi_\theta)\right]}{\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n} + (b_\ell\delta_\ell^n + b_s\delta_s^n)\,\theta\left(e^{d_{n+1}} - e^{f_{n+1}}\right)}\hat{s}_t.\end{aligned}$$

Then, matching the constant terms on both sides and the coefficients of $\hat{s}_t$ on both sides gives

$$\log\theta = \frac{1}{\eta}\log\left(\frac{B(1-\eta)}{\kappa}\right) + \frac{1}{\eta}\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n} + (b_\ell\delta_\ell^n + b_s\delta_s^n)\,\theta\left(e^{d_{n+1}} - e^{f_{n+1}}\right)\right] \quad (125)$$

and

$$\psi_\theta = \frac{1}{\eta}\sum_{n=0}^{\infty}\frac{(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n}b_n + (b_\ell\delta_\ell^n + b_s\delta_s^n)\,\theta\left[e^{d_{n+1}}(e_{n+1}+\psi_\theta) - e^{f_{n+1}}(g_{n+1}+\eta\psi_\theta)\right]}{\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)e^{a_n} + (b_\ell\delta_\ell^n + b_s\delta_s^n)\,\theta\left(e^{d_{n+1}} - e^{f_{n+1}}\right)}, \quad (126)$$

which is a nonlinear system of equations in θ and ψ_θ because c_ℓ, c_s, b_ℓ, b_s, δ_ℓ, and δ_s depend on θ and, as shown below, d_n, e_n, f_n, and g_n all depend on ψ_θ. (See Lopez et al. (2017) for details in the general case.)

The expressions for a_n and b_n are given in Lemma 2. To derive d_n and e_n, we use an analogous linear approximation to P_{nt}^θ,

$$\log\left(\frac{P_{nt}^\theta}{A_t\theta_t}\right) = d_n + e_n\hat{s}_t,$$

where, using $\log(\theta_t) = \log(\theta) + \psi_\theta\hat{s}_t$, we will show that d_n and e_n are given by

$$d_n = \log(\beta) + (1-\alpha)g_a + d_{n-1} + \frac{\sigma_a^2}{2}\left(1 - e_{n-1} - \psi_\theta + \frac{e_{n-1}+\psi_\theta-\alpha}{S}\right)^2 \quad (127)$$

$$e_n = \alpha(1-\rho_s) + e_{n-1}\rho_s + (\rho_s - 1)\psi_\theta - \left(1 - e_{n-1} - \psi_\theta + \frac{e_{n-1}+\psi_\theta-\alpha}{S}\right)\frac{e_{n-1}+\psi_\theta-\alpha}{S}\sigma_a^2 \quad (128)$$

with $d_0 = e_0 = 0$. To derive these formulas for d_n and e_n, note that we can write the argument of the exponent inside the expectation in

$$\frac{P_{nt}^\theta}{A_t\theta_t} = E_t\left(e^{q_{t,t+1}+\Delta a_{t+1}+\Delta\log(\theta_{t+1})}\frac{P_{n-1t+1}^\theta}{A_{t+1}\theta_{t+1}}\right) \quad (129)$$

using $\log(P_{n-1,t+1}^\theta/A_{t+1}\theta_{t+1}) = d_{n-1} + e_{n-1}\hat{s}_{t+1}$ as

$$\begin{aligned}
&q_{t,t+1} + \Delta a_{t+1} + \Delta\log(\theta_{t+1}) + d_{n-1} + e_{n-1}\hat{s}_{t+1}\\
&\quad= [\log(\beta) - \alpha\Delta a_{t+1} - \alpha\Delta s_{t+1}] + \Delta a_{t+1} + \psi_\theta\Delta s_{t+1} + d_{n-1} + e_{n-1}\hat{s}_{t+1}\\
&\quad= [\log(\beta) - \alpha\Delta a_{t+1} - (\alpha-\psi_\theta)\{(\rho_s-1)\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}\}] + \Delta a_{t+1} + d_{n-1}\\
&\qquad+ e_{n-1}\left[\rho_s\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}\right]\\
&\quad= \log(\beta) + (1-\alpha)(g_a + \sigma_a\varepsilon_{at+1}) + [e_{n-1}\rho_s - (\alpha-\psi_\theta)(\rho_s-1)]\,\hat{s}_t + d_{n-1}\\
&\qquad+ [e_{n-1} - \alpha + \psi_\theta]\,\lambda_a(s_t)\sigma_a\varepsilon_{at+1}\\
&\quad= \log(\beta) + (1-\alpha)g_a\\
&\qquad+ [e_{n-1}\rho_s - (\alpha-\psi_\theta)(\rho_s-1)]\hat{s}_t + d_{n-1} + \{1 - \alpha[1+\lambda_a(s_t)] + (e_{n-1}+\psi_\theta)\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1}. \quad (130)
\end{aligned}$$

Note that, except for the last term, all of the variables in (130) are known at t so that

$$\begin{aligned}&\log\left[\mathbb{E}_t\left(\exp\{\log(\beta)+(1-\alpha)g_a+\left[e_{n-1}\rho_s-(\alpha-\psi_\theta)(\rho_s-1)\right]\hat{s}_t+d_{n-1}\}\right)\right]\\&\quad=\log(\beta)+(1-\alpha)g_a+[e_{n-1}\rho_s-(\alpha-\psi_\theta)(\rho_s-1)]\hat{s}_t+d_{n-1}.\end{aligned}\tag{131}$$

For the last term in (130), we use that the conditional expectation of a log-normal random variable with mean 0 and variance σ^2 is $\exp\sigma^2/2$ so that

$$\begin{aligned}&\log(\mathbb{E}_t\left\{\exp\left[1-\alpha(1+\lambda_a(s_t))+(e_{n-1}+\psi_\theta)\lambda_a(s_t)\right]\sigma_a\varepsilon_{at+1}\right\})\\&\quad=\frac{\sigma_a^2}{2}\left[1-\alpha(1+\lambda_a(s_t))+(e_{n-1}+\psi_\theta)\lambda_a(s_t)\right]^2\\&\quad\cong\frac{\sigma_a^2}{2}\left[1-\alpha(1+\lambda_a(s))+(e_{n-1}+\psi_\theta)\lambda_a(s)\right]^2\\&\qquad+\left\{\sigma_a^2\left(e_{n-1}+\psi_\theta-\alpha\right)\lambda_a'(s)[1-\alpha(1+\lambda_a(s))+(e_{n-1}+\psi_\theta)\lambda_a(s)]\right\}\hat{s}_t\\&\quad=\left[1-\frac{\alpha}{S}+(e_{n-1}+\psi_\theta)\left(\frac{1}{S}-1\right)\right]^2\frac{\sigma_a^2}{2}+\sigma_a^2\frac{(\alpha-e_{n-1}-\psi_\theta)}{S}\left[1-\frac{\alpha}{S}+(e_{n-1}+\psi_\theta)\left(\frac{1}{S}-1\right)\right]\hat{s}_t\\&\quad=\left[1-e_{n-1}-\psi_\theta-\frac{\alpha-e_{n-1}-\psi_\theta}{S}\right]^2\frac{\sigma_a^2}{2}\\&\qquad+\sigma_a^2\left[1-e_{n-1}-\psi_\theta-\frac{\alpha-e_{n-1}-\psi_\theta}{S}\right]\left(\frac{\alpha-e_{n-1}-\psi_\theta}{S}\right)\hat{s}_t,\end{aligned}\tag{132}$$

where in the third line we have performed a first-order approximation around $s_t=s$ and in the fourth line we have used that in a steady state,

$$\lambda_a(s_t)=\frac{1}{S}\left[1-2\left(s_t-s\right)\right]^{1/2}-1$$

satisfies $1+\lambda_a(s)=1/S$ and $\lambda_a'(s)=-1/S$. Adding (131) and (132) and grouping together the constants and the terms in $\hat{s}_t$, we have that

$$\begin{aligned}&\log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1}+\Delta a_{t+1}+\Delta\log(\theta_{t+1})+d_{n-1}+e_{n-1}\hat{s}_{t+1}\right)\right]\right\}\\&\quad=\log(\beta)+(1-\alpha)g_a+d_{n-1}+\left[1-e_{n-1}-\psi_\theta-\frac{\alpha-e_{n-1}-\psi_\theta}{S}\right]^2\frac{\sigma_a^2}{2}\\&\qquad+\left\{(\alpha-\psi_\theta)(1-\rho_s)+\rho_s e_{n-1}+\left[1-e_{n-1}-\psi_\theta-\frac{\alpha-e_{n-1}-\psi_\theta}{S}\right]\left(\frac{\alpha-e_{n-1}-\psi_\theta}{S}\right)\sigma_a^2\right\}\hat{s}_t.\end{aligned}\tag{133}$$

Now we are ready to use the recursion in (129), namely,

$$d_n+e_n\hat{s}_t=\log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1}+\Delta a_{t+1}+\Delta\log(\theta_{t+1})+d_{n-1}+e_{n-1}\hat{s}_{t+1}\right)\right]\right\}.$$

Matching up the constants and the coefficients of $\hat{s}_t$ on both sides of this equation using (133) gives expressions (127) and (128) above.

To derive f_n and g_n, we use a similar argument for

$$\log\left(\frac{P_{nt}^{\theta\eta}}{A_t\theta^{1-\eta}\theta_t^{\eta}}\right) = f_n + g_n\hat{s},$$

where, using $(1-\eta)\log(\theta) + \eta\log(\theta_t) = \log(\theta) + \eta\psi_\theta\hat{s}_t$,

$$f_n = \log(\beta) + (1-\alpha)g_a + f_{n-1} + \frac{\sigma_a^2}{2}\left(1 - g_{n-1} - \eta\psi_\theta + \frac{g_{n-1}+\eta\psi_\theta-\alpha}{S}\right)^2 \tag{134}$$

$$g_n = \alpha(1-\rho_s) + g_{n-1}\rho_s + \eta(\rho_s-1)\psi_\theta - \left(1 - g_{n-1} - \eta\psi_\theta + \frac{g_{n-1}+\eta\psi_\theta-\alpha}{S}\right)\frac{g_{n-1}+\eta\psi_\theta-\alpha}{S}\sigma_a^2 \tag{135}$$

with $f_0 = g_0 = 0$. To derive these formulas for f_n and g_n, note that we can write the argument of the exponent inside the expectation in

$$\frac{P_{nt}^{\theta\eta}}{A_t\theta^{1-\eta}\theta_t^{\eta}} = E_t\left\{\exp\left[q_{t,t+1} + \Delta a_{t+1} + \eta\Delta\log(\theta_{t+1})\right]\frac{P_{n-1t+1}^{\theta}}{A_{t+1}\theta^{1-\eta}\theta_{t+1}^{\eta}}\right\} \tag{136}$$

using $\log(P_{n-1,t+1}^{\theta\eta}/A_{t+1}\theta^{1-\eta}\theta_{t+1}^{\eta}) = f_{n-1} + g_{n-1}\hat{s}_{t+1}$ as

$$\begin{aligned}
&q_{t,t+1} + \Delta a_{t+1} + \eta\Delta\log(\theta_{t+1}) + f_{n-1} + g_{n-1}\hat{s}_{t+1}\\
&= [\log(\beta) - \alpha\Delta a_{t+1} - \alpha\Delta s_{t+1}] + \Delta a_{t+1} + \eta\psi_\theta\Delta s_{t+1} + f_{n-1} + g_{n-1}\hat{s}_{t+1}\\
&= [\log(\beta) - \alpha\Delta a_{t+1} - (\alpha - \eta\psi_\theta)\{(\rho_s-1)\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}\}] + \Delta a_{t+1} + f_{n-1}\\
&\quad + g_{n-1}\left[\rho_s\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}\right]\\
&= \log(\beta) + (1-\alpha)(g_a + \sigma_a\varepsilon_{at+1}) + \left[g_{n-1}\rho_s - (\alpha-\eta\psi_\theta)(\rho_s-1)\right]\hat{s}_t + f_{n-1}\\
&\quad + \left[g_{n-1} - \alpha + \eta\psi_\theta\right]\lambda_a(s_t)\sigma_a\varepsilon_{at+1}\\
&= \log(\beta) + (1-\alpha)g_a + \left[g_{n-1}\rho_s - (\alpha-\eta\psi_\theta)(\rho_s-1)\right]\hat{s}_t + f_{n-1}\\
&\quad + \{1 - \alpha[1+\lambda_a(s_t)] + (g_{n-1}+\eta\psi_\theta)\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1}.
\end{aligned} \tag{137}$$

Note that, except for the last term, all of the variables in (137) are known at t so that

$$\begin{aligned}
&\log\left[\mathbb{E}_t\left(\exp\left\{\log(\beta) + (1-\alpha)g_a + \left[g_{n-1}\rho_s - (\alpha-\eta\psi_\theta)(\rho_s-1)\right]\hat{s}_t + f_{n-1}\right\}\right)\right]\\
&\quad = \log(\beta) + (1-\alpha)g_a + \left[g_{n-1}\rho_s - (\alpha-\eta\psi_\theta)(\rho_s-1)\right]\hat{s}_t + f_{n-1}.
\end{aligned} \tag{138}$$

For the last term in (137), we use that the conditional expectation of a log-normal random variable with mean 0 and variance σ^2 is $\exp\sigma^2/2$ so that

$$\begin{aligned}
&\log\left[\mathbb{E}_t\left(\exp\{1 - \alpha[1+\lambda_a(s_t)] + (g_{n-1}+\eta\psi_\theta)\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1})\right]\\
&\quad = \frac{\sigma_a^2}{2}\left\{1 - \alpha[1+\lambda_a(s_t)] + (g_{n-1}+\eta\psi_\theta)\lambda_a(s_t)\right\}^2.
\end{aligned}$$

Now this expression approximately equals

$$
\begin{aligned}
&\frac{\sigma_a^2}{2}\left\{1-\alpha[1+\lambda_a(s)]+(g_{n-1}+\eta\psi_\theta)\lambda_a(s)\right\}^2 \\
&\quad+\left\{\sigma_a^2\left(g_{n-1}+\eta\psi_\theta-\alpha\right)\lambda_a'(s)[1-\alpha(1+\lambda_a(s))+(g_{n-1}+\eta\psi_\theta)\lambda_a(s)]\right\}\hat{s}_t \\
&=\left[1-\frac{\alpha}{S}+(g_{n-1}+\eta\psi_\theta)\left(\frac{1}{S}-1\right)\right]^2\frac{\sigma_a^2}{2}+\sigma_a^2\frac{(\alpha-g_{n-1}-\eta\psi_\theta)}{S}\left[1-\frac{\alpha}{S}+(g_{n-1}+\eta\psi_\theta)\left(\frac{1}{S}-1\right)\right]\hat{s}_t \\
&=\left(1-g_{n-1}-\eta\psi_\theta-\frac{\alpha-g_{n-1}-\eta\psi_\theta}{S}\right)^2\frac{\sigma_a^2}{2} \\
&\quad+\sigma_a^2\left(1-g_{n-1}-\eta\psi_\theta-\frac{\alpha-g_{n-1}-\eta\psi_\theta}{S}\right)\left(\frac{\alpha-g_{n-1}-\eta\psi_\theta}{S}\right)\hat{s}_t, \qquad (139)
\end{aligned}
$$

where in the third line we have performed a first-order approximation around $s_t = s$ and in the fourth line we have used the steady-state relation (83). Adding (138) and (139) and grouping together the constants and the terms in $\hat{s}_t$, we obtain

$$
\begin{aligned}
&\log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1}+\Delta a_{t+1}+\eta\Delta\log(\theta_{t+1})+f_{n-1}+g_{n-1}\hat{s}_{t+1}\right)\right]\right\} \\
&\quad=\log(\beta)+(1-\alpha)g_a+f_{n-1}+\left(1-g_{n-1}-\eta\psi_\theta-\frac{\alpha-g_{n-1}-\eta\psi_\theta}{S}\right)^2\frac{\sigma_a^2}{2} \\
&\quad+\left[\begin{array}{c}(\alpha-\eta\psi_\theta)(1-\rho_s)+\rho_s g_{n-1}+\left(1-g_{n-1}-\eta\psi_\theta-\frac{\alpha-g_{n-1}-\eta\psi_\theta}{S}\right) \\ \left(\frac{\alpha-g_{n-1}-\eta\psi_\theta}{S}\right)\sigma_a^2\end{array}\right]\hat{s}_t. \qquad (140)
\end{aligned}
$$

Now we are ready to use the recursion in (136), namely,

$$
f_n+g_n\hat{s}_t=\log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1}+\Delta a_{t+1}+\eta\Delta\log(\theta_{t+1})+f_{n-1}+g_{n-1}\hat{s}_{t+1}\right)\right]\right\}.
$$

Matching up the constants and the coefficients of $\hat{s}_t$ on both sides of this equation using (140) gives expressions (134) and (135) above. □

We can now compare the accuracy of the linear approximation around the risky steady state in Proposition 4' with the linear approximation based on the assumption $\lambda_{wt+j} = \lambda_w$ behind Proposition 4. As Table A.7 shows, the approximation in Proposition 4 that uses the assumption $\lambda_{wt+j} = \lambda_w$ generates an approximate process for λ_{wt} that is 1.93 times as volatile as the process derived from the global solution of the model, and that correlates with it with a coefficient of 0.998. In contrast, the approximation in Proposition 4' that does not impose the assumption $\lambda_{wt+j} = \lambda_w$ generates an approximate process for λ_{wt} that has nearly exactly the same volatility (0.999 as volatile) as the process derived from the global solution of the model, and that correlates with it with a coefficient of 0.985. The more complicated expression behind Proposition 4' offers a more accurate solution as well as an alternative sufficient statistic, but in the interest of providing insight into our new mechanism, we used in the paper the simpler expression behind Proposition 4.

B Nominal and Real Bonds in the Model

Here we provide some details about the pricing of nominal and real bonds.

B.1 Pricing Real Zero-Coupon Bonds

The price at time t of the n-period zero-coupon *real* bond P^r_{nt} is defined as the expected value of one unit of goods in n periods. Hence it is given by $P^r_{nt} = \mathbb{E}_t(Q_{t,t+n})$. We can write this formula recursively as $P^r_{nt} = \mathbb{E}_t(Q_{t,t+1}P^r_{n-1,t+1})$ with $P^r_{0t} = 1$.

