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Asset Prices and Unemployment Fluctuations*

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ABSTRACT

Recent critiques have demonstrated that existing attempts to account for the unemployment volatility puzzle of search models are inconsistent with the procylicality of the opportunity cost of employment, the cyclicality of wages, and the volatility of risk-free rates. We propose a model that is immune to these critiques and solves this puzzle by allowing for preferences that generate *time-varying risk* over the cycle, and so account for observed asset pricing fluctuations, and for *human capital accumulation on the job*, consistent with existing estimates of returns to labor market experience. Our business cycle model reproduces the observed fluctuations in unemployment because hiring a worker is a risky investment with long-duration surplus flows. Since the price of risk in our model sharply increases in recessions as observed in the data, the benefit from creating new matches greatly drops, leading to a large decline in job vacancies and an increase in unemployment of the same magnitude as in the data.

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The most important theoretical contribution of search models of the labor market to the study of business cycles is that they interpret involuntary unemployment as an equilibrium phenomenon. The key insight of these models is that involuntary unemployment can arise even without any assumed inefficiencies in contracting, such as rigid wages. Despite its great promise, though, Shimer (2005) showed that the textbook search model cannot generate anywhere near the observed magnitude of the fluctuations in the job-finding rate and unemployment in response to shocks of plausible magnitude. A large body of work has attempted to address this *unemployment volatility puzzle* but, as we discuss, recent critiques of it have demonstrated that existing attempts are inconsistent with the procylicality of the opportunity cost of employment, the cyclicality of wages, and the volatility of risk-free rates. Hence, in this precise sense, the puzzle has not been solved.

In this paper, we propose a model that reproduces these features of the data, respects the original promise of search models by generating involuntary equilibrium unemployment without relying on inefficient contracting or wage rigidities, and solves this puzzle. We do so by allowing for preferences that give rise to *time-varying risk* over the cycle, as consistent with observed fluctuations in asset prices, and for *human capital accumulation on the job*, in line with the documented growth of wages with labor market experience.

Throughout most of our analysis, we abstract from physical capital simply to help illustrate our mechanism in the most transparent way. We also extend our model to incorporate physical capital and show that such an augmented model matches key observed patterns of job-finding rates, unemployment, output, consumption, investment, and asset prices. In this exercise, we build on the work of Merz (1995) and Andolfatto (1996), who integrated search theory into quantitative business cycle models, and the work of Jermann (1998) and Tallarini (2000), who embedded asset pricing preferences into a quantitative business cycle model. Interestingly, in contrast to the classic separation result by Tallarini (2000) whereby introducing asset pricing preferences into a standard real business cycle model has no effect on the fluctuations of real variables, in our model with human capital introducing such preferences creates an important interaction between the real and financial sides of the economy that greatly amplifies fluctuations.

The main idea of our model is that hiring a worker is akin to investing in an asset with *risky* dividend flows that have *long* durations. In our model, as in the data, the price of risk rises sharply in downturns. Because of human capital accumulation on the job, the surplus flows to matches between firms and workers have long durations and so are sensitive to variation in the price of risk. These two features then imply that the benefits of creating a match sharply drop in downturns, which induces firms to substantially reduce the number of job vacancies they create and, correspondingly, leads unemployment to increase as much as it does in the data.

We add to the textbook search model two simple ingredients that make it consistent with two salient aspects of the data: asset prices fluctuate over the cycle and wages increase with experience in the labor market. To reproduce the first feature, we augment the textbook model with preferences that generate time-varying risk, whereas to accommodate the second feature, we introduce human capital accumulation on the job and depreciation off the job. We choose parameters for preferences and technology that are consistent with key observed properties of asset prices and wage-experience profiles, and show that the resulting allocations display fluctuations in unemployment that are as large as those observed in the data.

We generate involuntary unemployment without exploiting inefficiencies in wage contracting by focusing on labor market outcomes generated by a competitive search equilibrium. We find this equilibrium concept appealing relative to common bargaining concepts such as Nash bargaining or alternating offer bargaining, since these bargaining schemes give rise to inefficient wage setting unless the parameters that characterize the bargaining process are suitably chosen. For instance, a well-known result is that equilibrium wage setting under Nash bargaining is efficient and, hence, leads to the same outcomes that arise under competitive search when the Hosios's (1990) condition holds. (In the Appendix, we derive analogous conditions for efficiency for the alternative offer bargaining protocol.) In light of these results, we can interpret our work as focused on economies with efficient wage setting, which can be achieved under any of the three most popular wage determination schemes: competitive search and, as long as appropriately parametrized, Nash bargaining with recontracting each period, and alternating offer bargaining. In this sense, our results do not depend on the specific wage determination scheme chosen.

We argue that our two simple ingredients are both necessary to account for the observed volatility of unemployment. In particular, we show that if we retain human capital accumulation but replace our asset pricing preferences with standard constant relative risk aversion preferences, then the model generates no fluctuations in unemployment regardless of the degree of human capital accumulation. Conversely, if we retain our asset pricing preferences but abstract from human capital accumulation, then the model generates almost no fluctuations in unemployment.

We turn to providing further details about our two additional ingredients. Consider first preferences. The asset pricing literature has developed several classes of preferences and stochastic processes for exogenous shocks that give rise to large increases in the price of risk in downturns and, hence, reproduce key features of the fluctuations of asset prices. As Cochrane (2011) emphasizes, all of these preferences and shocks generate variation in asset prices from variation in risk premia, consistent with the data. To emphasize that our results are robust to the specific details of the preferences and shocks that achieve this variation, in our extensions we show that our results hold for a wide range of the most popular specifications.