B.2 Pricing Nominal Zero-Coupon Bonds

Here we show how we price nominal bonds in our model, the inflation process we posit, and then briefly discuss why bonds carry an inflation-risk premium. Note that the price at time t of an n-period zero-coupon nominal bond is defined as the expected value of a claim to one dollar in n periods in period-t goods, $P^b_{nt} = \mathbb{E}_t(Q_{t,t+n}/\Pi_{t,t+n})$, where $\Pi_{t,t+n}$ is the gross inflation rate between t and $t+n$. We can write this formula recursively as

$$P^b_{nt} = \mathbb{E}_t\left(\frac{Q_{t,t+1}}{\Pi_{t,t+1}}P^b_{n-1,t+1}\right) \qquad \text{with } P^b_{0t} = 1. \tag{141}$$

We follow Wachter (2006) and posit an exogenous monthly process for inflation of the form

$$\pi_{t+1} = \bar{\pi} + x_t + w_{t+1} \text{ and } x_{t+1} = \phi x_t + \psi w_{t+1}, \tag{142}$$

which we estimate below. Note for later that we allow the nominal shock w_t to be correlated with the aggregate productivity shock ε_{at}. Observe also that the gross inflation rate between t and $t+1$, $\Pi_{t,t+1}$, and the net inflation rate between t and $t+1$, π_{t+1}, are related by

$$\Pi_{t,t+1} = \exp(\pi_{t+1}). \tag{143}$$

We first discuss how we solve for the price of a n-period nominal bond. Recall that the state of our baseline (real) model is (s_t, Z_{et}, Z_{ut}), which we refer to as the *real state*. The nominal model augments the equations of the baseline model with the exogenous inflation process (142). The nominal model thus adds the *nominal state* x_t so that the *state* of the nominal model is $(s_t, Z_{et}, Z_{ut}, x_t)$. We focus on a stationary equilibrium in which the price of the n-period nominal bond is a stationary function of this state and so has the form $P^b_{nt} = P^b_n(s_t, Z_{et}, Z_{ut}, x_t)$. To solve for such an equilibrium, we posit a solution in which the log of this price is the sum of a nominal part that is affine in the nominal state x_t and a real part that depends on the real state (s_t, Z_{et}, Z_{ut}), that is,

$$\log(P^b_n(s_t, Z_{et}, Z_{ut}, x_t)) = a_n + b_n x_t + \log(F^b_n(s_t, Z_{et}, Z_{ut}))$$

or

$$P^b_n(s_t, Z_{et}, Z_{ut}, x_t) = \exp(a_n + b_n x_t)F^b_n(s_t, Z_{et}, Z_{ut}). \tag{144}$$

Now, starting with (141), namely,

$$P^b_{nt} = \mathbb{E}_t(Q_{t,t+1}\Pi^{-1}_{t,t+1}P^b_{n-1,t+1}), \tag{145}$$

we substitute on the left side of (145) using (144), where we let $F^b_{n,t}$ denote $F^b_n(s_t, Z_{et}, Z_{ut})$, and on the right side using $\Pi_{t,t+1} = \exp(\pi_{t+1})$ to obtain

$$\begin{aligned}
\exp(a_n + b_n x_t)F^b_{nt} &= \mathbb{E}_t\left[Q_{t,t+1}\exp(-\pi_{t+1})P^b_{n-1,t+1}\right] \\
&= \mathbb{E}_t\left[Q_{t,t+1}\exp(-\pi_{t+1})\exp(a_{n-1} + b_{n-1}x_{t+1})F^b_{n-1,t+1}\right] \\
&= \mathbb{E}_t\left[Q_{t,t+1}\exp(-\bar{\pi} - x_t - w_{t+1} + a_{n-1} + b_{n-1}(\phi x_t + \psi w_{t+1}))F^b_{n-1,t+1}\right] \\
&= \exp(-\bar{\pi} + a_{n-1} - (1 - \phi b_{n-1})x_t)\mathbb{E}_t\left[Q_{t,t+1}\exp((b_{n-1}\psi - 1)w_{t+1})F^b_{n-1,t+1}\right], \qquad (146)
\end{aligned}$$

where in the second line we used a version (144) for a $n-1$-period bond in period $t+1$ and let $F^b_{n-1,t+1}$ denote $F^b_{n-1}(s_{t+1}, Z_{et+1}, Z_{ut+1})$, in the third line we substituted for π_{t+1} using the first equation in (142) and for x_{t+1} using the second equation in (142), and collected terms, and in the fourth line we rearranged terms. Hence, matching the constant terms and the terms in x_t on both sides of the last line gives

$$a_n = -\bar{\pi} + a_{n-1} \text{ and } b_n = -(1 - \phi b_{n-1}) \qquad (147)$$

with $a_0 = b_0 = 0$. Thus, solving (147) recursively gives

$$a_n = -n\bar{\pi} \text{ and } b_n = -\frac{1-\phi^n}{1-\phi}.$$

Now using from (147) that

$$\exp(a_n + b_n x_t) = \exp(-\bar{\pi} + a_{n-1} + (\phi b_{n-1} - 1)x_t), \qquad (148)$$

we can divide both sides of the fourth line of (146) by (148) to obtain

$$F^b_{nt} = \mathbb{E}_t\left[Q_{t,t+1}\exp((b_{n-1}\psi - 1)w_{t+1})F^b_{n-1,t+1}\right] \text{ with } F^b_{0t} = P^b_{0t}e^{-a_0 - b_0 x_t} = 1, \qquad (149)$$

where the expression for F^b_{0t} uses (144). We then solve (149) nonlinearly.

Finally, notice that if the nominal shock w_{t+1} were uncorrelated with the real shock ε_{at+1}, then we could write the product in the fourth line in (146) as

$$\mathbb{E}_t\left[Q_{t,t+1}\exp((b_{n-1}\psi - 1)w_{t+1})F^b_{n-1,t+1}\right] = \mathbb{E}_t\left[\exp((b_{n-1}\psi - 1)w_{t+1})\right]\mathbb{E}_t\left(Q_{t,t+1}F^b_{n-1,t+1}\right),$$

in which case the real interest rate would reduce to the nominal interest rate minus the (log of the) expected inflation rate. Here instead we have

$$\begin{aligned}
\mathbb{E}_t\left[Q_{t,t+1}\exp((b_{n-1}\psi - 1)w_{t+1})F^b_{n-1,t+1}\right] &= \mathrm{Cov}_t(\exp((b_{n-1}\psi - 1)w_{t+1}, Q_{t,t+1}F^b_{n-1,t+1}) \\
&+ \mathbb{E}_t\left[\exp((b_{n-1}\psi - 1)w_{t+1})\right]\mathbb{E}_t\left[Q_{t,t+1}F^b_{n-1,t+1}\right].
\end{aligned}$$

Given our estimates, w_{t+1} and ε_{at+1} are negatively correlated, so that a nominal bond tends to pay off fewer goods when the marginal utility of these goods tends to be high. Hence, nominal bonds carry an inflation-risk premium.

B.2.1 Estimation of Exogenous Inflation Process

The key way the process for inflation is connected to the real economy is that shocks to inflation are correlated with shocks to productivity. We start by estimating an ARMA monthly process for inflation. Since measures of aggregate productivity are only available quarterly whereas our model is monthly, we proceed as follows. We append a monthly process for inflation to our real model of the same form as the quarterly process in Wachter (2006), namely,

$$\pi_{t+1} = \bar{\pi} + x_t + w_{t+1} \text{ and } x_{t+1} = \phi x_t + \psi w_{t+1},$$

where w_t is an i.i.d. random variable that is distributed $N(0, \sigma_w^2)$ and correlated with productivity innovations with $corr(\epsilon_{at}, w_t) = \rho_{\pi a}$. This state-space representation implies an ARMA(1,1) univariate representation for monthly inflation. We estimate the model by maximum likelihood and recover the parameters $(\bar{\pi}, \phi, \psi, \sigma_w)$.

We now define quarterly inflation and productivity growth as

$$\pi_{q,t+1} \equiv \pi_{t+1} + \pi_{t+2} + \pi_{t+3} \text{ and } \Delta a_{q,t+1} \equiv \Delta a_{t+1} + \Delta a_{t+2} + \Delta a_{t+3}.$$

These quarterly rates can be written in terms of the monthly parameters and processes as

$$\pi_{t+3} + \pi_{t+2} + \pi_{t+1} = 3\bar{\pi} + (1 + \phi + \phi^2)x_t + w_{t+3} + (1 + \psi)w_{t+2} + [1 + \psi(1 + \phi)]w_{t+1}$$

and

$$\Delta a_{t+3} + \Delta a_{t+2} + \Delta a_{t+1} = 3g_a + \epsilon_{at+3} + \epsilon_{at+2} + \epsilon_{at+1}.$$

Thus, after estimating the monthly inflation process we can construct

$$w_{q,t+1} \equiv w_{t+3} + (1 + \psi)w_{t+2} + [1 + \psi(1 + \phi)]w_{t+1},$$

and hence we can compute

$$\begin{aligned}\text{Cov}(w_{q,t+1}, \Delta a_{q,t+1}) &= \text{Cov}(w_{t+3} + (1 + \psi)w_{t+2} + [1 + \psi(1 + \phi)]w_{t+1}, \epsilon_{at+3} + \epsilon_{at+2} + \epsilon_{at+1}) \\ &= (1 + (1 + \psi) + [1 + \psi(1 + \phi)])\, \rho_{\pi a}\sigma_w\sigma_a.\end{aligned}$$

Rearranging terms, we can recover the monthly correlation between inflation and productivity innovations as

$$\rho_{\pi a} = \frac{\text{Cov}(w_{q,t+1}, \Delta a_{q,t+1})}{(1 + (1 + \psi) + [1 + \psi(1 + \phi)])\, \sigma_w\sigma_a}.$$

In Table A.8, we display the results of this estimation.

C Results for Alternative Preferences

Here we inspect the amplification mechanism emanating from the alternative preferences we consider and emphasize that they formally all work in a nearly identical way.

C.1 The Mechanism for Alternative Preferences

Note first that Proposition 3 in the paper holds as stated for our models with Campbell-Cochrane preferences with external habit, Epstein-Zin preferences with long-run risk, Epstein-Zin preferences with variable disaster risk, and the affine discount factor. The reason is simply that this result depends only on the search side of the model and not on the discount factor that a particular preference and shock structure implies. It turns out that an analogue of Lemma 2 holds for each of these preferences as well. For Campbell-Cochrane preferences with external habit, the log-linear approximation in (73) holds with the same constants given in Lemma 2 except that the constant S in b_n is replaced by $\bar{S}$. Proposition 4 then applies as stated. For Epstein-Zin preferences with long-run risk, the analogue of Lemma 2 holds with $\log(P_{nt}/A_t) = a_n + b_n \Delta s_t + c_n x_t$, where $b_n = \rho_s(1-\rho_s^n)/(1-\rho_s)$, $c_n = (1-\rho)(1-\rho_x^n)/(1-\rho_x)$, and the constants a_n are given next. For the remaining preferences, the prices of claims to strips have the same form as (73) with constants provided here next. Then, Proposition 4 applies as stated.

In order to provide some intuition as to how these elasticities and the associated weights vary across models, in Figure A.2, we graph these elasticities, scaled by the volatility of the relevant state, and the corresponding weights. Notice that in all these models, these scaled elasticities increase with the horizon n. Hence, the intuition for the role of human capital is the same for all these models: the greater the degree of human capital accumulation, the larger the weights placed on long-horizon claims, which are relatively more sensitive to changes in the exogenous state of an economy, and so the larger the volatility of the job-finding rate. Therefore, as far as the volatility of the job-finding rate is concerned, all of these models work in the same way.

C.2 Versions of Lemma 2 and Proposition 4 for Alternative Preferences

We now state and prove a version of Lemma 2 and Proposition 4 for each of the alternative preferences we examine.

C.2.1 Campbell-Cochrane Preferences with External Habit

Under our approximations, Lemma 2 and Proposition 4 apply as stated for the external habit case.

C.2.2 Epstein-Zin Preferences with Long-Run Risk

We begin by deriving the risk-adjusted log-linear approximation to strips around the risky steady state.

Lemma 2b (Price of Productivity Claims for EZ Preferences with Long-Run Risk). *The price of a claim to productivity in n periods approximately satisfies*

$$\log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n x_t + c_n \Delta s_t,$$

with

$$a_n \;=\; a_{n-1}+\log(\beta)+(1-\alpha)g_a-(\alpha-\rho)\,(w-d)+\frac{(1-\alpha)^2}{2}\sigma_a^2$$
$$+\left(c_{n-1}-(\alpha-\rho)\psi_{wx}\right)^2\frac{\phi_x^2\sigma_a^2}{2}+\left(b_{n-1}+\frac{1-\alpha}{1-\rho}-(\alpha-\rho)\psi_{ws}\right)^2\frac{\phi_s^2\sigma_a^2}{2},$$

$$w-d=-g_a-\frac{1-\alpha}{2}\sigma_a^2-\frac{1-\alpha}{2}\psi_{wx}^2\phi_x^2\sigma_a^2-\frac{1-\alpha}{2}\left(\psi_{ws}-\frac{1}{1-\rho}\right)^2\phi_s^2\sigma_a^2,$$

and

$$\psi_{wx}=\frac{\delta}{1-\beta+\delta(1-\rho_x)}\;\text{ and }\;\psi_{ws}=\frac{\delta}{1-\beta+\delta(1-\rho_s)}\frac{\rho_s}{1-\rho},$$

where $a_0=b_0=c_0=0$*,*

$$b_n=\frac{1-\rho_s^n}{1-\rho_s}\rho_s,\;\text{ and }\;c_n=\frac{1-\rho_x^n}{1-\rho_x}(1-\rho).$$

Now, given this result, it is immediate that if we take a first-order approximation to the key equation from Proposition 3,

$$\log(\lambda_{wt})=\chi+\left(\frac{1-\eta}{\eta}\right)\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n+c_s\delta_s^n)\frac{P_{nt}}{A_t}\right],$$

using the approximate solution for productivity strips in Lemma 2b, the following proposition is immediate.

Proposition 4b (Sufficient Statistic for Job-Finding Rate Volatility for EZ Preferences with Long-Run Risk). *Under the approximation in Lemma 2b, the responses of the job-finding rate to a change in* x_t *and* Δs_t *evaluated at a risky steady state are given by*

$$\frac{d\log(\lambda_{wt})}{dx_t}=\left(\frac{1-\eta}{\eta}\right)\sum_{n=0}^{\infty}\omega_n b_n\;\text{and}\;\frac{d\log(\lambda_{wt})}{d\Delta s_t}=\left(\frac{1-\eta}{\eta}\right)\sum_{n=0}^{\infty}\omega_n c_n\;\text{with}\;\omega_n=\frac{e^{a_n}(c_\ell\delta_\ell^n+c_s\delta_s^n)}{\sum_{n=0}^{\infty}e^{a_n}(c_\ell\delta_\ell^n+c_s\delta_s^n)},\tag{150}$$

where a_n*,*b_n *and* c_n *are given in Lemma 2b and the variance of the job-finding rate* $\sigma(\lambda_{wt})$ *satisfies*

$$\sigma(\lambda_{wt})^2=\left(\frac{d\log(\lambda_{wt})}{dx_t}\right)^2\sigma(x_t)^2+\left(\frac{d\log(\lambda_{wt})}{d\Delta s_t}\right)^2\sigma(\Delta s_t)^2.\tag{151}$$

We first prove Lemma 2b. To this purpose, note that the shocks

$$\Delta a_{t+1}=g_a+x_t+\sigma_a\varepsilon_{at+1},\;x_{t+1}=\rho_x x_t+\phi_x\sigma_a\varepsilon_{xt+1},\;\text{and}\;\Delta s_{t+1}=\rho_s\Delta s_t+\phi_s\sigma_a\varepsilon_{st+1}\tag{152}$$

imply that

$$\mathbb{E}_t\Delta a_{t+1}=g_a+x_t,\;\mathbb{E}_t x_{t+1}=\rho_x x_t,\;\text{and}\;\mathbb{E}_t\Delta s_{t+1}=\rho_s\Delta s_t.\tag{153}$$

Rewrite the preferences

$$V_t=\left[(1-\beta)S_tC_t^{1-\rho}+\beta\left(\mathbb{E}_tV_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}\tag{154}$$

as

$$W_t^{1-\rho} \equiv \left(\frac{V_t}{S_t^{\frac{1}{1-\rho}} C_t} \right)^{1-\rho} = (1-\beta) + \frac{\beta}{S_t C_t^{1-\rho}} \left(\mathbb{E}_t V_{t+1}^{1-\alpha} \right)^{\frac{1-\rho}{1-\alpha}}, \tag{155}$$

which is equivalent to

$$e^{(1-\rho)w_t} = (1-\beta) + \frac{\beta}{S_t C_t^{1-\rho}} \left(\mathbb{E}_t V_{t+1}^{1-\alpha} \right)^{\frac{1-\rho}{1-\alpha}}, \tag{156}$$

with $W_t \equiv V_t / \left(S_t^{1/(1-\rho)} C_t \right)$ and $w_t = \log(W_t)$ so $e^{(1-\rho)w_t} = [V_t/(S_t^{1/(1-\rho)} C_t)]^{1-\rho}$. Using $V_{t+1} = W_{t+1} S_{t+1}^{1/(1-\rho)} C_{t+1}$ in (156) gives

$$\begin{aligned} e^{(1-\rho)w_t} &= (1-\beta) + \frac{\beta}{S_t C_t^{1-\rho}} \left\{ \mathbb{E}_t \left[W_{t+1} S_{t+1}^{1/(1-\rho)} C_{t+1} \right]^{1-\alpha} \right\}^{\frac{1-\rho}{1-\alpha}} \\ &= (1-\beta) + \beta \left\{ \mathbb{E}_t \left[W_{t+1} (S_{t+1}/S_t)^{\frac{1}{1-\rho}} (C_{t+1}/C_t) \right]^{1-\alpha} \right\}^{\frac{1-\rho}{1-\alpha}} \\ &= 1 - \beta + \beta \left[\mathbb{E}_t e^{(1-\alpha)\left(w_{t+1} + \frac{1}{1-\rho} \Delta s_{t+1} + \Delta c_{t+1} \right)} \right]^{\frac{1-\rho}{1-\alpha}} = 1 - \beta + \beta \left(e^{d_t} \right)^{1-\rho}, \end{aligned} \tag{157}$$

where $s_t = \log(S_t)$, $c_t = \log(C_t)$, and

$$d_t \equiv \frac{1}{1-\alpha} \log \left\{ \mathbb{E}_t \left[e^{(1-\alpha)(w_{t+1} + \frac{1}{1-\rho} \Delta s_{t+1} + \Delta c_{t+1})} \right] \right\}. \tag{158}$$

Then, taking logs of both sides of (157) and dividing by $1-\rho$ gives

$$w_t = \frac{1}{1-\rho} \log \left(1 - \beta + \beta e^{(1-\rho)d_t} \right) \approx \frac{1}{1-\rho} \log (1 - \beta + \delta) + \frac{\delta}{1-\beta+\delta} (d_t - d), \tag{159}$$

where we to obtain the right side we used a first-order approximation around $d_t = d$ with $\delta \equiv \beta e^{(1-\rho)d}$. We will use affine guesses in the states for the unknown variables, which make w_{t+1} conditionally normal distributed, and will use the approximation that $\Delta c_{t+1} = \Delta a_{t+1}$, which makes Δc_{t+1} normally distributed as well. Using the properties of normal distribution, we can rewrite (158) as

$$d_t = \mathbb{E}_t w_{t+1} + \frac{\mathbb{E}_t \Delta s_{t+1}}{1-\rho} + \mathbb{E}_t \Delta a_{t+1} + \frac{1-\alpha}{2} \mathrm{Var}_t \left(w_{t+1} + \frac{1}{1-\rho} \Delta s_{t+1} + \Delta a_{t+1} \right), \tag{160}$$

Then, under the affine guesses

$$w_t = w + \psi_{wx} x_t + \psi_{ws} \Delta s_t \text{ and } d_t = d + \psi_{dx} x_t + \psi_{ds} \Delta s_t, \tag{161}$$

we can substitute for w_t and d_t in (159) to obtain

$$w + \psi_{wx} x_t + \psi_{ws} \Delta s_t = \frac{1}{1-\rho} \log (1 - \beta + \delta) + \frac{\delta}{1-\beta+\delta} (\psi_{dx} x_t + \psi_{ds} \Delta s_t). \tag{162}$$

Matching coefficients on both sides of (162) gives

$$w = \frac{1}{1-\rho} \log\left(1 - \beta + \delta\right),$$

$$\psi_{wx} = \frac{\delta}{1-\beta+\delta} \psi_{dx}, \tag{163}$$

and

$$\psi_{ws} = \frac{\delta}{1-\beta+\delta} \psi_{ds}. \tag{164}$$

Now substitute for d_t on the left side of (160) using (161) and substitute for w_{t+1} on the right side using the affine guess for w_{t+1} to obtain

$$\begin{aligned} d + \psi_{dx} x_t + \psi_{ds} \Delta s_t \quad &= \quad w + \psi_{wx} \mathbb{E}_t x_{t+1} + \psi_{ws} \mathbb{E}_t \Delta s_{t+1} + \frac{1}{1-\rho} \mathbb{E}_t \Delta s_{t+1} + \mathbb{E}_t \Delta a_{t+1} \\ &+ \frac{1-\alpha}{2} \mathrm{Var}_t \left(w + \psi_{wx} x_{t+1} + \psi_{ws} \Delta s_{t+1} + \frac{1}{1-\rho} \Delta s_{t+1} + \Delta a_{t+1} \right). \end{aligned}$$

Using the processes for the shocks $\Delta a_{t+1} = g_a + x_t + \sigma_a \varepsilon_{at+1}$, $x_{t+1} = \rho_x x_t + \phi_x \sigma_a \varepsilon_{xt+1}$, and $\Delta s_{t+1} = \rho_s \Delta s_t + \phi_s \sigma_a \varepsilon_{st+1}$, we further obtain

$$\begin{aligned} d + \psi_{dx} x_t + \psi_{ds} \Delta s_t &= w + \psi_{wx} \rho_x x_t + \psi_{ws} \rho_s \Delta s_t + \frac{1}{1-\rho} \rho_s \Delta s_t + g_a + x_t \\ + \frac{1-\alpha}{2} \mathrm{Var}_t &\left(w + \psi_{wx} \phi_x \sigma_a \varepsilon_{xt+1} + \psi_{ws} \phi_s \sigma_a \varepsilon_{st+1} + \frac{1}{1-\rho} \phi_s \sigma_a \varepsilon_{st+1} + g_a + x_t + \sigma_a \varepsilon_{at+1} \right) \end{aligned} \tag{165}$$

or, by the independence of the three shocks,

$$\begin{aligned} &\mathrm{Var}_t \left(w + \psi_{wx} \phi_x \sigma_a \varepsilon_{xt+1} + \psi_{ws} \phi_s \sigma_a \varepsilon_{st+1} + \frac{1}{1-\rho} \phi_s \sigma_a \varepsilon_{st+1} + g_a + x_t + \sigma_a \varepsilon_{at+1} \right) \\ &= \mathrm{Var}_t \left(\psi_{wx} \phi_x \sigma_a \varepsilon_{xt+1} \right) + \mathrm{Var}_t \left[\left(\psi_{ws} + \frac{1}{1-\rho_s} \right) \phi_s \sigma_a \varepsilon_{st+1} \right] + \mathrm{Var}_t \left(\sigma_a \varepsilon_{at+1} \right). \end{aligned}$$

Hence, (165) can be rewritten as

$$\begin{aligned} d + \psi_{dx} x_t + \psi_{ds} \Delta s_t \quad &= \quad w + \psi_{wx} \rho_x x_t + \psi_{ws} \rho_s \Delta s_t + \frac{1}{1-\rho} \rho_s \Delta s_t + g_a + x_t \\ &+ \frac{1-\alpha}{2} \sigma_a^2 + \frac{1-\alpha}{2} \psi_{wx}^2 \phi_x^2 \sigma_a^2 + \frac{1-\alpha}{2} \left(\psi_{ws} + \frac{1}{1-\rho} \right)^2 \phi_s^2 \sigma_a^2. \end{aligned} \tag{166}$$

By using the affine guess for d_t in this equation and matching coefficients, we obtain

$$d = w + g_a + \frac{1-\alpha}{2} \sigma_a^2 + \frac{1-\alpha}{2} \psi_{wx}^2 \phi_x^2 \sigma_a^2 + \frac{1-\alpha}{2} \left(\psi_{ws} + \frac{1}{1-\rho} \right)^2 \phi_s^2 \sigma_a^2,$$

$\psi_{dx} = \psi_{wx} \rho_x + 1$, and $\psi_{ds} = \psi_{ws} \rho_s + \rho_s / (1-\rho)$. Observe for later use that these equations imply

$$\psi_{wx} \rho_x - \psi_{dx} = -1 \text{ and } \psi_{ws} \rho_s - \psi_d = -\frac{\rho_s}{1-\rho}. \tag{167}$$

Using (163) and (164), we see that

$$\psi_{dx} = \left[\frac{\delta}{1-\beta+\delta}\psi_{dx}\right]\rho_x + 1 \text{ and } \psi_{ds} = \left[\frac{\delta}{1-\beta+\delta}\psi_{ds}\right]\rho_s + \rho_s/(1-\rho)$$

so that

$$\psi_{dx} = \frac{1-\beta+\delta}{1-\beta+\delta(1-\rho_x)} \text{ and } \psi_{ds} = \left(\frac{\rho_s}{1-\rho}\right)\frac{1-\beta+\delta}{1-\beta+\delta(1-\rho_s)}.$$

and thus using (167)

$$\psi_{wx} = \frac{\delta}{1-\beta+\delta(1-\rho_x)} \text{ and } \psi_{ws} = \left(\frac{\rho_s}{1-\rho}\right)\frac{\delta}{1-\beta+\delta(1-\rho_s)}.$$

We now turn to the formulas for the strips. Using the approximation $\log(P_{nt}/A_t) = a_n + b_n\Delta s_t + c_n x_t$ in the extension of our main pricing equation (72),

$$\log\left(\frac{P_{nt}}{A_t}\right) = \log\left(\mathbb{E}_t\left\{\exp\left[q_{t,t+1} + \Delta a_{t+1} + \log(P_{n-1,t+1}/A_{t+1})\right]\right\}\right), \tag{168}$$

gives (168) so that

$$a_n + b_n\Delta s_t + c_n x_t = \log\left\{\mathbb{E}_t\left[\exp(q_{t,t+1} + \Delta a_{t+1} + a_{n-1} + b_{n-1}\Delta s_{t+1} + c_{n-1}x_{t+1})\right]\right\}. \tag{169}$$

The stochastic discount factor for Epstein-Zin preferences is

$$Q_{t,t+1} = \frac{\partial V_t/\partial C_{t+1}}{\partial V_t/\partial C_t} \text{ and } \frac{\partial V_t}{\partial C_{t+1}} = \frac{\partial V_{t+1}}{\partial C_{t+1}}\frac{\partial V_t}{\partial V_{t+1}} \text{ so } Q_{t,t+1} = \frac{\partial V_{t+1}/\partial C_{t+1}\partial V_t/\partial V_{t+1}}{\partial V_t/\partial C_t}.$$

Note from (154) that

$$\frac{\partial V_t}{\partial C_t} = (1-\beta)V_t^\rho C_t^{-\rho}S_t \text{ and } \frac{\partial V_t}{\partial V_{t+1}} = \beta V_t^\rho V_{t+1}^{-\alpha}\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{\alpha-\rho}{1-\alpha}},$$

so that the stochastic discount factor can be expressed as

$$Q_{t,t+1} = \frac{\left[V_{t+1}^\rho C_{t+1}^{-\rho}S_{t+1}\right]\left[\beta V_t^\rho V_{t+1}^{-\alpha}\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{\alpha-\rho}{1-\alpha}}\right]}{V_t^\rho C_t^{-\rho}S_t} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{S_{t+1}}{S_t}\left[\frac{V_{t+1}}{\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}}}\right]^{-(\alpha-\rho)}. \tag{170}$$

By the definition of the variables w_t and d, and substituting $\Delta c_{t+1} = \Delta a_{t+1}$, we have

$$q_{t,t+1} = \log(\beta) + \Delta s_{t+1} - \rho\Delta a_{t+1} - (\alpha-\rho)\left(v_{t+1} - \frac{1}{1-\alpha}\log\{\mathbb{E}_t\left[e^{(1-\alpha)v_{t+1}}\right]\}\right), \tag{171}$$

where the term in brackets on the right side can be expanded as

$$
\begin{aligned}
& v_{t+1} - \frac{1}{1-\alpha}\log\{\mathbb{E}_t[e^{(1-\alpha)v_{t+1}}]\} && (172)\\
&\quad = w_{t+1} + \frac{1}{1-\rho}s_{t+1} + a_{t+1} - \frac{1}{1-\alpha}\log\left(\mathbb{E}_t\left\{\exp\left[(1-\alpha)(w_{t+1} + \frac{1}{1-\rho}s_{t+1} + a_{t+1})\right]\right\}\right) \\
&\quad = w_{t+1} + \frac{1}{1-\rho}\Delta s_{t+1} + \Delta a_{t+1} - \frac{1}{1-\alpha}\log\left(\mathbb{E}_t\left\{\exp\left[(1-\alpha)(w_{t+1} + \frac{1}{1-\rho}\Delta s_{t+1} + \Delta a_{t+1})\right]\right\}\right) \\
&\quad = w_{t+1} + \frac{1}{1-\rho}\Delta s_{t+1} + \Delta a_{t+1} - d_t, && (173)
\end{aligned}
$$

where we used $V_{t+1} = W_{t+1}S_{t+1}^{1/(1-\rho)}A_{t+1}$, subtracted and summed $s_t/(1-\rho)+a_t$, and used the definition of d_t. Substituting (173) into (171) gives

$$
\begin{aligned}
q_{t,t+1} &= \log(\beta) + \Delta s_{t+1} - \rho\Delta a_{t+1} - (\alpha-\rho)\left[w_{t+1} + \frac{1}{1-\rho}\Delta s_{t+1} + \Delta a_{t+1} - d_t\right] \\
&= \log(\beta) - \alpha\Delta a_{t+1} + \frac{1-\alpha}{1-\rho}\Delta s_{t+1} - (\alpha-\rho)(w_{t+1} - d_t). \qquad (174)
\end{aligned}
$$

Now we turn to developing recursive formulas for a_n, b_n, and c_n starting from (169), namely,

$$
a_n + b_n\Delta s_t + c_n x_t = \log\left\{\mathbb{E}_t\left[\exp(q_{t,t+1} + \Delta a_{t+1} + a_{n-1} + b_{n-1}\Delta s_{t+1} + c_{n-1}x_{t+1})\right]\right\}. \qquad (175)
$$

Consider the argument of the exponential function on the right side of this equation, namely, $q_{t,t+1} + \Delta a_{t+1} + a_{n-1} + b_{n-1}\Delta s_{t+1} + c_{n-1}x_{t+1}$, and substitute for $q_{t,t+1}$ using (174) to obtain

$$
\begin{aligned}
& \log(\beta) - \alpha\Delta a_{t+1} + \frac{1-\alpha}{1-\rho}\Delta s_{t+1} - (\alpha-\rho)(w_{t+1} - d_t) + \Delta a_{t+1} + a_{n-1} + b_{n-1}\Delta s_{t+1} + c_{n-1}x_{t+1} \\
= \; & \log(\beta) + (1-\alpha)\left(g_a + x_t + \sigma_a\varepsilon_{a,t+1}\right) + \left(b_{n-1} + \frac{1-\alpha}{1-\rho}\right)\left(\rho_s\Delta s_t + \phi_s\sigma_a\varepsilon_{st+1}\right) \\
& -(\alpha-\rho)(\mathbb{E}_t w_{t+1} + \psi_{wx}\phi_x\sigma_a\varepsilon_{xt+1} + \psi_{ws}\phi_s\sigma_a\varepsilon_{st+1} - d_t) \\
& +a_{n-1} + c_{n-1}(\rho_x x_t + \phi_x\sigma_a\varepsilon_{xt+1}) \\
= \; & \log(\beta) + (1-\alpha)(g_a + x_t) + a_{n-1} + \left(b_{n-1} + \frac{1-\alpha}{1-\rho}\right)\rho\Delta s_t && (176)\\
& +c_{n-1}\rho_x x_t - (\alpha-\rho)(\mathbb{E}_t w_{t+1} - d_t) && (177)\\
& +(1-\alpha)(\sigma_a\varepsilon_{a,t+1}) + \left[b_{n-1} + \frac{1-\alpha}{1-\rho} - (\alpha-\rho)\psi_{ws})\right]\phi_s\sigma_a\varepsilon_{st+1} && (178)\\
& +\left[c_{n-1} - (\alpha-\rho)\psi_{wx}\right]\phi_x\sigma_a\varepsilon_{xt+1} && (179)
\end{aligned}
$$

where in the second line we used the affine guesses

$$
w_t = w + \psi_{wx}x_t + \psi_{ws}\Delta s_t, \text{ and } d_t = d + \psi_{dx}x_t + \psi_{ds}\Delta s_t,
$$

that $x_{t+1} = \rho_x x_t + \phi_x \sigma_a \varepsilon_{xt+1}$, $\Delta a_{t+1} = g_a + x_t + \sigma_a \varepsilon_{a,t+1}$, $\Delta s_{t+1} = \rho_s \Delta s_t + \phi_s \sigma_a \varepsilon_{st+1}$, and

$$\begin{aligned} w_{t+1} &= [w + \psi_{wx}\rho_x x_t + \psi_{ws}\rho_s \Delta s_t] + \psi_{wx}\phi_x\sigma_a\varepsilon_{xt+1} + \psi_{ws}\phi_s\sigma_a\varepsilon_{st+1} \\ &= \mathbb{E}_t w_{t+1} + \psi_{wx}\phi_x\sigma_a\varepsilon_{xt+1} + \psi_{ws}\phi_s\sigma_a\varepsilon_{st+1}. \end{aligned}$$

Since the terms in (176) and (177) are all known at t, the stochastic terms in (178) and (179) are such that

$$\begin{aligned} &\log\left(\mathbb{E}_t\{\exp(1-\alpha)(\sigma_a\varepsilon_{a,t+1}) + \left[b_{n-1} + \frac{1-\alpha}{1-\rho} - (\alpha-\rho)\psi_{ws})\right]\phi_s\sigma_a\varepsilon_{st+1}\right. \\ &+ [c_{n-1} - (\alpha-\rho)\psi_{wx}]\,\phi_x\sigma_a\varepsilon_{xt+1}\}) = \frac{(1-\alpha)^2}{2}\sigma_a^2 + [c_{n-1} - (\alpha-\rho)\psi_{wx}]^2\frac{\phi_x^2\sigma_a^2}{2} \\ &+ \left[b_{n-1} + \frac{1-\alpha}{1-\rho} - (\alpha-\rho)\psi_{ws}\right]^2\frac{\phi_s^2\sigma_a^2}{2}. \end{aligned}$$

Therefore, substituting into (175) gives

$$a_n + b_n\Delta s_t + c_n x_t \tag{180}$$

$$= \log(\beta) + (1-\alpha)(g_a + x_t) + a_{n-1} + \left(b_{n-1} + \frac{1-\alpha}{1-\rho}\right)\rho\Delta s_t \tag{181}$$

$$\begin{aligned} &+ c_{n-1}\rho_x x_t - (\alpha-\rho)\mathbb{E}_t(w_{t+1} - d_t) \\ &+ \frac{(1-\alpha)^2}{2}\sigma_a^2 + [c_{n-1} - (\alpha-\rho)\psi_{wx}]^2\frac{\phi_x^2\sigma_a^2}{2} + \left[b_{n-1} + \frac{1-\alpha}{1-\rho} - (\alpha-\rho)\psi_{ws}\right]^2\frac{\phi_s^2\sigma_a^2}{2}. \end{aligned}$$

Now, substitute into (181), to obtain $\mathbb{E}_t(w_{t+1}) = w + \psi_{wx}\rho_x x_t + \psi_{ws}\rho_s\Delta s_t$ and $d_t = d + \psi_{dx}x_t + \psi_{ds}\Delta s_t$. Matching the constant terms, the coefficients of the terms in Δs_t, and the coefficients of the terms in x_t on both sides of (180) and (181) yields

$$\begin{aligned} a_n &= a_{n-1} + \log(\beta) + (1-\alpha)g_a - (\alpha-\rho)(w-d) \\ &+ \frac{(1-\alpha)^2}{2}\sigma_a^2 + [c_{n-1} - (\alpha-\rho)\psi_{wx}]^2\frac{\phi_x^2\sigma_a^2}{2} + \left[b_{n-1} + \frac{1-\alpha}{1-\rho} - (\alpha-\rho)\psi_{ws}\right]^2\frac{\phi_s^2\sigma_a^2}{2}, \end{aligned}$$

$$b_n = b_{n-1}\rho_s + \frac{1-\alpha}{1-\rho}\rho_s - (\alpha-\rho)(\psi_{ws}\rho_s - \psi_d), \tag{182}$$

and

$$c_n = c_{n-1}\rho_x + 1 - \alpha - (\alpha-\rho)(\psi_{wx}\rho_x - \psi_{dx}). \tag{183}$$

Now using $\psi_{ws}\rho_s - \psi_d = -\rho_s/(1-\rho_s)$ derived in (167) in (182), we obtain

$$b_n = b_{n-1}\rho_s + \frac{1-\alpha}{1-\rho}\rho_s + \frac{\rho_s(\alpha-\rho)}{1-\rho} = b_{n-1}\rho_s + \rho_s$$

so $b_n = (1-\rho_s^n)\rho_s/(1-\rho_s)$ and using $\psi_{wx}\rho_x - \psi_{dx} = -1$ derived in (167) in (183), we further obtain

$$c_n = c_{n-1}\rho_x + 1 - \alpha + (\alpha-\rho) = c_{n-1}\rho_x + 1 - \rho$$

so $c_n = (1-\rho_x^n)(1-\rho)/(1-\rho_x)$. This concludes the proof. $\square$

C.2.3 Epstein-Zin Preferences with Variable Disaster Risk

We begin with the risk-adjusted log-linear approximation to strips around the risky steady state.