In our baseline model, we use a variant of the original preferences in Campbell and Cochrane (1999) in which we eliminate the associated consumption externality by making the habit in consumption a function of exogenous shocks. We find these preferences appealing because they incorporate the idea that the price of risk rises in recessions in a transparent and intuitive way. Moreover, as we show, their implications for asset prices and unemployment fluctuations are nearly identical to those of the original preferences in Campbell and Cochrane (1999).

Consider now human capital accumulation. For simplicity, we assume that a worker's human capital grows at a constant rate during employment and depreciates at a constant rate during unemployment, and that market production, home production, and the cost of posting job vacancies are proportional to human capital. This formulation is particularly convenient because it implies that only the aggregate levels of human capital of employed and unemployed workers, rather than their distributions, need to be recorded as state variables.[1]

We then characterize the mechanism generating our quantitative results. We show that the job-finding rate is proportional to the present value of the surplus flows from a match between a firm and a worker scaled by aggregate productivity. This present value, in turn, can be expressed as a weighted average of the prices of claims to aggregate productivity at each future time horizon, referred to as *claims to aggregate productivity* or *strips*. The *weights* of this weighted average of prices of claims are determined by the degree of human capital accumulation whereas the *prices* of these claims are determined by the preference and shock structure. Intuitively, since human capital accumulation increases the duration of surplus flows, the greater is the amount of human capital accumulation, the slower is the decay of the surplus flows from a match between a firm and a worker, and, hence, the larger are the weights attached to strips at longer horizons. Since strips are more volatile at longer horizons—a feature shared by all the asset pricing models we consider—the larger the weights on longer horizons, the more sensitive is the job finding rate to aggregate shocks.

Formally, we prove that the volatility of the job-finding rate can be well approximated by a single sufficient statistic: a weighted average, $\sum_n \omega_n b_n \sigma(s_t)$, over different horizons n of the elasticity b_n of the price of a strip with respect to the exogenous stochastic state of an economy, s_t, multiplied by the volatility of this state, $\sigma(s_t)$. The weights $\{\omega_n\}$ decay more slowly the greater is human capital accumulation on the job. Further, the elasticity b_n increases with the horizon n so that strips become more sensitive to the state s_t as the horizon of a strip increases.

This sufficient statistic further allows us to characterize the roles of time-varying risk and human capital accumulation in our results. First, we show that when there is little time-varying risk, the elasticity b_n of the price of strips with respect to the state s_t is small regardless of the horizon n. Hence, the model cannot generate much volatility in the job-finding rate regardless of the weights $\{\omega_n\}$ on strips. Second, we show that when there is little or no human capital accumulation on the job, the weights $\{\omega_n\}$ are nearly all concentrated on short-horizon claims, which display little volatility under all of our asset pricing specifications. Intuitively, these weights are small because, absent human capital accumulation, the duration of surplus flows from a match is very short. In this case, the problem of hiring a worker is nearly static, so variation in time-varying risk has little effect on the present value of surplus flows from a match.

[1] In the Appendix, we consider a more general formulation of the human capital process in which the rates of human capital accumulation and depreciation are stochastic and vary with the level of acquired human capital. This richer version of the model better reproduces the shape of empirical wage-experience profiles and yields results very similar to those under our baseline for the volatility of the job-finding rate and unemployment.

Therefore, the model cannot generate much volatility in the job-finding rate in this case either. Only when both features are present, namely, time variation in the price of risk and human capital accumulation on the job, can our model produce sizable volatility in the job-finding rate and unemployment.

We conclude by considering three extensions. First, we augment our model with physical capital subject to adjustment costs along the lines of Jermann (1998), and construct a business cycle model in the spirit of the seminal work by Merz (1995) and Andolfatto (1996). As Shimer (2005) points out, though, these latter two papers miss a key feature of the data, namely, the strong negative correlation between vacancies and unemployment. Our model, instead, not only reproduces this feature but also matches salient patterns of job-finding rates, unemployment, output, consumption, investment, and asset prices in the data.

Second, we extend our model to a simple life-cycle setting with young and mature consumers. In the data, both the growth rate of human capital during employment and the volatility of unemployment are higher for young workers than for mature ones. Such an extension of our model can account for these patterns as well.

Third, we show that our results hold for a variety of popular preference structures. We first consider a version of the long-run risk setup of Bansal and Yaron (2004) with preferences as in Epstein and Zin (1989), modified along the lines suggested by Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Schorfheide, Song, and Yaron (2018) to allow for long-run risk and preference shocks in order to better reproduce observed asset prices. Following the setup of Wachter (2013), we then consider the preferences in Epstein and Zin (1989) augmented with a time-varying risk of disasters, defined as episodes of unusually large decreases in aggregate consumption associated with marked declines in productivity. Finally, given the popularity of reduced-form asset pricing models that simply specify a discount factor as a function of shocks, we explore a version of the affine discount factor model of Ang and Piazzesi (2003) as a representative model of this class.

We find that all these economies imply analogous results for the volatility of the job-finding rate and unemployment. As in the case of our baseline model, each of these models' implications for these volatilities depends only on our single sufficient statistic, which captures the volatility of the exogenous state, the implied variation in the price of risk, and the persistence of the returns to hiring workers. All of these preference and shock structures also have broadly similar implications for asset prices with the exception of the two models with Epstein-Zin preferences, which are slightly less successful at replicating some of the features of stock and bond returns. In the Appendix, we further show that the asset pricing implications of our search model with endogenous production under each of these preference structures are essentially identical to those of the original versions of these models, which were developed for pure exchange economies. In this sense, our various models' implications for asset prices are simply inherited from their original versions: augmenting them with labor market search makes them neither better nor worse.