Lemma 2c (Price of Productivity Claims for EZ Preferences with Variable Disaster Risk). *The price of a claim to productivity in n periods approximately satisfies*

$$\log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n(s_t - s),$$

with

$$a_n = a_{n-1} + \log(\beta) + (1-\alpha)g_a - (\alpha-\rho)(w-d) + \frac{(1-\alpha)^2}{2}\sigma^2 + \frac{\left[(\rho-\alpha)\psi_{ws} + b_{n-1}\right]^2 \phi_s^2 s}{2}\sigma^2 + \left[e^{(\alpha-1)\theta} - 1\right]s,$$

$$w - d = -g_a - \frac{1-\alpha}{2}(1+\psi_{ws}^2\phi_s^2 s)\sigma^2 - \frac{e^{(\alpha-1)\theta}-1}{1-\alpha}s,$$

and ψ_{ws} is the negative root of the quadratic equation

$$\psi_{ws} = \frac{\delta}{1-\beta+\delta(1-\rho_s)}\left[\frac{e^{(\alpha-1)\theta}-1}{1-\alpha} + \frac{(1-\alpha)}{2}\psi_{ws}^2\phi_s^2\sigma^2\right],$$

where $a_0 = b_0 = 0$ and

$$b_n = b_{n-1}\rho_s - (1-\rho)\left[\frac{e^{(\alpha-1)\theta}-1}{\alpha-1}\right] + \frac{(\alpha-\rho)(1-\alpha)}{2}\psi_{ws}^2\phi_s^2\sigma^2 + \frac{\left[(\rho-\gamma)\psi_{ws} + b_{n-1}\right]^2\phi_s^2}{2}\sigma^2.$$

By taking a first-order approximation to the key equation in Proposition 3,

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right)\log\left[\sum_{n=0}^{\infty}(c_\ell\delta_\ell^n + c_s\delta_s^n)\frac{P_{nt}}{A_t}\right],$$

and using the approximate solution for productivity strips in Lemma 2c, the following proposition is immediate.

Proposition 4c (Sufficient Statistic for Job-Finding Rate Volatility for EZ Preferences with Variable Disaster Risk). *Under the approximation in Lemma 2c, the response of the job-finding rate to a change in s_t evaluated at a risky steady state is given by*

$$\frac{d\log(\lambda_{wt})}{ds_t} = \left(\frac{1-\eta}{\eta}\right)\sum_{n=0}^{\infty}\omega_n b_n \text{ with } \omega_n = \frac{e^{a_n}(c_\ell\delta_\ell^n + c_s\delta_s^n)}{\sum_{n=0}^{\infty}e^{a_n}(c_\ell\delta_\ell^n + c_s\delta_s^n)}, \tag{184}$$

where a_n and b_n are given in Lemma 2c and the standard deviation of the job-finding rate $\sigma(\lambda_{wt})$

satisfies

$$\sigma(\lambda_{wt}) = \frac{d\log(\lambda_{wt})}{ds_t}\sigma(s_t). \tag{185}$$

We now turn to the proof of Lemma 2c. The proof is as follows. In this case, the forcing process are

$$\Delta a_{t+1} = g_a + \sigma_a \varepsilon_{at+1} - \theta j_{t+1} \text{ and } s_{t+1} = (1-\rho_s)s + \rho_s s_t + \phi_s \sigma_s \sqrt{s_t}\varepsilon_{st+1}$$

with $j_{t+1} \sim Poisson(s_t)$, and $(\varepsilon_{a,t}, \varepsilon_{s,t}) \sim N(0, I_2)$ i.i.d. and independent of j_{t+1}. Note that $\mathbb{E}_t j_{t+1} = s_t$. Recall that the conditional (log) moment generating function of a standard normal random variable ε_{t+1} is $\log[\mathbb{E}_t(e^{\alpha\varepsilon_{t+1}})] = \alpha^2/2$ and that of a Poisson random variable j_{t+1} is $\log[\mathbb{E}_t(e^{\alpha j_{t+1}})] = (e^\alpha - 1)s_t$. It follows from the independence of $(\varepsilon_{a,t+1}, \varepsilon_{s,t+1}, j_{t+1})$ that their joint conditional (log) moment generating function is

$$\log\left\{\mathbb{E}_t\left[\exp(\alpha_a\varepsilon_{a,t+1} + \alpha_s\sqrt{s_t}\varepsilon_{s,t+1} + \alpha_j j_{t+1})\right]\right\} = \frac{\alpha_a^2}{2} + \left(\frac{\alpha_s^2}{2} + e^{\alpha_j} - 1\right)s_t. \tag{186}$$

Like in the previous Epstein-Zin case, rewrite $V_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}$ as

$$\left(\frac{V_t}{C_t}\right)^{1-\rho} = 1-\beta + \frac{\beta}{C_t^{1-\rho}}\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}},$$

which can be equivalently expressed as

$$e^{(1-\rho)w_t} = 1-\beta + \frac{\beta}{C_t^{1-\rho}}\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}},$$

where $W_t \equiv V_t/C_t$ and $w_t = \log(W_t)$ so $e^{(1-\rho)w_t} = (V_t/C_t)^{1-\rho}$. Using that $V_{t+1} = W_{t+1}C_{t+1}$ gives

$$\begin{aligned} e^{(1-\rho)w_t} &= (1-\beta) + \frac{\beta}{C_t^{1-\rho}}\left(\mathbb{E}_t\left[W_{t+1}C_{t+1}\right]^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}} = (1-\beta) + \beta\left(\mathbb{E}_t\left[W_{t+1}(C_{t+1}/C_t)\right]^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}} \\ &= 1-\beta+\beta\left(\mathbb{E}_t e^{(1-\alpha)(w_{t+1}+\Delta c_{t+1})}\right)^{\frac{1-\rho}{1-\alpha}} = 1-\beta+\beta\left(e^{d_t}\right)^{1-\rho}, \end{aligned}$$

where $c_t = \log(C_t)$ and

$$d_t \equiv \frac{1}{1-\alpha}\log\left(\mathbb{E}_t e^{(1-\alpha)(w_{t+1}+\Delta c_{t+1})}\right).$$

Using the definitions of the variables w and d, the stochastic discount factor is $q_{t,t+1} = \log(\beta) - \alpha\Delta c_{t+1} - (\alpha-\rho)(w_{t+1} - d_t)$ with

$$w_t = \frac{1}{1-\rho}\log\left(1-\beta+\beta e^{(1-\rho)d_t}\right) \approx \frac{1}{1-\rho}\log\left(1-\beta+\beta e^{(1-\rho)d}\right) + \frac{\delta}{1-\beta+\delta}\hat{d}_t, \tag{187}$$

$$d_t = \frac{1}{1-\alpha}\log\left(\mathbb{E}_t e^{(1-\alpha)(w_{t+1}+\Delta c_{t+1})}\right), \tag{188}$$

and $\delta \equiv \beta e^{(1-\rho)d}$. By the usual notation, the log price of the n-th productivity strip is

$$a_n + b_n\hat{s}_t = \log\left[\mathbb{E}_t\left(e^{q_{t,t+1}+\Delta a_{t+1}+a_{n-1}+b_{n-1}\hat{s}_{t+1}}\right)\right]. \tag{189}$$

Under the simplifying assumption that $\Delta c_{t+1} \approx \Delta a_{t+1}$ and using the affine guesses $w_t = w + \psi_{ws}\hat{s}_t$ and $d_t = d + \psi_{ds}\hat{s}_t$, we can approximate the solution to equations (187), (188), and (189) as

$$w + \psi_{ws}\hat{s}_t = \frac{1}{1-\rho}\log\left(1-\beta+\delta\right) + \frac{\delta}{1-\beta+\delta}\psi_{ds}\hat{s}_t,$$

$$\begin{aligned} d + \psi_{ds}\hat{s}_t &= E_t w_{t+1} + g_a + \frac{1}{1-\alpha}\log\left\{\mathbb{E}_t\left[e^{(1-\alpha)(\psi_{ws}\sigma_s\sqrt{s_t}\varepsilon_{st+1}+\sigma_a\varepsilon_{a,t+1})-\theta j_{t+1}}\right]\right\} \\ &= w + \psi_{ws}\rho_s\hat{s}_t + g_a + \frac{1-\alpha}{2}(\sigma_a^2 + \psi_{ws}^2\sigma_s^2 s_t) + \frac{e^{(\alpha-1)\theta}-1}{1-\alpha}(s+\hat{s}_t), \end{aligned}$$

and

$$\begin{aligned} a_n + b_n\hat{s}_t &= \log(\beta) - (\alpha-\rho)(E_t w_{t+1} - d_t) + (1-\alpha)g_a + a_{n-1} + b_{n-1}E_t\hat{s}_{t+1} \\ &\quad + \log\left(\mathbb{E}_t\left\{\exp\left[(b_{n-1} - (\alpha-\rho)\psi_{ws})\sigma_s\sqrt{s_t}\varepsilon_{st+1} + (1-\alpha)(\sigma_a\varepsilon_{a,t+1}) - \theta j_{t+1})\right]\right\}\right) \\ &= \log(\beta) + (1-\alpha)g_a - (\alpha-\rho)(w-d) + a_{n-1} - (\alpha-\rho)(\psi_{ws}\rho_s - \psi_{ds})\hat{s}_t + b_{n-1}\rho_s\hat{s}_t \\ &\quad + \frac{(1-\alpha)^2}{2}\sigma_a^2 + \frac{\left[(\rho-\alpha)\psi_{ws} + b_{n-1}\right]^2\sigma_s^2}{2}(s+\hat{s}_t) + [e^{(\alpha-1)\theta}-1](s+\hat{s}_t), \end{aligned}$$

where we used the log of the moment generating function in (186) to characterize the expectations. Hence, by matching coefficients, we obtain

$$\psi_{ws} = \frac{\delta}{1-\beta+\delta}\psi_{ds} = \frac{\delta}{1-\beta+\delta(1-\rho_s)}\left[\frac{e^{(\alpha-1)\theta}-1}{1-\alpha} + \frac{(1-\alpha)}{2}\psi_{ws}^2\phi_s^2\sigma^2\right],$$

$$\psi_{ds} = \psi_{ws}\rho_s + \frac{e^{(\alpha-1)\theta}-1}{1-\alpha} + \frac{1-\alpha}{2}\psi_{ws}^2\phi_s^2\sigma^2 = \frac{1-\beta+\delta}{1-\beta+\delta(1-\rho_s)}\left(\frac{e^{(\alpha-1)\theta}-1}{1-\alpha} + \frac{1-\alpha}{2}\psi_{ws}^2\phi_s^2\sigma^2\right),$$

$$w = \frac{1}{1-\rho}\log\left(1-\beta+\delta\right),$$

and

$$d = w + g_a + \frac{1-\alpha}{2}(1+\psi_{ws}^2\phi_s^2 s)\sigma^2 + \frac{e^{(\alpha-1)\theta}-1}{1-\alpha}s.$$

Also,

$$\begin{aligned} a_n &= a_{n-1} + \log(\beta) + (1-\alpha)g_a - (\alpha-\rho)\left(w-d\right) + \frac{(1-\alpha)^2}{2}\sigma^2 \\ &\quad + \frac{\left[(\rho-\alpha)\psi_{ws} + b_{n-1}\right]^2\phi_s^2 s}{2}\sigma^2 + \left[e^{(\alpha-1)\theta}-1\right]s \end{aligned}$$

and

$$\begin{aligned} b_n &= b_{n-1}\rho_s + (\alpha-\rho)\left[\frac{e^{(\alpha-1)\theta}-1}{1-\alpha} + \frac{1-\alpha}{2}\psi_{ws}^2\phi_s^2\sigma^2\right] + \frac{\left[(\rho-\alpha)\psi_{ws} + b_{n-1}\right]^2\phi_s^2}{2}\sigma^2 + e^{(\alpha-1)\theta} - 1 \\ &= b_{n-1}\rho_s - (1-\rho)\left[\frac{e^{(\alpha-1)\theta}-1}{\alpha-1}\right] + \frac{(\alpha-\rho)(1-\alpha)}{2}\psi_{ws}^2\phi_s^2\sigma^2 + \frac{\left[(\rho-\gamma)\psi_{ws} + b_{n-1}\right]^2\phi_s^2}{2}\sigma^2. \end{aligned}$$

This concludes the proof. □

C.2.4 Affine Stochastic Discount Factor

We begin with the risk-adjusted log-linear approximation to strips around the risky steady state.

Lemma 2d (Price of Productivity Claims for the Affine Stochastic Discount Factor). *The price of a claim to productivity in n periods approximately satisfies*

$$\log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n s_t,$$

with $a_0 = b_0 = 0$,

$$a_n = a_{n-1} + g_a - \mu_0 + (1 + b_{n-1})^2 \frac{\sigma_a^2}{2} - \gamma_0(1 + b_{n-1})\sigma_a^2,$$

and

$$b_n = (\gamma_1 \sigma_a^2 + \mu_1)\left[\frac{1 - (\rho_s + \gamma_1 \sigma_a^2)^n}{1 - (\rho_s + \gamma_1 \sigma_a^2)}\right].$$

Next, by taking a first-order approximation to the equation in Proposition 3,

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right) \log\left[\sum_{n=0}^{\infty}(c_\ell \delta_\ell^n + c_s \delta_s^n)\frac{P_{nt}}{A_t}\right],$$

and using the approximate solution for productivity strips in Lemma 2d, the following proposition is immediate.

Proposition 4d (Sufficient Statistic for Job-Finding Rate Volatility for the Affine Stochastic Discount Factor). *Under the approximation in Lemma 2d, the response of the job-finding rate to a change in s_t evaluated at a risky steady state is given by*

$$\frac{d\log(\lambda_{wt})}{ds_t} = \left(\frac{1-\eta}{\eta}\right)\sum_{n=0}^{\infty}\omega_n b_n \text{ with } \omega_n = \frac{e^{a_n}(c_\ell \delta_\ell^n + c_s \delta_s^n)}{\sum_{n=0}^{\infty} e^{a_n}(c_\ell \delta_\ell^n + c_s \delta_s^n)}, \tag{190}$$

where a_n and b_n are given in Lemma 2d and the standard deviation of the job-finding rate $\sigma(\lambda_{wt})$ satisfies

$$\sigma(\lambda_{wt}) = \frac{d\log(\lambda_{wt})}{ds_t}\sigma(s_t). \tag{191}$$

We now turn to the proof of Lemma 2d. The proof is as follows. The affine model implies that the price in period t of a claim to productivity growth A_{t+n}/A_t in n periods is

$$\mathbb{E}_t\left(Q_{t,t+n}\frac{A_{t+n}}{A_t}\right) = e^{a_n + b_n s_t}. \tag{192}$$

By the pricing equation (72), we have

$$
\begin{aligned}
a_n + b_n s_t &= \log\left\{\mathbb{E}_t\left[e^{q_{t,t+n}+\Delta a_{t+1}+a_{n-1}+b_{n-1}\hat{s}_{t+1}}\right]\right\} \\
&= \log\left(\mathbb{E}_t\left\{e^{[-(\mu_0-\mu_1 s_t)-\frac{1}{2}(\gamma_0-\gamma_1 s_t)^2\sigma_a^2-(\gamma_0-\gamma_1 s_t)\sigma_a\varepsilon_{at+1}]+\Delta a_{t+1}+a_{n-1}+b_{n-1}\rho_s s_t+b_{n-1}\sigma_a\varepsilon_{at+1}}\right\}\right) \\
&= \log\left\{e^{-(\mu_0-\mu_1 s_t)-\frac{1}{2}(\gamma_0-\gamma_1 s_t)^2\sigma_a^2+a_{n-1}+b_{n-1}\rho_s s_t}\mathbb{E}_t\left[e^{-(\gamma_0-\gamma_1 s_t)\sigma_a\varepsilon_{at+1}+\Delta a_{t+1}+a_{n-1}+b_{n-1}\sigma_a\varepsilon_{at+1}}\right]\right\} \\
&= -\mu_0+\mu_1 s_t+g_a+a_{n-1}+b_{n-1}\rho_s s_t+(1+b_{n-1})^2\frac{\sigma_a^2}{2}-(1+b_{n-1})(\gamma_0-\gamma_1 s_t)\sigma_a^2, \qquad (193)
\end{aligned}
$$

where in the first line we used that $q_{t,t+n} = \log(Q_{t,t+1})$ and in the second line we used that

$$
q_{t,t+n} = -(\mu_0-\mu_1 s_t) - \frac{1}{2}(\gamma_0-\gamma_1 s_t)^2\sigma_a^2 - (\gamma_0-\gamma_1 s_t)\sigma_a\varepsilon_{at+1}.
$$

By matching the constants on both sides of (193), we obtain

$$
a_n = a_{n-1} + g_a - \mu_0 + (1+b_{n-1})^2\frac{\sigma_a^2}{2} - \gamma_0(1+b_{n-1})\sigma_a^2
$$

and by matching the coefficients on s_t on both sides of (193), we obtain

$$
b_n = b_{n-1}(\rho_s+\gamma_1\sigma_a^2) + \gamma_1\sigma_a^2 + \mu_1 = (\gamma_1\sigma_a^2+\mu_1)\left[\frac{1-(\rho_s+\gamma_1\sigma_a^2)^n}{1-(\rho_s+\gamma_1\sigma_a^2)}\right].
$$

This completes the proof of the claim. □

D Analogues of Proposition 2 for Epstein-Zin Preferences

Here we show that our model with Epstein-Zin preferences with variable disaster risk and $\rho = 1$ implies no fluctuations in unemployment. We also show that absent preference shocks, our model with Epstein-Zin preferences with long-run risk and $\rho = 1$ also leads to no fluctuations in unemployment.

Epstein-Zin Preferences with Variable Disaster Risk. Consider our model with Epstein-Zin preferences

$$
V_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}, \qquad (194)
$$

in which the process for productivity growth is given by

$$
\Delta a_{t+1} = g_a + \sigma_a\varepsilon_{at+1} - \theta j_{t+1}, \qquad (195)
$$

where the disaster component j_{t+1} is a Poisson random variable with intensity s_t, which evolves as

$$
s_{t+1} = (1-\rho_s)s + \rho_s s_t + \sqrt{s_t}\sigma_s\varepsilon_{st+1}. \qquad (196)
$$

The pricing kernel is given by

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left[\frac{V_{t+1}}{\left(\mathbb{E}_t V_{t+1}^{1-\alpha} \right)^{\frac{1}{1-\alpha}}} \right]^{\rho-\alpha}. \tag{197}$$

Proposition 2A (Constant Job-Finding Rate Under EZ Preferences with Variable Disaster Risk). *Starting from the steady-state values of the total human capital of employed and unemployed workers, Z_e and Z_u, with preferences of the form in (194), $\rho = 1$, and a process for productivity shocks that follows (195) and (196), both the job-finding rate and unemployment are constant.*

Proof. Here we show that with Epstein-Zin preferences and productivity shocks that follow (195) and (196), both the job-finding rate and unemployment are constant. More generally, the multipliers and allocations satisfy

$$\tilde{\mu}_{ut} = \tilde{\mu}_u, \tilde{\mu}_{et} = \tilde{\mu}_e, \theta_t = \theta, \tilde{C}_t = \tilde{C}, Z_{et} = Z_e, \text{ and } Z_{ut} = Z_u, \tag{198}$$

where variables with ˜ are scaled by productivity. We do so by showing that the equations characterizing the solution to the planning problem, namely, (26)-(28) along with that problem's constraints, admit a solution of the form just described. Note that Epstein-Zin preferences with $\rho = 1$ reduce to

$$v_t = (1-\beta)c_t + \frac{\beta}{1-\alpha} \log \left(\mathbb{E}_t \left\{ \exp\left[(1-\alpha) v_{t+1} \right] \right\} \right). \tag{199}$$

Now, let us simplify the term in the exponential function and use the notation $\tilde{v}_{t+1} = v_{t+1} - a_{t+1}, \tilde{c}_t = c_t - a_t$, and $e_{at+1} = \Delta a_{t+1} - \mathbb{E}_t \Delta a_{t+1}$. Then,

$$\begin{aligned} \exp\left[(1-\alpha)v_{t+1}\right] &= \exp\left[(1-\alpha)(v_{t+1} - a_{t+1} + a_{t+1} - \mathbb{E}_t a_{t+1} + \mathbb{E}_t a_{t+1} - a_t + a_t)\right] \\ &= \exp\left[(1-\alpha)(\tilde{v}_{t+1} + \Delta a_{t+1} - \mathbb{E}_t \Delta a_{t+1} + \mathbb{E}_t a_{t+1})\right] \\ &= \exp\left[(1-\alpha)\mathbb{E}_t a_{t+1}\right] \exp\left[(1-\alpha)(\tilde{v}_{t+1} + \Delta a_{t+1} - \mathbb{E}_t \Delta a_{t+1})\right] \\ &= \exp\left[(1-\alpha)\mathbb{E}_t a_{t+1}\right] \exp\left[(1-\alpha)(\tilde{v}_{t+1} + e_{at+1})\right]. \end{aligned} \tag{200}$$

Using $c_t = \tilde{c}_t + a_t$, and $\log\left(\mathbb{E}_t \left\{ \exp\left[(1-\alpha)\mathbb{E}_t a_{t+1}\right] \right\}\right) = (1-\alpha)\mathbb{E}_t a_{t+1}$, (199) can be written as

$$\begin{aligned} v_t &= (1-\beta)\tilde{c}_t + (1-\beta)a_t + \beta \mathbb{E}_t a_{t+1} + \frac{\beta}{1-\alpha} \log\left(\mathbb{E}_t \left\{ \exp\left[(1-\alpha)(\tilde{v}_{t+1} + e_{at+1})\right] \right\}\right) \\ &= (1-\beta)\tilde{c}_t + a_t + \beta \mathbb{E}_t \Delta a_{t+1} + \frac{\beta}{1-\alpha} \log\left(\mathbb{E}_t \left\{ \exp\left[(1-\alpha)(\tilde{v}_{t+1} + e_{at+1})\right] \right\}\right). \end{aligned}$$

Recall from (170) that the stochastic discount factor for Epstein-Zin preferences under $\rho = 1$ is

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left[\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha} \right)^{\frac{1}{1-\alpha}}} \right]^{1-\alpha},$$

which we can rewrite as

$$
\begin{aligned}
Q_{t,t+1} &= \beta \exp -\Delta c_{t+1} + (1-\alpha)v_{t+1} - \log\left(\mathbb{E}_t \left\{\exp\left[(1-\alpha)v_{t+1}\right]\right\}\right) \\
&= \beta \exp -\Delta \tilde{c}_{t+1} - \Delta a_{t+1} + (1-\alpha)(\tilde{v}_{t+1} + \Delta a_{t+1}) - \log\left(\mathbb{E}_t \left\{\exp\left[(1-\alpha)(\tilde{v}_{t+1} + \Delta a_{t+1})\right]\right\}\right) \\
&= \beta \exp -\Delta \tilde{c}_{t+1} - \Delta a_{t+1} + (1-\alpha)(\tilde{v}_{t+1} + e_{at+1}) - \log\left(\mathbb{E}_t \left\{\exp\left[(1-\alpha)(\tilde{v}_{t+1} + e_{at+1})\right]\right\}\right),
\end{aligned}
$$

where to obtain the second equality we used $\tilde{v}_{t+1} = v_{t+1} - a_{t+1}$ and added and subtracted a_t, and to obtain the third equality we added and subtracted $\mathbb{E}_t \Delta a_{t+1}$.

We now show that this system admits a solution in which $\tilde{\mu}_{ut}$, $\tilde{\mu}_{et}$, θ_t, $\tilde{C}_t$, Z_{et}, and Z_{ut} are constant. First note that, under our conjectured solution,

$$
\mathbb{E}_t\left(Q_{t,t+1}e^{\Delta a_{t+1}}\right) = \beta \mathbb{E}_t\left[\exp\left(-\Delta \tilde{c}_{t+1} + (1-\alpha)(\tilde{v}_{t+1} + e_{at+1}) - \log\left\{\mathbb{E}_t\left[e^{(1-\alpha)(\tilde{v}_{t+1}+e_{at+1})}\right]\right\}\right)\right]
$$

reduces to

$$
\mathbb{E}_t\left(Q_{t,t+1}e^{\Delta a_{t+1}}\right) = \beta \mathbb{E}_t\left[\exp\left((1-\alpha)(\tilde{v}_{t+1} + e_{at+1}) - \log\left\{\mathbb{E}_t\left[e^{(1-\alpha)(\tilde{v}_{t+1}+e_{at+1})}\right]\right\}\right)\right] = \beta.
$$

So under this conjectured solution,

$$
\tilde{\mu}_{ut} = b + \phi(1+g_u)\mathbb{E}_t\left(Q_{t,t+1}e^{\Delta a_{t+1}}\left\{\lambda_w(\theta)\eta(\theta)\tilde{\mu}_{et+1} + [1-\eta(\theta_{t+1})\lambda_w(\theta_{t+1})]\tilde{\mu}_{ut+1}\right\}\right)
$$

reduces to

$$
\tilde{\mu}_u = b + \phi(1+g_u)\beta\left[\lambda_w \eta \tilde{\mu}_e + (1-\eta\lambda_w)\tilde{\mu}_u\right]
$$

and

$$
\tilde{\mu}_{et} = 1 + \phi(1+g_e)\mathbb{E}_t\left\{Q_{t,t+1}e^{\Delta a_{t+1}}\left[(1-\sigma)\tilde{\mu}_{et+1} + \sigma\tilde{\mu}_{ut+1}\right]\right\}
$$

reduces to

$$
\mu_e = 1 + \phi(1+g_e)\beta\left[(1-\sigma)\tilde{\mu}_e + \sigma\tilde{\mu}_u\right],
$$

which, by the free-entry condition, implies that θ_t is constant. It follows that the human capital stocks

$$
Z_e = \phi(1+g_e)(1-\sigma)Z_e + \phi(1+g_u)\lambda_w Z_u \text{ and } Z_u = 1 - \phi + \phi(1+g_e)\sigma Z_e + \phi(1+g_u)(1-\lambda_w)Z_u
$$

are also constant, where we have assumed that the initial conditions for Z_{et} and Z_{ut} are equal to the posited constants Z_e and Z_u. Hence, aggregate consumption is constant $\tilde{C} = Z_e + bZ_u - \kappa\phi(1+g_u)Z_u$. This verifies the conjectured constant solution. □

Epstein-Zin Preferences with Long-Run Risk and no Preference Shocks. Consider our model with Epstein-Zin preferences and long-run risk, but without discount rate shocks. In particular, preferences are now given by

$$
V_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}. \tag{201}
$$

Productivity growth now has a long-run risk component x_t in that

$$\Delta a_{t+1} = g_a + x_t + \sigma_a \varepsilon_{at+1} \text{ and } x_{t+1} = \rho_x x_t + \phi_x \sigma_a \varepsilon_{xt+1}, \tag{202}$$

where the shocks ε_{at} and ε_{xt} are standard normal i.i.d. and orthogonal to each other. The pricing kernel is

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left[\frac{V_{t+1}}{\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}}}\right]^{\rho-\alpha}.$$

Proposition 2B (Constant Job-Finding Rate Under EZ Preferences with Long-Run Risk). *Starting from the steady-state values of the total human capital of employed and unemployed workers, Z_e and Z_u, with preferences of the form (201), $\rho = 1$, and a process for productivity shocks that follows (202), both the job-finding rate and unemployment are constant.*

Proof. The proof is identical to the proof of Proposition 2A. Simply note that $e_{at+1} = \Delta a_{t+1} - E_t \Delta a_{t+1}$ equals $\sigma_a \varepsilon_{at+1} - \theta j_{t+1}$ in Proposition 2A and it equals $\sigma_a \varepsilon_{at+1}$ in Proposition 2B, but that distinction is inconsequential for the proof. □

E Numerical Solution of the Model

Here we describe the global algorithm that we employ to solve the model. We adopt a global numerical strategy because asset prices are highly nonlinear under the asset-pricing preferences we consider. (Petrosky-Nadeau, Zhang, and Kuehn 2018 highlight the importance of the model's nonlinearities when search frictions are present, even in the absence of risk-sensitive preferences.) Specifically, we solve the model by projecting the global solution of our model onto the space spanned by a basis of high-order Chebyshev polynomials and evaluate expectations by a multidimensional Gauss-Hermite quadrature with a sufficiently large number of nodes so that results are not sensitive to any small increase or decrease in the number of nodes. We turn to providing a few details for each of our models.

Baseline Preferences. Recall that the sensitivity function is given by

$$\lambda_a(s_t) = \frac{1}{S}\left[1 - 2\left(s_t - s\right)\right]^{1/2} - 1. \tag{203}$$

Rather than using the surplus consumption $s_t - s$ as the state, it is convenient to use the term $g(s_t) \equiv \left[1 - 2\left(s_t - s\right)\right]^{1/2}$ in (203) as the state. Indeed, doing so allows for an accurate solution for a smaller Chebyshev polynomial order rather than simply using the surplus consumption state. Intuitively, the function $g(s_t)$, which appears in the volatility of the surplus consumption process, is hard to approximate with a polynomial in $s_t - s$ due to its kink at $s_t - s = 1/2$. It is therefore more appropriate to use $g(s_t)$ as the state, from which we can recover $s_t - s$ using powers of the transformed state. We use Chebyshev polynomials of degree twenty for the transformation $g(s_t)$ of the surplus consumption state $s_t - s$ and of degree five for the human capital states Z_{et} and Z_{ut}.

As shown by Wachter (2005), the best practice in solving models with Campbell-Cochrane preferences is to consider a large and fine grid over the surplus consumption space that, importantly, places many grid points close to zero. Accordingly, we construct a grid for the state $g(s_t)$ that ranges from

0 (the minimum value possible) to 8.4, reflecting a minimum value of S of 10^{-16} and so very close to zero. We chose the maximum value by progressively widening the grid until results no longer change. For the human capital states, we adopt instead an adaptive grid, namely, a grid that covers minimum and maximum values of long simulations of the variables from the solved model.

After having set the grid $\mathbb{S}_t$ and the basis functions, our algorithm proceeds as follows. Let $\mathbb{S}_{ht}$, $h = 1, \ldots, H$, denote the h-th element of the grid, which spans the space of state $(g(s_t), Z_{et}, Z_{ut})$. We start from a constant guess for the functional $\theta_t = \theta^{(0)}(\mathbb{S}_t)$ that maps elements of the state space into values for market tightness. We then use that guess and the h nodes for tomorrow's productivity shocks used in the Gauss-Hermite quadrature to construct tomorrow's values of the state $\mathbb{S}_{h,t+1}(j)$, for $j = 1, \ldots, J$ and $h = 1, \ldots, H$, using the laws of motion of the state variables. We can then evaluate tomorrow's values for θ_{t+1} and hence compute μ_{et} and μ_{ut} using (26) and (27). With those values in hand, we can use (28) to compute a new value for the functional $\theta^{(1)}(\mathbb{S}_t)$ for each grid point. We repeat this algorithm to calculate the n-th functional $\theta^{(n)}(\mathbb{S}_t)$ for all grid points. The algorithm stops when their maximum differences are small, technically, when $\|\theta^{(n)}(\mathbb{S}_t) - \theta^{(n-1)}(\mathbb{S}_t)\|_\infty < 10^{-6}$, where $\|\cdot\|_\infty$ denotes the sup norm.

Campbell and Cochrane with External Habit. The global routine for preferences with external habit is the same as for our baseline preferences with one modification. As before, we approximate θ_t by Chebyshev polynomials in the states $(g(s_t), Z_{et}, Z_{ut})$. However, for Campbell-Cochrane preferences with external habit, s_t is an endogenous variable in that the innovations to it are endogenous, since they depend on the process for consumption. In particular, to compute s_{t+1}, we need to determine $\varepsilon_{ct+1} \equiv \Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1}$, which is a different function of $(g(s_t), Z_{et}, Z_{ut})$ for every realization of ε_{at+1}. We allow for this dependence by specifying a different Chebyshev polynomial in the states $(g(s_t), Z_{et}, Z_{ut})$ for each realization of ε_{at+1}. That is, we set a 100-point grid for productivity shocks ε_{at+1}, and for each value of ε_{at+1} on this grid, we approximate $s_{t+1}(\varepsilon_{at+1})$ by Chebyshev polynomials in the states $(g(s_t), Z_{et}, Z_{ut})$. Letting j denote a grid point for ε_{at+1} and $\mathbb{S}_t$ denote the grid for $(g(s_t), Z_{et}, Z_{ut})$, our algorithm starts from guesses for the functionals $\theta_t = \theta^{(0)}(\mathbb{S}_t)$ and $s_{t+1} = s^{(0)}(\mathbb{S}_t, j)$ for each grid point $j = 1, \ldots, 100$ for ε_{at+1} and constructs $\theta^{(n)}(\mathbb{S}_t)$ and $s^{(n)}(\mathbb{S}_t, j)$. The algorithm stops when both $\theta^{(n)}$ and $s^{(n)}$ converge. (Technically, when under the sup norm, $\|(\theta^{(n)}(\mathbb{S}_t) - \theta^{(n-1)}(\mathbb{S}_t), s^{(n)}(\mathbb{S}_t, j) - s^{(n-1)}(\mathbb{S}_t, j))\|_\infty < 10^{-6}$.)

Baseline Preferences and Physical Capital. The degree of accuracy for surplus consumption and the human capital states is the same as for the baseline model without physical capital. We span the additional dimension of the state space, K_t/A_t, by a Chebyshev-polynomial of order five and an adaptive grid. In solving the model, we iterate also over the functional for the investment-capital ratio $I_t/K_t = IK(\mathbb{S}_t)$ using the optimality condition for investment.