Based on all of these results, we view our exercise as a promising first step toward developing an integrated theory of real and financial business cycles.

1. Relation to the Literature

Our model relies on a fundamentally different mechanism than that isolated by Ljungqvist and Sargent (2017) in their survey of attempts to solve the unemployment volatility puzzle, which include Hagedorn and Manovskii (2008), Hall and Milgrom (2008), and Pissarides (2009). In particular, Ljungqvist and Sargent (2017) show that all of these models feature an acyclical opportunity cost of employment. In a recent paper, though, Chodorow-Reich and Karabarbounis (2016) critique this literature and argue that none of these attempts are consistent with the data. Specifically, these authors document that the opportunity cost of employment in the data is procyclical with an elasticity close to one rather than, as assumed in these models, zero. These authors further demonstrate that once these models are made consistent with this aspect of the data, they are incapable of generating volatile unemployment.

A second critique of the literature that has addressed this puzzle by introducing some form of wage rigidity is by Kudlyak (2014). This work builds on the insight of Becker's (1962) classic paper that only the present value of the wages paid by firms to workers over the course of an employment relationship is allocative for employment. Kudlyak (2014) establishes that the appropriate measure of rigidity of the allocative wage for a large class of search models is the cyclicality of the user cost of labor, defined as the difference in the present values of wages between two firm-worker matches that are formed in two consecutive periods. As Kudlyak (2014) estimates and Basu and House (2016) confirm, the user cost of labor is highly cyclical in that it sharply falls when unemployment rises. Both of these papers also argue that reproducing the observed cyclicality of the user cost of labor is the key litmus test for the cyclicality of wages implied by any business cycle model. As these authors discuss, early attempts to solve the unemployment volatility puzzle fail this test. Here we show that our model, instead, passes it.

Finally, a third critique of the literature on the unemployment volatility puzzle has been formulated by Borovicka and Borovickova (2019), who argue that the literature is grossly at odds with robust patterns of asset prices. In contrast to existing work, our model incorporates standard asset pricing preferences, which generate movements in risk-free rates and risk premia in accord with the data. In this sense, our model overcomes this final critique as well.

The important related contribution of Hall (2017) accounts for the observed volatility of unemployment within a model that features alternating wage offer bargaining, a reduced-form discount factor, and no human capital accumulation. This paper is immune to the critique by Chodorow-Reich and Karabarbounis (2016) but not to those by Kudlyak (2014) and Borovicka and Borovickova (2019). In particular, as we show in the Appendix, Hall (2017) relies on a parametrization of wage setting that yields highly inefficient allocations associated with a counterfactually low degree of cyclicality of the user cost of labor. Hence, in this precise sense, the wages in Hall (2017) are much more rigid than those in the data. Moreover, Borovicka and Borovickova (2019) show that in Hall's model, fluctuations in unemployment arise not from time-variation in the price of risk, as in our model, but rather from strongly countercyclical movements in the risk-free rate, which are counterfactual. Thus, although Hall (2017) provides critical insights, it is inconsistent with the evidence in Kudlyak (2014) and Basu and House (2016) and is subject to the critique by Borovicka and Borovickova (2019).

Also related to ours is the work of Kilic and Wachter (2018). These authors embed a reduced-form version of the mechanism in Hall (2017) within a model with preferences as in Epstein and Zin (1989) with variable disaster risk. Although the resulting model's pricing kernel does not generate a risk-free rate puzzle, it generates volatile unemployment by heavily relying on a form of inefficient real wage stickiness as in Hall (2017). In contrast, we show that variable disaster risk can generate realistic fluctuations in the job-finding rate under efficient wage setting without rigid wages, provided human capital is incorporated.

Finally, our mechanism is fundamentally different from those in the large literature discussed by Ljungqvist and Sargent (2017) that can account for the unemployment volatility puzzle under certain key assumptions. This literature focuses on changes in unemployment across steady states in response to changes in aggregate productivity because, in the class of models they consider, steady-state changes well approximate stochastic fluctuations in productivity. The class of models we propose differs from these in two dimensions. First, steady-state changes in aggregate productivity in our models do not well approximate stochastic fluctuations in productivity. Second, the main result of Ljungqvist and Sargent (2017) on the conditions for unemployment to be volatile does not apply.

To elaborate, Ljungqvist and Sargent (2017) proved that existing search models generate large fluctuations in unemployment only if these models feature what they term a small *fundamental surplus fraction*, which is a scaled measure of the steady-state surplus from a match between a firm and a worker. We show that this result does not hold in our model by contrasting the implications of two classes of preferences: CRRA preferences and our baseline preferences with time-varying risk. Both classes lead to identical steady states and, hence, identical fundamental surplus fractions, but in response to productivity shocks, CRRA preferences lead to no fluctuations in unemployment whereas our baseline model produces large ones. The reason is that fluctuations in unemployment in our model are driven by time-varying risk arising from shocks with constant variance, which cannot be captured through either comparisons of deterministic steady states or preferences, like CRRA preferences, that do not give rise to time-varying risk.[2]

[2] In the Appendix, we first prove that in all of these models, the change in unemployment across steady states resulting from a change in aggregate productivity is identically zero, once we modify them to be consistent with the critique by Chodorow-Reich and Karabarbounis (2016) and the insight of Shimer (2010) that if recruiting workers or bargaining takes time away from production, then the cost of doing so is proportional to the opportunity cost of a worker's time in production. We then show that the commonly used cross-steady-state comparisons in response to a change in aggregate productivity are a poor approximation to those over time. In particular, in our model, the change in unemployment across steady states in response to a change in aggregate productivity is identically zero. In contrast, the stochastic version of our model generates the same volatility of unemployment as in the data. See the Appendix for details.

2. Economy

We embed a Diamond-Mortenson-Pissarides (DMP) model of the labor market with competitive search within a general equilibrium model of an economy in which households are composed of employed and unemployed workers and own firms. The economy is subject to both aggregate shocks, including productivity shocks, and idiosyncratic shocks. We extend the DMP model to include two key features: asset-pricing preferences that generate time-varying risk and human capital accumulation with experience. In our baseline model, we use a version of Campbell and Cochrane (1999) preferences with an exogenous consumption habit and in our extensions, we consider several other popular preference structures.

The economy consists of a continuum of firms and consumers. Each consumer belongs to one of a large number of families that insure their members against idiosyncratic risks. Each consumer survives from one period to the next with probability ϕ. A measure $1-\phi$ of new consumers is born each period so that the measure of consumers in the economy is constant over time and equal to one. Individual consumers accumulate human capital when employed. Firms post vacancies to hire consumers with any desired level of human capital.

A. Technologies and Resource Constraints

Consumers are indexed by a state variable that summarizes their ability to produce output. The variable z_t, referred to as *human capital*, captures any increase in a consumer's productivity with experience in the labor market. A consumer with state variable z_t produces $A_t z_t$ units of output when employed and $bA_t z_t$ units of output when unemployed in period t. Hence, the opportunity cost of employment is $bA_t z_t$ with an elasticity to aggregate productivity of one, consistent with the findings in Chodorow-Reich and Karabarbounis (2016). Here we follow Hall (2017), who incorporates these findings by assuming that the opportunity cost of employment is proportional to aggregate productivity; see the discussion in Hall (2017, p. 324). We assume that aggregate productivity follows a random walk process with drift g_a given by

$$(1) \quad \log(A_{t+1}) = g_a + \log(A_t) + \sigma_a \varepsilon_{at+1},$$

where $\varepsilon_{at+1} \sim N(0,1)$. Newly born consumers draw their initial human capital from a distribution $\nu(z)$ with mean 1 and enter the labor market unemployed. After entry, when a consumer is employed, human capital evolves according to

$$(2) \quad z_{t+1} = (1+g_e)z_t,$$

and when a consumer is unemployed, it evolves according to

$$(3) \quad z_{t+1} = (1+g_u)z_t,$$

where $g_e \geq 0$ and $g_u \leq 0$ are constant rates of human capital accumulation on the job and depreciation off the job. Posting a vacancy directed at a consumer with human capital z costs a firm $\kappa A_t z$ in lost production in period t. This specification of the cost of posting vacancies is consistent with the argument in Shimer (2010) that to recruit workers, existing workers must reduce their time devoted to production, which costs a firm lost output. Under this view, the cost of recruiting workers moves one-for-one with the productivity of a worker engaged in market production.[3] Note that scaling home production and vacancy posting costs by z is convenient because, as we show later, it implies that all value functions are linear in z. This scaling assumption, though, is not necessary for our results and is purely motivated by analytical tractability and computational convenience. (In the Appendix, we consider a more general human capital process that does not scale with z and show that the resulting model works very similarly to our baseline model.)

The realization of the productivity innovation ε_t is the aggregate event. Let $\varepsilon^t = (\varepsilon_0, \dots, \varepsilon_t)$ be the history of aggregate events at time t. An allocation is a set of stochastic processes for consumption $\{C(\varepsilon^t)\}$ and measures of employed consumers, unemployed consumers, and vacancies posted for each level of human capital z, $\{e(z, \varepsilon^t), u(z, \varepsilon^t), v(z, \varepsilon^t)\}$. For notational simplicity, we suppress any explicit dependence on ε^t and express these allocations in shorthand notation as $\{C_t, e_t(z), u_t(z), v_t(z)\}$ from now on. The measures of employed and unemployed consumers satisfy

$$(4) \qquad \int_z [e_t(z) + u_t(z)]\, dz = 1.$$

The timing of events is as follows. At the beginning of period t, current productivity A_t is realized, firms post vacancies and wage offers, and unemployed workers from the end of period $t-1$ search for jobs. Then, new matches are formed and employed consumers immediately begin to work. At the end of the period, a fraction σ of employed consumers separate from their firms and enter the unemployment pool of period t, and consumption takes place.

To understand the law of motion for the measure of employed and unemployed consumers, consider unemployed consumers searching for a job at the beginning of period t with human capital z, denoted by $u_{bt}(z)$. These consumers were unemployed at the end of period $t-1$, had human capital $z/(1+g_u)$ that grew at rate $1+g_u$ to z between $t-1$ and t, and survived. Therefore,

$$(5) \qquad u_{bt}(z) \equiv \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right).$$

The term $1/(1+g_u)$ that multiplies u_{t-1} in (5) arises from the change of variable in the density over $z/(1+g_u)$ to derive the density over z. At the beginning of period t, firms post a measure of vacancies $v_t(z)$ to target consumers with human capital z thus creating a measure $m_t(u_{bt}(z), v_t(z))$ of matches, where $m_t(\cdot)$ is a constant returns-to-scale matching function increasing in both arguments. The transition laws

[3] Note that since we maintain that aggregate productivity follows a random walk with positive drift, it would not make sense to assume that home production b and the vacancy cost κ are constant, because then the ratios b/A_t and κ/A_t would (in a precise stochastic sense) converge to zero and all agents would always work.

for employed and unemployed workers' human capital are then given, respectively, by

$$(6) \quad e_t(z) = \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \lambda_{wt}(\theta_t(z))\, u_{bt}(z)$$

and

$$(7) \quad u_t(z) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + [1-\lambda_{wt}(\theta_t(z))]u_{bt}(z) + (1-\phi)\,\nu(z),$$

where $\lambda_{wt}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/u_{bt}(z)$ is the *job-finding rate* of an unemployed consumer with human capital z and $\theta_t(z) = v_t(z)/u_{bt}(z)$ is the *tightness* of the labor market for consumers with human capital z.