Epstein-Zin Preferences with Long-Run Risk. We use Chebyshev polynomials of degree five and an adaptive grid in all four dimensions of the state space $(x_t, s_t, Z_{et}, Z_{ut})$, which consists of the persistence component of the productivity process x_t, the discount-rate shock s_t, and the human capital stocks of employed and unemployed workers, Z_{et} and Z_{ut}. After having set the grid $\mathbb{S}_t$ and the basis functions, our algorithm proceeds as follows. Let $\mathbb{S}_{ht}$, $h = 1, \ldots, H$ denote the h-th element of the grid, which spans the space of the state, $(x_t, s_t, Z_{et}, Z_{ut})$. We start from a constant guess for the functionals for the market tightness $\theta_t = \theta^{(0)}(\mathbb{S}_t)$ on and the detrended value function $w_t = w^{(0)}(\mathbb{S}_t)$ where $w_t = \log(W_t) \equiv \log[V_t/(S_t^{1/(1-\rho)} C_t)]$ on the grid $\mathbb{S}_t$. We then use that guess and the h nodes for tomorrow's shocks for the Gauss-Hermite quadrature to construct tomorrow's values of the state $\mathbb{S}_{h,t+1}(j)$ for $j = 1, \ldots, J$ and $h = 1, \ldots, H$, using the laws of motion of the state variables. We can then evaluate tomorrow's values for θ_{t+1} and w_{t+1} and hence compute μ_{et} and μ_{ut} using (26) and (27) and d_t using (158). Once we have those values, we can use (28) and (157) to compute a new value for the functionals $\theta^{(1)}(\mathbb{S}_t)$ and $w^{(1)}(\mathbb{S}_t)$ for each grid point. We repeat this algorithm to calculate the n-th functionals $\theta^{(n)}(\mathbb{S}_t)$ and $w^{(n)}(\mathbb{S}_t)$ for all grid points. The algorithm stops when $\|(\theta^{(n)}(\mathbb{S}_t) - \theta^{(n-1)}(\mathbb{S}_t), w^{(n)}(\mathbb{S}_t) - w^{(n-1)}(\mathbb{S}_t))\|_\infty < 10^{-6}$.

Epstein-Zin Preferences with Variable Disaster Risk. We use Chebyshev polynomials of degree five and an adaptive grid in all three dimensions of the state space, which consists of the time-varying disaster intensity s_t and the stocks of human capital of employed and unemployed workers, Z_{et} and Z_{ut}. The algorithm is analogous to the one for Epstein-Zin preferences with long-run risk, except that the state space is now three-dimensional.

Affine Discount Factor Model. We use Chebyshev polynomials of degree twenty in the s-dimension and of degree five for the stocks of human capital of employed and unemployed workers, Z_{et} and Z_{ut}, over an adaptive grid. The algorithm is analogous to the one for Campbell-Cochrane preferences with exogenous habit.

F A More General Human Capital Process

So far, we have considered a simple process of human capital accumulation such that human capital grows at a constant rate when a consumer is employed and decays at a constant rate when a consumer is unemployed. In the data, though, wage growth tends to decline as experience in the labor market accumulates. To accommodate this feature of the data, we consider a more general human capital process as in Kehoe, Midrigan, and Pastorino (2019) in the spirit of that in Ljungqvist and Sargent (1998, 2008), whereby human capital z_t evolves according to the autoregressive process

$$\log(z_{t+1}) = (1 - \rho_z)\log(\bar{z}_e) + \rho_z \log(z_t) + \sigma_z \varepsilon_{zt+1} \tag{204}$$

when a consumer is employed, whereas it evolves according to

$$\log(z_{t+1}) = (1 - \rho_z)\log(\bar{z}_u) + \rho_z \log(z_t) + \sigma_z \varepsilon_{zt+1} \tag{205}$$

when a consumer is unemployed, where ε_{zt+1} is a standard Normal random variable. Newborn consumers start as unemployed with general human capital z, where $\log(z)$ is drawn from the normal distribution $N(\log(\bar{z}_u), \sigma_z^2/(1-\rho_z^2))$. We assume that $\bar{z}_u < \bar{z}_e$ so that when a consumer is employed, on average, human capital z_t drifts up toward a high level of productivity $\bar{z}_e$ from the low average level of productivity $\bar{z}_u$ of newborn consumers. Analogously, when a consumer is unemployed, on average, human capital z_t depreciates and hence drifts down toward a low level of productivity, $\bar{z}_u$, which we normalize to one so that $\log(\bar{z}_u) = 0$. The parameter ρ_z governs the rate at which human capital converges toward $\bar{z}_e$ when a consumer is employed and toward $\bar{z}_u$ when a consumer is unemployed. Hence, the higher ρ_z is, the slower human capital accumulates during employment, the slower it depreciates during unemployment, and the slower wages grow with experience. Incorporating idiosyncratic shocks ε_{zt+1} allows the model to reproduce the dispersion in wage growth rates observed in the data. (See Rubinstein and Weiss 2006.)

A consumer with human capital z_t produces $A_t z_t$ when employed but, in contrast to our baseline model, bA_t when unemployed. Also in contrast to our baseline model, we assume that a firm incurs the cost κA_t to recruit a consumer with any level of human capital. (Recall that the earlier scaling of home production and the cost of posting vacancies by z_t was purely motivated by analytical convenience to allow the model to aggregate.) To ensure that the job-finding rate $\lambda_{wt}(z)$ lies between zero and one, we assume that the matching function is $m_t(u_{bt}(z), v_t(z)) = \min\{u_{bt}(z), Bu_{bt}(z)^\eta v_t(z)^{1-\eta}\}$. A competitive search equilibrium is defined as before with the free-entry condition for market z now given by

$$\kappa A_t \geq \lambda_{ft}(\theta_t(z))[Y_t(z) - W_t(z)], \tag{206}$$

with equality if vacancies are created in an active market z in that the measure of vacancies $v_t(z)$ is strictly positive. Here we focus on our baseline preferences with exogenous habit.

We parametrize the model as before with few modifications. With $\bar{z}_u$ normalized to one, the parameters of the human capital process are $\bar{z}_e$, ρ_z, and σ_z. We target a net annual wage growth over the first 10 years in the labor market of 5.5%, based on the estimates by Rubinstein and Weiss (2006) discussed in the paper, and a difference in the log real wages between workers with 30 years of experience and those with 1 year of experience of 1.2, based on the estimates by Elsby and Shapiro (2012) also discussed in the paper. These two targets help pin down ρ_z and $\bar{z}_e$. We choose σ_z to match the standard deviation of annual wage growth for workers with up to 10 years of labor market experience, which is 1.2 percentage points according to the estimates by Rubinstein and Weiss (2006) from the National Longitudinal Survey of Youth (NLSY).

Since, unlike our baseline model, this version of the model is not amenable to aggregation, we need to record the measures of human capital among employed and unemployed workers, $(e_t(z), u_t(z))$, as part of the endogenous state of the economy. This feature makes the model much more difficult to solve numerically than our baseline model. For this reason, we use a variant of the algorithm by Krusell and Smith (1996) that, unlike in typical applications such as those in Winberry (2018), needs to accurately capture time-varying risk in aggregate variables.

Notwithstanding this complexity, this version of the model too successfully solves the unemployment volatility puzzle. In particular, the model produces only a slightly lower volatility for the job-finding rate and unemployment than in the data, respectively, 6.38 versus 6.66 and 0.65 versus 0.75. In this sense, our results based on a simple model of human capital accumulation are robust to extensions that capture additional features of the micro data on returns to labor market experience.

G Alternating Offer Bargaining

In the introduction of the paper, we claim that under alternating offer bargaining, wage setting can be efficient. Here we show that this is the case when two conditions hold: the exogenous rate of breakdown of bargaining between workers and firms converges to one and the probability that a worker makes the first offer equals the elasticity of the matching function with respect to the measure of unemployed workers. We now briefly lay out the alternating offer bargaining game of Hall (2017) and then prove our claim.

Using our notation, the formulae for the resource constraint, the post-match value W_{pt}, the unemployment value U_t, the present value of output in a match Y_t, the value of a vacancy V_t, and the free-entry condition in the alternating offer bargaining equilibrium are identical to those in the competitive search equilibrium, but without human capital accumulation in that $g_e = g_u = 0$ and $z = 1$ for all consumers. The only two remaining differences between Hall's model and our model is that wages in Hall's model are set in an imperfectly competitive rather than a competitive way and the stochastic discount factor $Q_{t,t+1}$ is an exogenous rather than an endogenous one.

The game can be described as follows. The worker makes the first wage offer with probability p and the firm makes the first wage offer with probability $1 - p$. In each subsequent period, firms and workers deterministically alternate making offers each period, if bargaining has not broken down, until an offer is accepted. If period t is one in which the firm makes the offer, we denote the offer by W_{ft}, whereas if period t is one in which the worker makes the offer, we denote it by W_{wt}—these offers are contingent on the exogenous state ε^t, but we have suppressed their explicit dependence on ε^t for simplicity. In each period, with probability δ bargaining exogenously breaks down, in which case the firm returns to the market with an unfilled vacancy and the worker enters unemployment. When the firm offers W_{ft} in period t, then the worker can either accept it, reject it and make a counteroffer W_{wt+1} in period $t + 1$ if bargaining does not exogenously break down, or abandon negotiations and immediately return to unemployment. The firm has symmetric options if it is the worker's turn to make an offer. The cost of bargaining to the worker is that in each period of bargaining, the worker only receives the value of home production bA_t rather than a wage, so the implicit delay cost is the difference between foregone wages and home production. The cost of bargaining to the firm is the cost ψA_t of making a counteroffer to the worker at t; we refer to ψ as the *haggling cost*. Thus, the three parameters that characterize this bargaining scheme are (p, δ, ψ).

As explained in Hall and Milgrom (2008), standard logic implies that the firm will make the best possible offer from its perspective so that the worker will prefer to accept it rather than to make a counteroffer, in the event of no exogenous breakdown, or to abandon negotiations. Thus, the firm's offer W_{ft} satisfies

$$W_{ft} + W_{pt} = \max\left\{bA_t + \phi(1-\delta)\mathbb{E}_t\left[Q_{t,t+1}(W_{wt+1} + W_{pt+1})\right] + \phi\delta\mathbb{E}_t\left(Q_{t,t+1}U_{t+1}\right), U_t\right\}, \tag{207}$$

where the maximum ensures that the worker does not strictly prefer unemployment today to accepting such an offer. Of course, the firm's offer W_{ft} must be smaller than the discounted value of output from the match with the worker, Y_t, or else the firm would prefer to stay idle. Thus, $W_{ft} \leq Y_t$. In turn, the worker will make the best possible offer from the worker's perspective so that the firm will prefer to accept it rather than to make a counteroffer, in the event of no exogenous breakdown, or to abandon negotiations. Therefore, the worker's offer satisfies

$$Y_t - W_{wt} = \max\{-\psi A_t + \phi(1-\delta)\mathbb{E}_t[Q_{t,t+1}(Y_{t+1} - W_{ft+1})], 0\}, \tag{208}$$

where the maximum ensures that the firm does not strictly prefer to abandon negotiations rather than to accept the offer. Clearly, the worker will only make offers such that employment is preferable to unemployment, that is, $W_{wt} + W_{pt} \geq U_t$ must hold—recall that W_{pt} is a worker's post-match value. Since a family consists of a large number of consumers who are independently drawn to make the first offer in bargaining, the value to a family of the wages of all its consumers who are bargaining at t is $W_{mt} = pW_{wt} + (1-p)W_{ft}$. Likewise, W_{mt} is the value to the firm of the present value of wages from bargaining.

G.1 Efficiency

We now show that when the duration of bargaining is short, allocations are close to efficient and thus close to the competitive search ones, but when the duration is long, allocations are very inefficient.

Proposition A.1. (Efficiency of Alternating Offer Bargaining). *When the probability p that the worker makes the first offer equals the elasticity of the matching function with respect to the measure of unemployed workers, then the allocations in a sequence of bargaining games indexed by the breakdown probabilities $\{\delta_n\}_{n=1}^{\infty}$ converge to the constrained efficient allocations as δ_n converges to one.*

Recall that an allocation is efficient if it solves the planning problem in the paper. Proposition A.1 directly applies to the model in Hall (2017) with an exogenous discount factor. It also applies to our model if we modify the equilibrium concept from that of competitive search equilibrium to that of alternating offer bargaining equilibrium.

Proof. For each $\delta(n)$ we have

$$W_{ft}(n) + W_{pt}(n) = \max \left\{ \begin{array}{c} bA_t + [1-\delta(n)]\phi \mathbb{E}_t \left\{ Q_{t,t+1}(n)[W_{wt+1}(n) + W_{pt+1}(n)] \right\} \\ +\delta(n)\phi \mathbb{E}_t \left[Q_{t,t+1}(n) U_{t+1}(n) \right], U_t(n) \end{array} \right\} \tag{209}$$

and

$$Y_t - W_{wt}(n) = \max\{-\psi A_t + \phi[1-\delta(n)]\mathbb{E}_t Q_{t,t+1}(n)(Y_{t+1} - W_{ft+1}(n)), 0\}. \tag{210}$$

Clearly, all of these sequences are continuous in n. Taking the limit of both sides of these equations in n yields

$$W_{ft} + W_{pt} = U_t \tag{211}$$

and

$$Y_t - W_{wt} = 0, \tag{212}$$

where in (211) we have used that for $\delta(n)$ sufficiently close to 1, the two terms in the maximum converge whereas in (212) the first term in the maximum is strictly negative. By continuity, the participation constraints $W_{ft} \leq Y_t$ and $W_{wt} + W_{pt} \geq U_t$ also clearly hold. Hence, substituting for W_{wt} and W_{ft} from (211) and (212) into $W_{mt} = pW_{wt} + (1-p)W_{ft}$ and using that $p = \eta$, we obtain

$$W_{mt} = \eta \left(U_t - W_{pt}\right) + (1-\eta)Y_t. \tag{213}$$

Adding $W_{pt} - U_t$ to both sides and collecting terms gives that

$$W_{mt} + W_{pt} - U_t = (1-\eta)(Y_t + W_{pt} - U_t), \tag{214}$$

that is, a worker receives a share $1-\eta$ of the surplus in that $W_{mt} + W_{pt} - U_t = (1-\eta)X_t$, where the surplus is defined as

$$X_t = (W_{mt} + W_{pt} - U_t) + (Y_t - W_{mt}) = Y_t + W_{pt} - U_t. \tag{215}$$

Hence, the firm's receives a share η of the surplus in that $Y_t - W_{mt} = \eta X_t$. But this surplus sharing rules are precisely the conditions for the efficiency of Nash bargaining when the Hosios condition holds. Hence, the allocations are efficient. □

G.2 The Cyclicality of the User Cost of Labor

Here we show that Hall (2017) generates sizable fluctuations in unemployment only under a parametrization of wage setting that yields very rigid and inefficient wages. It turns out that the critical parameter governing the stickiness of wages in Hall (2017) is the probability of exogenous breakdown of bargaining, δ. It is not easy to interpret this exogenous breakdown probability based on actual bargaining behavior because, in equilibrium, the first offer is accepted regardless of the value of δ. We find it therefore useful to translate δ into units of time by calculating the mean duration of the opportunity to bargain to form a match, if bargaining continues until it exogenously breaks down. Correspondingly, we refer to $1/\delta$ as the duration of a job opportunity during bargaining. It turns out that the longer is the duration of a job opportunity, the stickier are real wages. In Hall's baseline model, this duration is 77 months.

In Table A.6, the third column illustrates the parameters and results in Hall (2017) reproduced from the replication code for Hall (2017). Note that when the duration of a job opportunity is 77 months, the cyclicality of the user cost of labor is 0.1%. That is, after a one percentage point increase in the unemployment rate, the user cost of labor actually slightly *increases*. Recall that Kudlyak (2014) estimates that after a one percentage point increase in the unemployment rate, the user cost of labor *falls* by 5.2%—Basu and House (2016) obtain a similar estimate of 5.8%. In this sense, Hall's model generates an extreme degree of wage rigidity that is at odds with the estimated cyclicality of the user cost of labor.

We now turn to determine the duration of a job opportunity that generates the observed degree of wage cyclicality. As the second column in Table A.6 shows, at $1/\delta = 2.6$ months, the model generates the observed cyclicality of the user cost of labor. With this degree of rigidity, however, the model generates 1/25th of the volatility of unemployment in the data (0.03/0.75). (For this exercise, as we vary the duration of a job opportunity, we adjust the vacancy posting cost in Hall's (2017) replication code to keep the mean unemployment rate unchanged.)

The idea behind Hall's mechanism is simple: in downturns, the user cost of labor does not fall, even though the present value of what a worker will produce over the course of a match greatly falls. Hence, firms greatly contract their vacancies in recessions. Such a mechanism, though, is inconsistent with the evidence on the cyclicality of the user cost of labor.

We have shown that the results in Hall (2017) depend critically on the duration of a job opportunity, $1/\delta$. When this duration is short, the model generate very small fluctuations in unemployment, whereas when it is long, the model generates large fluctuations. Here we further link this key parameter to the efficiency of the resulting allocations: when the duration is short, allocations are close to efficient and thus close to the competitive search ones, but when the duration is long, allocations are very inefficient.

Proposition A.1 offers an additional interpretation of the results in Table A.6, namely, that inefficiencies are central to the amplification mechanism in Hall (2017): the lower is $1/\delta$, the more efficient are the allocations in Hall (2017), the smaller is the impact of changes in the stochastic discount factor on the volatility of the job-finding rate, and so the lower is the volatility of unemployment—by Proposition A.1, allocations are efficient when $\delta = 1$. Indeed, for Hall's model to generate the observed volatility of unemployment, the economy has to be very inefficient in that the duration of a job opportunity has to be 6.2 years rather than one month.

We can shed light on the mechanism in Hall (2017) also by solving for the time-varying Nash bargaining weights of firms and workers that produce the job-finding rates in the alternating offer bargaining equilibrium. Recall that the efficient allocations are achieved under Nash bargaining with a constant bargaining weight equal to η, which equals 1/2 in both Hall's and our parametrizations. In Figure A.3, we plot this time-varying Nash bargaining weight for a worker in Hall's economy. We see that in deep downturns, the worker's bargaining weight *increases* sharply relative to its level in booms. Thus, a key intuition for Hall's mechanism is that firms understand that during downturns workers will demand much larger surplus shares in order to accept a job. Anticipating such behavior, firms drastically cut vacancies and so unemployment drops.

H Comparison with the Differential Productivity Mechanism of Search Models

In the paper, we argue that our mechanism is fundamentally different from those in the large literature discussed by Ljungqvist and Sargent (2017) that addresses the unemployment volatility puzzle. Here we establish this claim. The literature reviewed by Ljungqvist and Sargent (2017) builds in a mechanism of *differential productivity across sectors*. Specifically, it assumes that an increase in productivity leads to an increase in the productivity of working in the market relative to both the productivity of working at home and the cost of posting vacancies. Then, as Shimer (2005, p. 25) explains, "an increase in labor productivity relative to the value of nonmarket activity and to the cost of advertising a job vacancy makes unemployment relatively expensive and vacancies relatively cheap. The market substitutes toward vacancies." That is, in a boom, the differential increase in productivity in the market draws workers out of nonmarket activity and into the market.