To understand these expressions, consider (7), for instance. Observe first that new entrants into the unemployment pool include the measure $\phi\sigma e_{t-1}(z/(1+g_e))/(1+g_e)$ of consumers with $z/(1+g_e)$ units of human capital in $t-1$ and z units of human capital in t who worked in period $t-1$, separated from their firms at the end of the period (an event with probability σ), and survived (an event with probability ϕ). New entrants into unemployment also include all newborn consumers with human capital z of measure $(1-\phi)\nu(z)$. Note that a proportion $1-\lambda_{wt}(\theta_t(z))$ of unemployed consumers at the beginning of period t remain unemployed.

For later use, it is convenient to define the *job-filling rate* for a firm that posts a vacancy for consumers with human capital z as $\lambda_{ft}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/v_t(z)$. It follows that $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$. We also define the elasticity of the job-filling rate with respect to $\theta_t(z)$ as $\eta_t(\theta_t(z)) = -\theta_t(z)\lambda'_{ft}(\theta_t(z))/\lambda_{ft}(\theta_t(z))$ so that $1-\eta_t(\theta_t(z)) = \theta_t(z)\lambda'_{wt}(\theta_t(z))/\lambda_{wt}(\theta_t(z))$.[4] Note that when we later assume a Cobb-Douglas matching function, the elasticity $\eta_t(\theta_t(z))$ is a constant.

The aggregate resource constraint in period t is

$$(8) \quad C_t \leq A_t \int_z z e_t(z)dz + bA_t \int_z z u_t(z)dz - \kappa A_t \int_z z v_t(z)dz,$$

where the right side of this constraint adds the total output of the employed, the total output of the unemployed, and subtracts the total cost of posting vacancies.

B. A Family's Problem

We represent the insurance arrangements in the economy by assuming that each consumer belongs to one of a large number of identical families, each consisting of a continuum of household members, that have access to complete one-period contingent claims against aggregate risk. Risk sharing within a family implies that each household member consumes the same amount C_t of goods at date t regardless of the idiosyncratic shocks that such a member experiences. (This type of risk-sharing arrangement is familiar from the work of Merz 1995 and Andolfatto 1996.)

Given this setup, we can separate a family's problem into two parts. The first part is at the level of the family and determines the family's choice of assets and common consumption level of each member.

[4] To see this, substitute $\lambda_{ft}(\theta_t) = \lambda_{wt}(\theta_t)/\theta_t$ and $\theta_t\lambda'_{ft}(\theta_t) = \lambda'_{wt}(\theta_t) - \lambda_{wt}(\theta_t)/\theta_t$ into the expression for $1-\eta_t(\theta_t(z))$.

The second part is at the level of individual consumers and firms in the family. The individual consumer problem determines the employment and unemployment status of each consumer in the family whereas the individual firm problem determines the vacancies created and the matches formed by each firm that the family owns.

In our baseline model, we replace the external consumption habit in Campbell and Cochrane (1999) with an exogenous habit in order to eliminate the consumption externality generated by their external habit but retain the desirable asset pricing properties of their specification. (See Ljungqvist and Uhlig 2015 for the implications of this externality.) We show later that our specification implies results nearly identical to those implied by their specification. With the *exogenous habit* X_t, a family's utility is given by

$$(9) \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - X_t)^{1-\alpha}}{1-\alpha}.$$

In a symmetric equilibrium, the consumption of the representative family C_t equals aggregate consumption and we can define the *aggregate surplus consumption ratio* as $S_t = (C_t - X_t)/C_t$ so that aggregate marginal utility is $\beta^t (C_t - X_t)^{-\alpha} = \beta^t C_t^{-\alpha} S_t^{-\alpha}$. As in Campbell and Cochrane (1999), we specify the law of motion for the exogenous habit X_t indirectly by specifying a law of motion for the aggregate surplus consumption ratio S_t. Specifically, we assume that $s_t = \log(S_t)$ is an autoregressive process given by

$$(10) \quad s_{t+1} = (1 - \rho_s)\, s + \rho_s s_t + \lambda_a(s_t)(\Delta a_{t+1} - \mathbb{E}_t \Delta a_{t+1}),$$

where $a_t = \log(A_t)$ and s denotes the mean of s_t. The *sensitivity function* $\lambda_a(s_t)$ is defined as

$$(11) \quad \lambda_a(s_t) = \frac{1}{S} \left[1 - 2\,(s_t - s)\right]^{1/2} - 1$$

when the right side of (11) is nonnegative and zero otherwise. As in Campbell and Cochrane (1999), the function $\lambda_a(s_t)$ is chosen so that in a downturn following a technology shock, risk aversion rises sharply but the risk-free rate does not. The *pricing kernel* for the economy is

$$(12) \quad Q_{t,t+1} = \beta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\alpha}.$$

This kernel determines the intertemporal price of consumption goods and is the discount factor used by individual consumers and firms. Using similar notation, we let $Q_{t,r} = \beta^{r-t} [S_r C_r / (S_t C_t)]^{-\alpha}$ denote the discount factor for period $r \geq t+1$ in units of the period-t consumption good.