In such a literature, authors compute the steady-state response of the job-finding rate and unemployment to a steady-state change in aggregate productivity. We show that our model works differently by proving two results. First, if we perform the same steady-state experiment in our model, we obtain no change in the job-finding rate. Second, once we modify the models in Ljungqvist and Sargent (2017) so that productivity enters those models as it does ours, then in both the basic matching model and the alternating offer bargaining model of Hall and Milgrom (2008), a change in steady-state productivity has similarly no effect on the job-finding rate. We show an analogous result for the training cost model of Pissarides (2009), also reviewed by Ljungqvist and Sargent (2017). (Note that our results are reminiscent of the result on the neutrality of productivity shocks by Shimer 2010. See also a related intuition by Ljungqvist and Sargent 2017 in footnote 28 of their paper, page 2664.)

H.1 Steady-State Change in Aggregate Productivity in Baseline Model

We consider the experiment conducted by Ljungqvist and Sargent (2017) in our model, namely a steady-state increase in A, and obtain the following result. For simplicity, we abstract from growth.

Proposition A.2 (Zero Elasticity of Job-Finding Rate in Baseline Model). *In our baseline model, the steady-state levels of the job-finding rate and unemployment are independent of steady-state productivity, A.*

To see why, note that in the baseline model $Q_{t,t+1} = \beta$ at a steady state where $S_t = S$ and $C_t = C$. Evaluating the expression for the job-finding rate in (120) at a steady state gives $\log(\lambda_w) = \chi + (1-\eta)\log\left((\mu_e - \mu_u)/A\right)/\eta$, where μ_e and μ_e are the steady-state versions of (22) and (23), namely,

$$\frac{\mu_e}{A} = 1 + \phi(1+g_e)\beta\left[(1-\sigma)\left(\frac{\mu_e}{A}\right) + \sigma\left(\frac{\mu_u}{A}\right)\right] \text{ and } \left(\frac{\mu_u}{A}\right) = b + \phi(1+g_u)\beta\left[\eta\lambda_w\left(\frac{\mu_e}{A}\right) + (1-\eta\lambda_w)\left(\frac{\mu_u}{A}\right)\right].$$

Clearly, $(\mu_e - \mu_u)/A$ is independent of A and so is the job-finding rate. Notice that key to this result is that the steady-state value of the discount factor does not vary with the steady-state value of A. Since this same property holds for a broad class of consumption-based discount factors, including all of those considered here, all of these discount factors are consistent with this proposition.

H.2 Basic Matching Model

Consider the *basic matching model* in Ljungqvist and Sargent (2017). Using notation similar to ours, in this model consumers are risk neutral with discount factor β. A consumer produces A units of output when employed and b units of output when unemployed. The cost of posting a vacancy is κ, the exogenous separation rate is σ, the worker's bargaining weight is γ, and the job-filling rate for a firm is $\lambda_f(\theta)$ given market tightness θ. Equation (12) in Ljungqvist and Sargent (2017, p. 2635) shows that the equilibrium value of market tightness is determined by the free-entry condition, which we rearrange and express as

$$\kappa = (1-\gamma)\lambda_f(\theta)\frac{\beta(A-b)}{1-\beta[1-\sigma-\gamma\theta\lambda_f(\theta)]}. \tag{216}$$

These authors then differentiate this equation to derive $d\log(\theta)/d\log(A)$ and explain how their measure of *fundamental surplus* given by $A-b$ is critical for understanding the magnitude of this derivative. In contrast, in our model, the output produced in the market and the cost of posting a vacancy are proportional to productivity so that b and κ are replaced by bA and κA, respectively. Observe that scaling home production b by A is consistent with the findings in Chodorow-Reich and Karabarbounis (2016), as discussed in the paper. Scaling κ by A is consistent with the view in Shimer (2010) that posting vacancies absorbs a fixed amount of workers' time in recruiting that could otherwise be devoted to producing goods. When this is the case, the free-entry condition becomes

$$\kappa A = (1-\gamma)\lambda_f(\theta)\frac{\beta(1-b)A}{1-\beta[1-\sigma-\gamma\theta\lambda_f(\theta)]}. \tag{217}$$

Since A cancels out from both sides of this equality, θ is constant and thus $d\log(\theta)/d\log(A) = 0$.

Proposition A.3 (Zero Elasticity of Job-Finding Rate in Basic Matching Model). *In* *the basic matching model, if home produced output and the cost of posting a vacancy are proportional to productivity, then the change in steady-state unemployment with respect to a change in steady-state productivity is zero regardless of all other parameters.*

Note that this result holds regardless of the size of the home production parameter b, which plays an important role in the debate that originated with Shimer (2005) and Hagedorn and Manovskii (2008). More generally, this property holds independently of the size of the fundamental surplus, which, instead, is central to the analysis in Ljungqvist and Sargent (2017).

H.3 Hall and Milgrom (2008): Alternating Offer Bargaining Model

A similar result also applies to alternating offer bargaining models. Consider the exposition in Ljungqvist and Sargent (2017) of Hall and Milgrom (2008). In this model, firms and workers make alternating offers and after each unsuccessful bargaining round, the firm incurs a haggling cost of ψ of making a new offer while the worker receives b. There is a probability δ that the job opportunity exogenously expires across bargaining rounds and the worker reenters unemployment. Ljungqvist and Sargent (2017) assume that $\delta = \sigma$ so the probability that a job opportunity expires equals the probability of exogenous separation between a firm and a worker. Under this assumption, the free-entry condition (equation (36), p. 2648 of Ljungqvist and Sargent 2017) can be rearranged to obtain

$$\kappa = \frac{\lambda_f(\theta)\beta}{1-\beta(1-\sigma)}\left[A - \frac{b+\beta(1-\sigma)(A+\psi)}{1+\beta(1-\sigma)}\right]. \tag{218}$$

Now, suppose we extend the earlier idea in Shimer (2010) that recruiting workers takes a fixed amount of an existing worker's time to the idea that each round of bargaining also absorbs a fixed amount of a worker's time in haggling. Under this interpretation, it is natural to scale both κ and ψ by A, since both parameters reflect the foregone opportunity of producing goods for a worker engaged in either recruiting or bargaining. Hence, (218) becomes

$$\kappa A = \frac{\lambda_f(\theta)\beta}{1-\beta(1-\sigma)}\left[1 - \frac{b+\beta(1-\sigma)(1+\psi)}{1+\beta(1-\sigma)}\right] A. \tag{219}$$

Since A cancels out from both sides of this equality, θ is constant and so $d\log(\theta)/d\log(A) = 0$. Note that this same result holds even if δ does not equal σ because all value functions are proportional to A.

Proposition A.4 (Zero Elasticity of Job-Finding Rate in Alternating Offer Bargaining Model). *In the alternating offer bargaining model, if home produced output, the cost of posting a vacancy, and the haggling cost are proportional to productivity, then the change in steady-state unemployment with respect to a change in steady-state productivity is zero regardless of all other parameters.*

H.4 Pissarides (1999): Training Costs

Here we revisit the analysis of Ljungqvist and Sargent (2017) for the Pissarides's (2009) model with training costs. Pissarides (2009) shows that the presence of fixed training costs that are incurred after bargaining has been concluded can make unemployment more responsive to productivity changes.

Formally, firms pay a cost κ to post a vacancy and when a match with a worker is formed, they pay a fixed cost h to train the worker for the job. In this case, the value of match surplus is reduced by the fixed cost training and hence the free-entry condition becomes

$$\kappa = (1-\gamma)\lambda_f(\theta)\beta\left\{\frac{A-b}{1-\beta\left[1-\sigma-\gamma\theta\lambda_f(\theta)\right]}-h\right\}. \tag{220}$$

Now suppose that we extend the logic of Shimer (2010) to that of training a new worker. Specifically, we assume that an existing worker must reduce the time devoted to production by h units to train a new worker. Since an existing worker could devote the same amount of time to producing output and the output produced by a worker is proportional to A, then a doubling of productivity also doubles the training cost. Hence, here h is replaced by hA. The analogous free-entry condition is

$$A\kappa = (1-\gamma)\lambda_f(\theta)\beta\left\{\frac{(1-b)A}{1-\beta\left[1-\sigma-\gamma\theta\lambda_f(\theta)\right]}-hA\right\}. \tag{221}$$

Since A cancels from both sides of this equality, θ does not depend on A and so $d\log(\theta)/d\log(A)=0$.

Proposition A.5 (Zero Elasticity of Job-Finding Rate in Training Cost Model). *In the matching model with fixed training costs, if home produced output, the cost of posting a vacancy, and training costs are proportional to productivity, then the change in steady-state unemployment with respect to a change in steady-state productivity is zero regardless of all other parameters.*

In sum, our model produces large movements in response to productivity changes but works differently from those analyzed by Ljungqvist and Sargent (2017) in their excellent synthesis of the work on the unemployment volatility puzzle. All of these models depend critically on the differential productivity mechanism, while ours does not.

I Data

This appendix contains data details omitted from the paper.

I.1 Wage Growth on the Job and Wage Loss off the Job

Here we describe how we infer the rate of human capital accumulation on the job and depreciation off the job, respectively, from observed wage growth during employment and wage changes associated with spells of nonemployment.

Evidence on Wage Growth. We measure the rate of human capital accumulation on the job based on the estimates of wage growth by Rubinstein and Weiss (2006) from a sample of workers from the 1979-2000 waves of the NLSY, who are between 14 and 21 years of age in 1979 and are surveyed annually since the initial wave of the survey in 1979. These authors exclude from their sample the military sub-sample and the non-black, non-Hispanic disadvantaged samples. They also omit observations on workers with missing data on own or parents' education, Armed Forces Qualification Test score, or labor market outcomes, and on individuals enrolled in schooling in a given year. These authors further exclude workers with a reported hourly wage lower than \$4 or higher than \$2,000 (adjusted for the 2000 CPI) and individuals who work less than 35 weeks or less than \$1,000 annual hours. To be able to estimate wage growth, the authors restrict their sample according to these criteria in two consecutive years. For each individual in the resulting sample, the authors calculate average annual wage growth by experience.

As reported in their Table 2b, Rubinstein and Weiss (2006) estimate that the average annual growth rate of real hourly wages is 7.7% for individuals with up to 10 years of labor market experience, 3.3% for individuals with 11 to 15 years of labor market experience, and 4.9% for individuals with 16 to 25 years of labor market experience, gross of the annual growth rate of aggregate productivity that we calculate as equal to 2.2%. Given the impossibility of estimating growth rates at higher levels of experience, for the life-cycle model, we interpolate these estimated growth rates by nonlinear least squares according to the following specification,

$$\Delta w_t \equiv f(x_t) = \log\left\{\exp(g_w) + \exp\left[\beta_0 + \beta_1 \exp(-\beta_2 x_t)\right]\right\} + \varepsilon_t, \tag{222}$$

where Δw_t denotes the growth rates of wages estimated by Rubinstein and Weiss (2006) and x_t denotes labor market experience. We extrapolate growth rates for the missing experience years based on the estimates of the parameters of (222). To understand (222), note that the expression on the right-hand side of (222) is the so-called soft maximum between the arithmetic mean of the three growth rates estimated by Rubinstein and Weiss (2006), namely, $g_w = 5.3\%$, and the function $\beta_0 + \beta_1 \exp(-\beta_2 x_t)$. To see how the parameters $(\beta_0, \beta_1, \beta_2)$ are identified, observe that

$$m(x_t) \equiv \frac{\exp(\mathbb{E}f(x_t+1)) - \exp(g_w)}{\exp(\mathbb{E}f(x_t)) - \exp(g_w)} = \frac{\exp\left[\beta_0 + \beta_1 \exp(-\beta_2 x_t - \beta_2)\right]}{\exp\left[\beta_0 + \beta_1 \exp(-\beta_2 x_t)\right]} = e^{\beta_1 \exp(-\beta_2 x_t - \beta_2) - \beta_1 \exp(-\beta_2 x_t)}.$$

Thus, the ratio $\log(m(x_t'))/\log(m(x_t))$ for any two x_t and x_t',

$$\frac{\log(m(x_t'))}{\log(m(x_t))} = \frac{\beta_1[\exp(-\beta_2) - 1]\exp(-\beta_2 x_t')}{\beta_1[\exp(-\beta_2) - 1]\exp(-\beta_2 x_t)} = \exp(-\beta_2(x_t' - x_t)),$$

identifies β_2. Once β_2 is identified, $m(x_t)$ or $m(x_t')$ identifies β_1 and $\exp(\mathbb{E}f(x_t))$ evaluated at any other x_t identifies β_0. Based on this procedure, we estimate that the average annual growth rate of real hourly wages is 4.86% up to the first 10 years of labor market experience, and is 3.22% for the remaining years (up to 40).

Evidence on Wage Loss off the Job. We recover the rate of human capital depreciation off the job from the wage changes resulting from spells of nonemployment, defined as episodes of either unemployment or nonparticipation. We measure the change in the wages of workers who experience a transition from employment to nonemployment and back to employment as the percentage difference between the first wage in the first employment spell *after* nonemployment and the last wage in the last employment spell *before* nonemployment.

To this purpose, given the typical short duration of nonemployment spells in U.S. labor markets, we use monthly data from the Panel Study of Income Dynamics (PSID) family and individual (merged) files—see Krolikowski (2017) for a similar strategy. To elaborate, the PSID starts in 1968 with an interview of approximately 5,000 families, and follows any new families formed from the original group of families. In the survey years prior to 1988, though, the PSID did not collect monthly information on employment status at different employers so it is not possible to calculate monthly employment spells for these years. Moreover, although between 1968 and 1997 interviews were conducted annually, since 1997 interviews have been biennial. For these reasons, we only use data from the 1988-1997 waves. In order to obtain results that are comparable to alternative data sources and for consistency with the sample we used to estimate wage growth on the job, we restrict the sample to working-age males aged 18 through 60. We omit observations on individuals who are self-employed, and use individual weights to account for the PSID's poverty over-sample and nonrandom attrition.[1]

An important feature of the data for our purposes is that for the years between 1988 and 1997, respondents were asked to report their employment status in each month of the previous calendar year, as well as monthly employment information for up to two main employers. From these monthly information, it is then possible to determine the actual length of employment and nonemployment spells experienced by workers.[2] The data also include detailed information on wages. Specifically, for employed individuals, the data provide the starting wage at a worker's current employer as well as the ending wage at the worker's former employer. Similarly, for nonemployed individuals, the data provide the starting wage at a worker's former employer and the ending wage at the worker's employer before the former one. Combined with information on workers' labor force status, this information allows us to calculate wage changes associated with transitions from employment to nonemployment and back to employment (E-N-E) of interest.

Net of the annual growth rate of aggregate productivity of 2.2%, we estimate the average change in hourly wages between the *first* wage after a complete spell of nonemployment and the *last* wage before such a spell to be 1.43% for workers with less than 10 years of labor market experience and -12.26% for workers with more than 10 years of labor market experience.

I.2 Job-Finding Rate and Unemployment Rate

We rely on the method in Shimer (2012) described in the paper to compute the mean and standard deviation of the job-finding rate and (constant-separation) unemployment rate overall and by experience groups. Specifically, we use monthly data from the Bureau of Labor Statistics (BLS) between 1948 and 2007 and compute quarterly average of monthly job-finding and employment rates. For the experience-specific counterparts of these statistics, we use BLS monthly data between 1976 and 2007 since information on short-term unemployment is unavailable before 1976. Given that the BLS provides these statistics only for selected age ranges, we consider only those age groups that we can map into the experience groups of interest as defined by workers with less than 10 or more than 10 years of labor market experience (up to 40). Moreover, since the age-specific employment data from the BLS are available only as non-deseasonalized, we apply to these data the same de-seasonalization procedure that the BLS applies to the aggregate data using the 13ARIMA-SEATS (X-13) seasonal adjustment program that the BLS provides.

We then compute the mean and standard deviation of the job-finding rate and the unemployment rate for workers with up to 25 and up to 35 years of age and for workers over 25 and over 35 years of age. Since the mean and median number of years of education in the United States are approximately 12,

[1] We include the supplementary low income sample and the 1997 immigrant sample in the analysis, but exclude the Latino sample introduced in 1990. We are grateful to Pawel Krolokowski for assistance in the use of these data.

[2] Transition probabilities based on such a sample are broadly consistent with those obtained from the Survey of Income and Program Participation (SIPP) and the Current Population Survey (CPS).

we interpret the average mean and standard deviation of the job-finding rate and the unemployment rate among workers with up to 25 and up to 35 years of age as the mean and standard deviation of the job-finding rate and the unemployment rate for workers with less than 10 years of labor market experience, reported in Table 4 in the paper. Similarly, we interpret the average mean and standard deviation of the job-finding rate and the unemployment rate among workers with over 25 and over 35 years of age as the mean and standard deviation of the job-finding rate and the unemployment rate for individuals with more than 10 years of labor market experience, also reported in Table 4. We repeat a similar procedure for the autocorrelation of the job-finding rate and the unemployment rate and for the correlation between the job-finding rate and the unemployment rate for workers with less than 10 years of labor market experience and more than 10 years of labor market experience.