Since each family is identical, has access to complete one-period contingent claims against aggregate risk, and the prices of contingent claims are related in the usual fashion to the marginal rate of substitution in (12), for notational simplicity we do not explicitly include these claims in the budget constraint of a family, which can then be written as

$$(13) \quad C_t + I_t = W_t + \Pi_t + H_t,$$

where I_t are the total resources invested by a family to create new vacancies, W_t are the total wages of employed consumers of the family, Π_t are the profit flows of the firms that the family owns, and H_t is the total home production of unemployed consumers of the family. In equilibrium, $I_t = \kappa A_t \int_z z v_t(z)dz$, $W_t + \Pi_t = A_t \int_z z e_t(z)dz$, and $H_t = bA_t \int_z z u_t(z)dz$.

Note for later that the risk-free rate $R_{ft} = \exp(r_{ft})$, namely, the return on a claim purchased at t to one unit of consumption in all states at $t+1$, satisfies $R_{ft} = 1/\mathbb{E}_t Q_{t,t+1}$. More generally, the return R_{t+1} on any asset in $t+1$ must satisfy the first-order condition $1 = \mathbb{E}_t Q_{t,t+1} R_{t+1}$. By a standard argument in Hansen and Jagannathan (1991), this fact implies that the (log) *Sharpe ratio* of any asset, defined here as the ratio of the log of the conditional mean excess return on an asset, $\log(\mathbb{E}_t(R_{t+1}/R_{ft}))$, to the conditional standard deviation of the log excess return, $\sigma_t(\log(R_{t+1}/R_{ft}))$, must satisfy

$$(14)\quad \left|\frac{\log(\mathbb{E}_t(R_{t+1}/R_{ft}))}{\sigma_t(\log(R_{t+1}/R_{ft}))}\right| \leq \sigma_t(\log(Q_{t,t+1})) = \alpha[1+\lambda_a(s_t)]\sigma_t(\Delta c_{t+1})$$

when returns are lognormally distributed.[5] The right side of this Hansen-Jagannathan bound, $\alpha[1 + \lambda_a(s_t)]\sigma_t(\Delta c_{t+1})$, is the highest possible Sharpe ratio in this economy, the *maximum Sharpe ratio*, which is a common measure of the *price of risk*. As Campbell and Cochrane (1999) showed, a critical feature of these type of preferences is that the price of risk varies with the exogenous state s_t so that when the state is low, the price of risk is high, and risky investments are not that attractive. This feature of the price of risk will prove critical to generating volatility in the job-finding rate and unemployment in our model.

C. Comparison with Original Campbell-Cochrane Preferences

Our preferences with exogenous habit are very similar to those in Campbell and Cochrane (1999). The differences are that the exogenous habit X_t in the utility function (9) is replaced in Campbell and Cochrane (1999) by the *external habit* $\bar{X}_t$ whose law of motion is indirectly determined by the process for the corresponding aggregate surplus consumption ratio $\bar{S}_t = \left(\bar{C}_t - \bar{X}_t\right)/\bar{C}_t$,

$$(15)\quad \bar{s}_{t+1} = (1-\rho_s)\,\bar{s} + \rho_s \bar{s}_t + \lambda(\bar{s}_t)(\Delta\bar{c}_{t+1} - \mathbb{E}_t \Delta\bar{c}_{t+1}),$$

where $\bar{s}_t = \log(\bar{S}_t)$ and the associated sensitivity function is given by $\lambda(\bar{s}_t) = \left[1 - 2\left(\bar{s}_t - \bar{s}\right)\right]^{1/2}/\bar{S} - 1$ when $\lambda(\bar{s}_t)$ is nonnegative and by zero otherwise. Note that the law of motion for surplus consumption in (10) in the exogenous habit specification is driven by innovations to the growth rate of aggregate productivity, Δa_t, whereas the corresponding law of motion in Campbell and Cochrane (1999) is driven by innovations to the growth rate of aggregate consumption, $\Delta\bar{c}_t$. In the economy in Campbell and Cochrane (1999), consumption is exogenous so that $\Delta\bar{c}_t = \Delta a_t$ and these two specifications are identical. In our production

[5] Alternatively, the same first-order condition implies that the *level* of the Sharpe ratio for any asset return R_{t+1} satisfies

$$\mathbb{E}_t(R^e_{t+1})/\sigma_t(R^e_{t+1}) = -\mathrm{Corr}_t(Q_{t,t+1}, R^e_{t+1})\sigma_t(Q_{t,t+1})/\mathbb{E}_t(Q_{t,t+1}) \leq \sigma_t(Q_{t,t+1})/\mathbb{E}_t(Q_{t,t+1}),$$

where $R^e_{t+1} = R_{t+1} - R_{ft+1}$ is the excess return. With $Q_{t,t+1}$ conditionally lognormal, the maximal Sharpe ratio in levels is

$$\max_{\{\text{all assets}\}}[\mathbb{E}_t(R^e_{t+1})/\sigma_t(R^e_{t+1})] = \left\{\exp\left\{\alpha^2[1+\lambda_a(s_t)]^2\sigma_t^2(\Delta c_{t+1})\right\} - 1\right\}^{1/2} \cong \alpha[1+\lambda_a(s_t)]\sigma_t(\Delta c_{t+1}).$$

Our definition of the (log) Sharpe ratio implies $\alpha[1+\lambda_a(s_t)]\sigma_t(\Delta c_{t+1})$ is an exact, rather than an approximate, upper bound.

economy, in contrast, consumption is not identical to productivity. As we show later, though, these two specifications lead to nearly identical quantitative results.