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Table A.1: Implications of Search Economies vs. Endowment Economies for Stock and Bond Prices

	Data	Baseline	CC	EZ w/ LRR	EZ w/ Disasters	Affine SDF
Search Economies						
Mean excess return (%p.a.)	6.96	6.30	6.38	4.61	4.80	6.96
S.d. excess return (%p.a.)	15.6	14.1	15.2	10.3	10.7	15.6
Mean excess return / s.d. excess return (p.a.)	0.45	0.45	0.45	0.45	0.45	0.45
Mean log price-dividend ratio	3.51	3.36	3.37	3.77	3.24	3.24
S.d. log price-dividend ratio	0.44	0.36	0.36	0.36	0.36	0.36
Mean 20-year real yield (%p.a.)	4.81	3.75	3.84	2.80	-1.38	4.36
S.d. 20-year real yield (%p.a.)	2.00	2.20	2.28	1.25	2.19	2.11
Mean 20-year nominal yield (%p.a.)	7.71	7.73	7.81	6.48	2.28	8.43
S.d. 20-year nominal yield (%p.a.)	2.41	2.28	2.37	1.27	2.20	2.24
Endowment Economies						
Mean excess return (%p.a.)	6.96	6.85	6.74	4.48	4.84	6.94
S.d. excess return (%p.a.)	15.6	15.3	15.1	9.9	10.8	15.5
Mean excess return / s.d. excess return (p.a.)	0.45	0.45	0.45	0.45	0.45	0.45
Mean log price-dividend ratio	3.51	3.29	3.30	3.78	3.69	3.23
S.d. log price-dividend ratio	0.44	0.37	0.36	0.31	0.36	0.36
Mean 20-year real yield (%p.a.)	4.81	4.34	4.47	2.82	-1.42	4.36
S.d. 20-year real yield (%p.a.)	2.00	2.34	2.36	1.26	2.20	2.11
Mean 20-year nominal yield (%p.a.)	7.71	8.30	8.42	6.50	2.20	8.43
S.d. 20-year nominal yield (%p.a.)	2.41	2.41	2.42	1.28	2.21	2.24

Table A.2: Parametrization and Results for Model with Campbell-Cochrane Preferences with External Habit

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
κ, hiring cost	0.975	Mean unemployment rate	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (%p.a.)	0.92	0.92
$\bar{S}$, mean of state S_t	0.2087	S.d. risk-free rate (%p.a.)	2.31	2.31
α, inverse EIS	5.0	Maximum Sharpe ratio (p.a.)	0.45	0.45
Assigned		*Labor Market Results*		
B, efficiency of matching technology	0.46	Mean job-finding rate	0.46	0.46
b, home production parameter	0.6	S.d. job-finding rate	6.66	6.69
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.75
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, persistence of state	0.9944	Correlation unemployment, job-finding rate	-0.96	-0.98
g_e, human capital growth when employed (%p.a.)	3.5			
		Asset Market Results		
		Mean excess return (%p.a.)	6.96	6.38
		S.d. excess return (%p.a.)	15.6	15.2
		Mean excess return / s.d. excess return (p.a.)	0.45	0.45
		Mean log price-dividend ratio	3.51	3.37
		S.d. log price-dividend ratio	0.44	0.36

Table A.3: Parametrization and Results for Model with Epstein-Zin Preferences with Long-Run Risk

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.80	S.d. productivity growth (%p.a.)	1.84	1.84
κ, hiring cost	1.31	Mean unemployment rate	5.9	5.9
β, time preference factor	0.998	Mean risk-free rate (%p.a.)	0.92	0.92
ϕ_s, relative s.d. s_t	0.0379	S.d. risk-free rate (%p.a.)	2.31	2.31
α, risk aversion coefficient	4.3	Maximum Sharpe ratio (p.a.)	0.45	0.45
Assigned		*Labor Market Results*		
B, efficiency of matching technology	0.46	Mean job-finding rate	0.46	0.46
b, home production parameter	0.6	S.d. job-finding rate	6.66	6.36
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.69
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_x, persistence of x_t	0.9977	Correlation unemployment, job-finding rate	-0.96	-0.98
ρ_s, persistence of s_t	0.9944			
g_e, human capital growth when employed (%p.a.)	3.5	*Asset Market Results*		
ρ, inverse EIS	0.1			
rel. size x_t component of productivity	0.0445	Mean excess return (%p.a.)	6.96	4.61
		S.d. excess return (%p.a.)	15.6	10.3
		Mean excess return / s.d. excess return (p.a.)	0.45	0.45
		Mean log price-dividend ratio	3.51	3.77
		S.d. log price-dividend ratio	0.44	0.36

Table A.4: Parametrization and Results for Model with Epstein-Zin Preferences with Variable Disaster Risk

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
κ, hiring cost	1.22	Mean unemployment rate	5.9	5.9
β, time preference factor	0.998	Mean risk-free rate (%p.a.)	0.92	0.92
σ_s, disaster intensity volatility parameter	0.0083	S.d. risk-free rate (%p.a.)	2.31	2.31
α, risk aversion coefficient	2.65	Maximum Sharpe ratio (p.a.)	0.45	0.45
Assigned		*Labor Market Results*		
B, efficiency of matching technology	0.46	Mean job-finding rate	0.46	0.46
b, home production parameter	0.6	S.d. job-finding rate	6.66	5.66
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.77
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, persistence of disaster intensity	0.9966	Correlation unemployment, job-finding rate	-0.96	-0.98
g_e, human capital growth when employed (%p.a.)	3.5			
ρ, inverse EIS	0.1	*Asset Market Results*		
s, disaster intensity (%p.a.)	3.55			
θ, disaster impact	0.26	Mean excess return (%p.a.)	6.96	4.80
		S.d. excess return (%p.a.)	15.6	10.7
		Mean excess return / s.d. excess return (p.a.)	0.45	0.45
		Mean log price-dividend ratio	3.51	3.24
		S.d. log price-dividend ratio	0.44	0.36

Table A.5: Parametrization and Results for Model with Affine Stochastic Discounts

Panel A: Parameters

Endogenously Chosen	
g_a, mean productivity growth (%p.a.)	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84
κ, hiring cost	0.90
μ_0	0.00077
μ_1	-0.042
γ_0	25.6
γ_1	0.83
Assigned	
B, efficiency of matching technology	0.46
b, home production parameter	0.6
σ, probability of separation	0.028
η, matching function elasticity	0.5
ϕ, survival probability	0.9972
ρ_s, persistence of state	0.9944
g_e, human capital growth when employed (%p.a.)	3.5

Panel B: Moments

Targeted	Data	Model
Mean productivity growth (%p.a.)	2.22	2.22
S.d. productivity growth (%p.a.)	1.84	1.84
Mean unemployment rate	5.9	5.9
Mean risk-free rate (%p.a.)	0.92	0.92
S.d. risk-free rate (%p.a.)	2.31	2.31
Maximum Sharpe ratio (p.a.)	0.45	0.45
S.d. excess return	15.6	15.6
Labor Market Results		
Mean job-finding rate	0.46	0.46
S.d. job-finding rate	6.66	7.52
Autocorrelation job-finding rate	0.94	0.99
S.d. unemployment rate	0.75	0.73
Autocorrelation unemployment rate	0.97	0.99
Correlation unemployment, job-finding rate	-0.96	-0.98
Asset Market Results		
Mean excess return (%p.a.)	6.96	6.96
S.d. excess return (%p.a.)	15.6	15.6
Mean excess return / s.d. excess return (p.a.)	0.45	0.45
Mean log price-dividend ratio	3.51	3.24
S.d. log price-dividend ratio	0.44	0.36

Table A.6: Hall (2017) with Alternative Durations of Job Opportunities

	Data	Model in Hall (2017) $1/\delta = 2.6$	Original
Parameters			
Avg. duration of job opportunity during bargaining (months)	–	2.6	77
Per-round probability bargaining ends, δ	–	1/2.6	1/77
Bargaining delay cost, ψ	–	1.01	0.57
Results			
S.d. quarterly unemployment rate (pp)	0.75	0.03	0.97
Cyclicality of user cost of labor to unemployment (%)	-5.2	-5.2	0.10

Note: The probability that a job opportunity breaks down after n rounds of bargaining is $\delta(1-\delta)^n$ so the expected duration of a job opportunity during bargaining is $\delta + 2\delta(1-\delta) + \ldots + n\delta(1-\delta)^{n-1} + \ldots = 1/\delta$ rounds.

Table A.7: Accuracy of Approximations for Job-Finding Rate

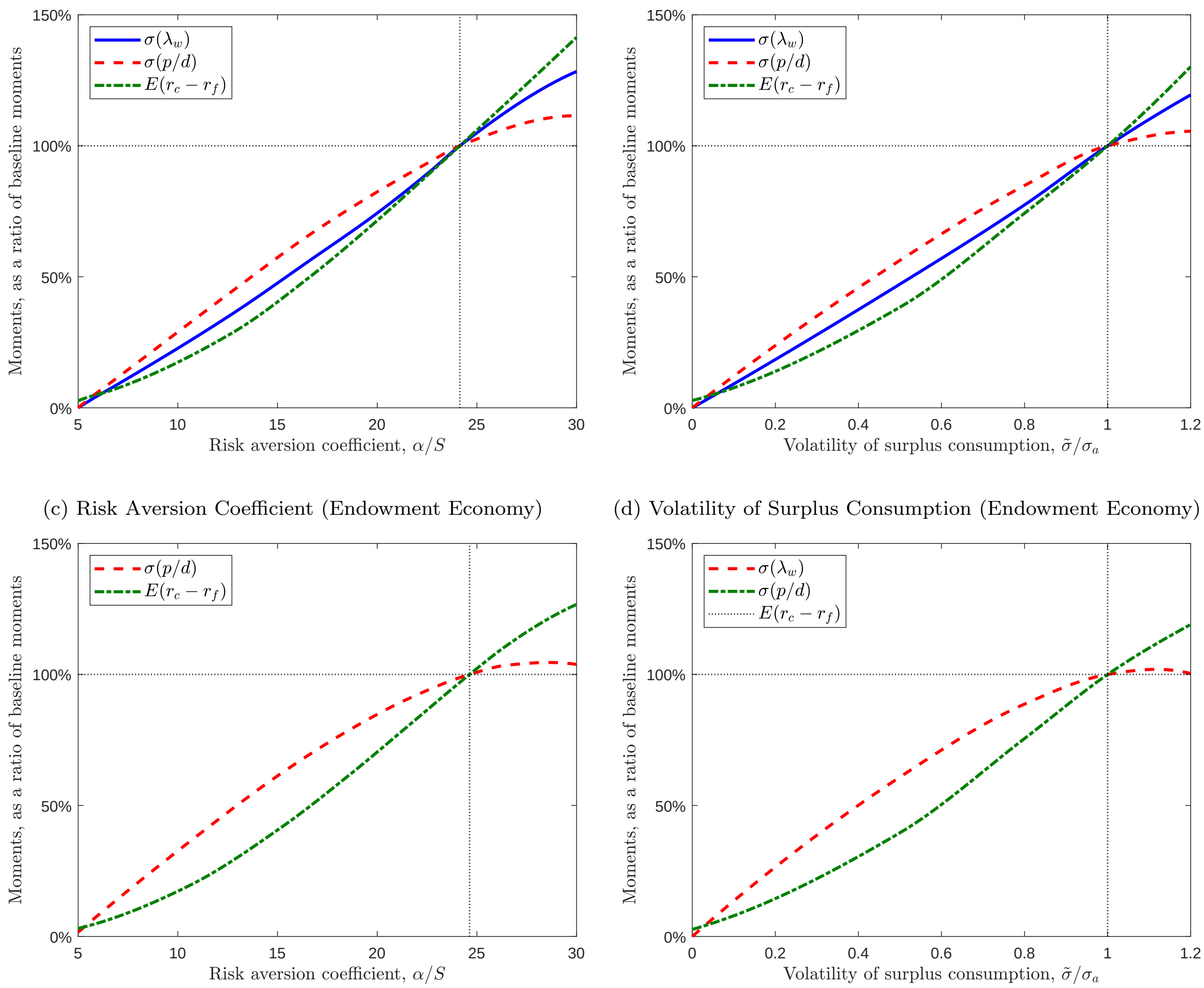
Approximation	Linear Around RSS	Linear Around RSS and $\lambda_{wt+n} \approx \lambda_w$
$\sigma(\lambda_{wt}^{approx})/\sigma(\lambda_{wt}^{global})$	0.999	1.93
$corr(\lambda_{wt}^{approx}, \lambda_{wt}^{global})$	0.985	0.998

Note: All of these approximations use the assumption that $\Delta c_{t+1} \approx \Delta a_{t+1}$.

Table A.8: Estimated Parameters for Inflation Process

Parameters	Estimates
$\bar{\pi}$, mean inflation rate	0.0031
ϕ, autocorrelation coefficient	0.9479
ψ, relative volatility of latent variable	0.2499
σ_w, volatility of inflation shocks	0.0027
$\rho_{\pi a}$, correlation inflation shocks, productivity shocks	-0.1620

Figure A.1: Sensitivity of Key Moments to Preference Parameters in Baseline and Endowment Economies



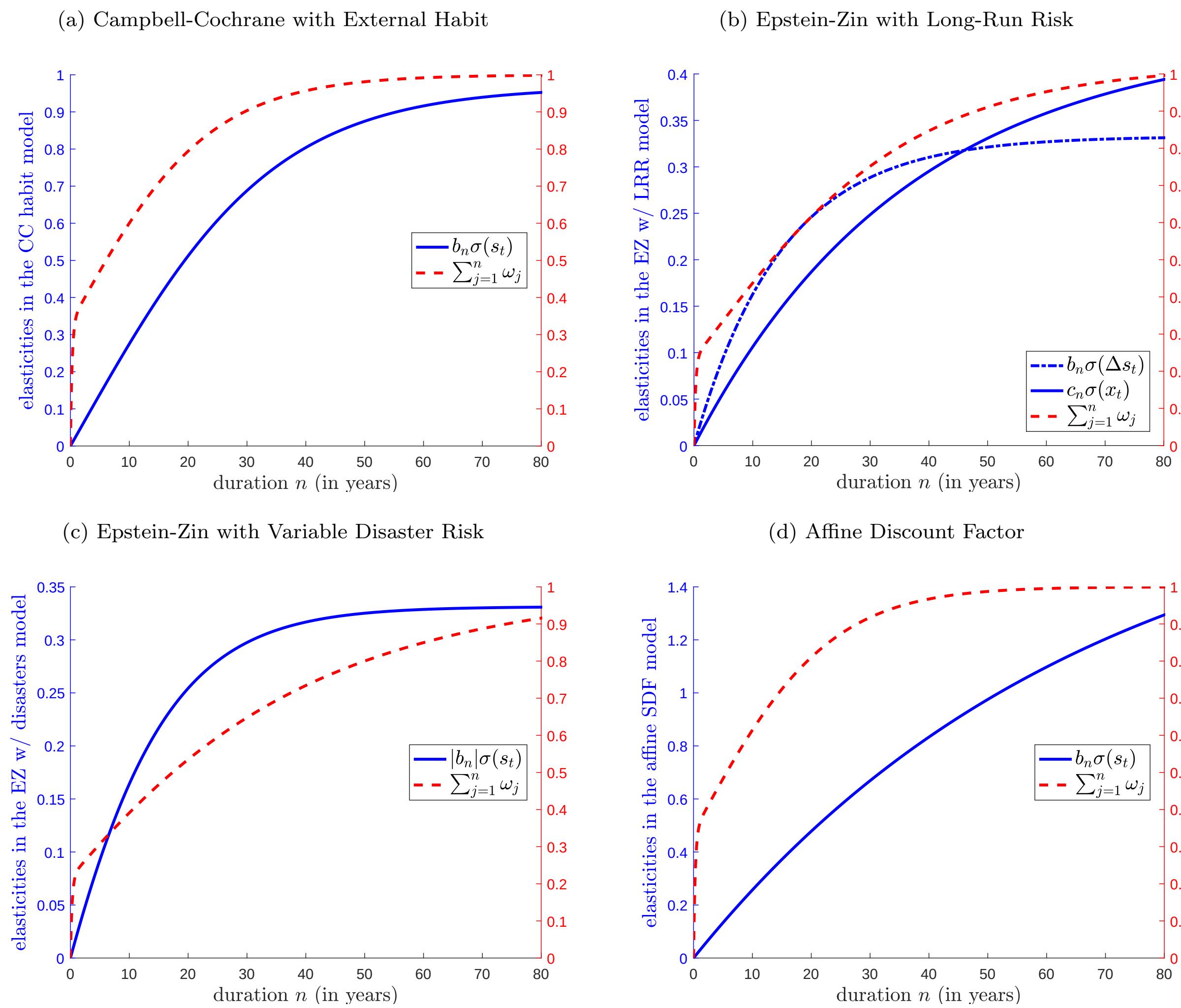
(a) Risk Aversion Coefficient (Search Economy)

(b) Volatility of Surplus Consumption (Search Economy)

(c) Risk Aversion Coefficient (Endowment Economy)

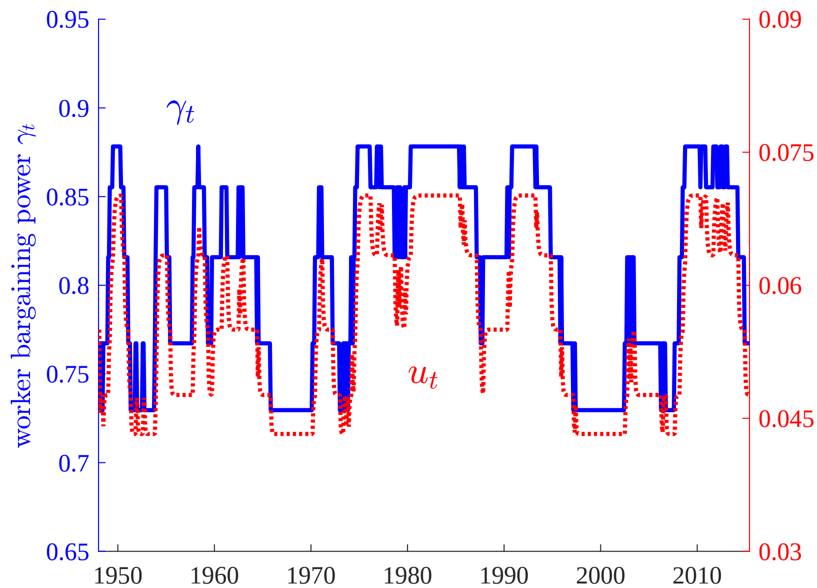
(d) Volatility of Surplus Consumption (Endowment Economy)

Figure A.2: Determinants of Volatility of Job-Finding Rate



Note: $\sigma(\lambda_{wt}) = \left|\sum_{n=1}^{\infty}\omega_n b_n\right|\sigma(s_t)$ for Campbell-Cochrane preferences with external habit, affine stochastic discount factor, and Epstein-Zin preferences with variable disaster risk, and $\sigma(\lambda_{wt}) = \sqrt{\left(\sum_{n=1}^{\infty}\omega_n b_n\right)^2\sigma(\Delta s_t)^2 + \left(\sum_{n=1}^{\infty}\omega_n c_n\right)^2\sigma(x_t)^2}$ for Epstein-Zin preferences with long-run risk.

Figure A.3: Time-Varying Worker Bargaining Power in Hall (2017)



Note: Constructed from data from Hall (2017).