D. Competitive Search Equilibrium

We set up a competitive search equilibrium in the spirit of the market utility approach in Montgomery (1991). See also Moen (1997) and, for an extensive review of the literature, Wright et al. (forthcoming). Let $\mathbb{Z}_t$ be the set of human capital levels among the unemployed in period t. Since we assume free entry, we can think of there being a large number of firms that in period t search for workers with any given level of human capital $z \in \mathbb{Z}_t$. Each period t consists of two stages. In stage 1, any firm that searches for workers with human capital z posts vacancies for such workers and commits to a *wage offer* for a resulting match, $W_{mt}(z)$, defined as the present value of the wages paid over the course of the match with a worker of type z. In stage 2, after having observed all offers, workers of type z choose which market to search in. A market is defined by $(z, W_{mt}(z))$, namely, a skill level and a wage offer for that skill level.[6] These two stages should be thought of as occurring at the beginning of each period t right after aggregate productivity is realized.[7] Then, matches are formed, output is produced, and, at the end of the period, consumption takes place. We now turn to set up and characterize a symmetric equilibrium starting from stage 2.

Stage 2: Consumers Choose Labor Market in Which to Search

We start by considering symmetric histories in which all firms have made the same offers in stage 1 of period t, so that there is only one wage offer $W_{mt}(z)$ for each level of human capital z. We refer to $(z, W_{mt}(z))$ as the *common market*. We refer to the present value of all payments to a worker with human capital z from future home production and future employment after a match formed at t dissolves as the *post-match value* at t, denoted by $W_{pt}(z)$, which is given recursively by

$$(16) \quad W_{pt}(z) = \sigma \mathbb{E}_t Q_{t,t+1} U_{t+1}(z') + (1-\sigma)\mathbb{E}_t Q_{t,t+1} W_{pt+1}(z')$$

with $z' = (1+g_e)z$. Of course, the total value of a new match to a worker is $W_t(z) = W_{mt}(z) + W_{pt}(z)$, since the current match pays $W_{mt}(z)$ and the worker's post-match value is $W_{pt}(z)$. We decompose the total value of a match to a worker into these two terms so as to clearly distinguish the part that a firm chooses, namely, $W_{mt}(z)$, and the part that a firm takes as given, namely, $W_{pt}(z)$. The value of unemployment $U_t(z)$ is

$$(17) \quad U_t(z) = bA_t z + \mathbb{E}_t Q_{t,t+1}\{\lambda_{wt+1}(\theta_{t+1}(z'))[W_{mt+1}(z') + W_{pt+1}(z')] + [1-\lambda_{wt+1}(\theta_{t+1}(z'))]U_{t+1}(z')\}$$

[6] Rather than envisioning one large market with many firms that make the same wage offer, we find it useful to think of every firm as potentially creating its own market through its wage offer and of workers as freely flowing between these markets until the value of search $\mathcal{W}_t(z)$, defined later, is equated across them. Given a set of wage offers from all markets, the associated levels of market tightness are determined by the equality of the value of search across markets. As a convention, we interpret two or more markets with identical human capital and offers as sub-markets of the same market.

[7] In a monthly model like ours, one might think of these stages as all occurring early on the morning of the first day of a month. Then, on the same day, consumers and firms match and produce that day and for the rest of the month.

with $z' = (1+g_u)z$. The *value of search* for a worker with human capital z in market $(z, W_{mt}(z))$ is

$$(18) \quad \mathcal{W}_t(z) = \lambda_{wt}(\theta_t(z))[W_{mt}(z) + W_{pt}(z)] + [1 - \lambda_{wt}(\theta_t(z))]U_t(z).$$

Since a firm needs to anticipate workers' behavior in stage 2 when it contemplates an arbitrary wage offer in stage 1, we also need to determine outcomes in stage 2 for any such offer. Given that we focus on a symmetric equilibrium, we need only consider asymmetric histories at the beginning of stage 2 in which all firms but one have offered $W_{mt}(z)$ and one has offered, say, $\tilde{W}_{mt}(z)$. Consider then markets $(z, W_{mt}(z))$ and $(z, \tilde{W}_{mt}(z))$. The tightness $\theta_t(z)$ of market $(z, W_{mt}(z))$ satisfies the free-entry condition defined later in (22). The tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_{mt}(z))$ is determined as follows. As long as the wage offer $\tilde{W}_{mt}(z)$ is sufficiently attractive, workers flow between markets $(z, W_{mt}(z))$ and $(z, \tilde{W}_{mt}(z))$ until the value of search in the two markets is equated. In this case, $\tilde{\theta}_t(z)$ is determined by the *worker participation constraint* $\tilde{\mathcal{W}}_t(z) = \mathcal{W}_t(z)$, which can be expressed as

$$(19) \quad \lambda_{wt}(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z)+W_{pt}(z)]+[1-\lambda_{wt}(\tilde{\theta}_t(z))]U_t(z) = \lambda_{wt}(\theta_t(z))[W_{mt}(z)+W_{pt}(z)]+[1-\lambda_{wt}(\theta_t(z))]U_t(z)$$

with $\tilde{\mathcal{W}}_t(z)$ defined by the left side of this equality. Alternatively, if the wage offer $\tilde{W}_{mt}(z)$ is so low that the left side of (19) is less than the right side even with a job-finding rate $\lambda_{wt}(\tilde{\theta}_t(z))$ of one in that $\tilde{W}_{mt}(z)+W_{pt}(z) < \mathcal{W}_t(z)$, then $\tilde{\theta}_t(z) = 0$ and no workers flow to market $(z, \tilde{W}_{mt}(z))$. In this case, although workers can find a job with probability one in market $(z, \tilde{W}_{mt}(z))$, they prefer to search in the common market $(z, W_{mt}(z))$.

Note that by the one-shot deviation principle, we have maintained that after period t, regardless of whether a worker accepts the offer $W_{mt}(z)$ in market $(z, W_{mt}(z))$ or the offer $\tilde{W}_{mt}(z)$ in market $(z, \tilde{W}_{mt}(z))$, the worker takes as given the same set of value functions $\{U_r(z)\}_{r=t}^{\infty}$ and so $\{W_{pr}(z)\}_{r=t}^{\infty}$ in any period $r \geq t$ resulting from future home production and employment. Note for later that if a firm makes the symmetric wage offer $\tilde{W}_{mt}(z) = W_{mt}(z)$, then by the participation constraint (19), the tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_{mt}(z))$ is the symmetric one $\theta_t(z)$. Thus, we can think of workers' optimal search strategies as specifying the behavior that firms in stage 1 anticipate will determine the tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_{mt}(z))$ in stage 2 given any offer $\tilde{W}_{mt}(z)$ such that $\tilde{W}_{mt}(z) + W_{pt}(z) \geq \mathcal{W}_t(z)$.

Finally, at the end of stage 2 of period t, each family consumes C_t.

Stage 1: Firms Choose Contingent Wage Offers and Post Vacancies

Consider the problem of any given firm targeting a worker with human capital z in stage 1 of period t when the state is ε^t and aggregate productivity is $A_t = A(\varepsilon^t)$. To set up this problem, given that we focus on a symmetric equilibrium, we allow a firm to choose any possible wage offer $\tilde{W}_{mt}(z)$ when all other firms that search for workers with human capital z make the *symmetric wage offer* $W_{mt}(z)$.

Consider market $(z, W_{mt}(z))$. Any firm targeting a worker of type $z \in \mathbb{Z}_t$ incurs the cost $\kappa A_t z$ to post a vacancy. Denote by $Y_t(z)$ the present value of output produced by a match between a firm and a worker of type z and let $z' = (1+g_e)z$. Since a match dissolves with exogenous probability σ, the present value $Y_t(z)$ can be expressed recursively as

$$(20)\quad Y_t(z) = A_t z + (1-\sigma)\mathbb{E}_t Q_{t,t+1} Y_{t+1}(z').$$

Given a wage offer $W_{mt}(z)$ for workers of type z, the value of a vacancy aimed at such workers is

$$(21)\quad V_t(z) = -\kappa A_t z + \lambda_{ft}(\theta_t(z))[Y_t(z) - W_{mt}(z)] + [1-\lambda_{ft}(\theta_t(z))] \max_{z'}[\mathbb{E}_t Q_{t,t+1} V_{t+1}(z')].$$

Note that the last term in (21) captures the idea that if a firm is unsuccessful in hiring a worker with human capital z in period t, then the firm can search again in period $t+1$ for a worker with any human capital level z' that it chooses. Free entry into market $(z, W_{mt}(z))$ implies that $V_t(z) = 0$ for any t and z so that

$$(22)\quad \kappa A_t z = \lambda_{ft}(\theta_t(z))[Y_t(z) - W_{mt}(z)].$$

Consider now the problem of a firm choosing an offer $\tilde{W}_{mt}(z)$ possibly different from $W_{mt}(z)$. We use the specification of workers' behavior in stage 2 to derive the tightness $\tilde{\theta}_t(z)$ associated with market $(z, \tilde{W}_{mt}(z))$ and restrict attention to *serious offers*, namely, offers that satisfy

$$(23)\quad \mathcal{W}_t(z) \leq \tilde{W}_{mt}(z) + W_{pt}(z)$$

and hence lead to a positive job-filling rate, as discussed earlier. When a firm makes a (serious) offer of $\tilde{W}_{mt}(z)$, the value of a vacancy is

$$(24)\quad \tilde{V}_t(z) = -\kappa A_t z + \lambda_{ft}(\tilde{\theta}_t(z))[Y_t(z) - \tilde{W}_{mt}(z)] + [1-\lambda_{ft}(\tilde{\theta}_t(z))] \max_{z'}[\mathbb{E}_t Q_{t,t+1} \tilde{V}_{t+1}(z')],$$

where $\lambda_{ft}(\tilde{\theta}_t(z))$, determined from $\lambda_{wt}(\tilde{\theta}_t(z))$ in the worker participation constraint (19), is the job-filling rate in market $(z, \tilde{W}_{mt}(z))$. The problem of a firm that posts a vacancy for a worker of type z is then

$$(25)\quad \max_{\{\tilde{W}_{mt}(z), \tilde{\theta}_t(z)\}} \tilde{V}_t(z),$$

subject to the participation constraint (19) and the serious offer constraint (23). The first-order conditions for this problem and the free-entry condition for period $t+1$, namely, $V_{t+1}(z) = 0$, give

$$(26)\quad \frac{\lambda'_{ft}(\tilde{\theta}_t(z))}{\lambda_{ft}(\tilde{\theta}_t(z))}[Y_t(z) - \tilde{W}_{mt}(z)] = -\frac{\lambda'_{wt}(\tilde{\theta}_t(z))}{\lambda_{wt}(\tilde{\theta}_t(z))}[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)].$$

In a symmetric equilibrium, this condition becomes

$$(27)\quad \frac{\lambda'_{ft}(\theta_t(z))}{\lambda_{ft}(\theta_t(z))}[Y_t(z) - W_{mt}(z)] = -\frac{\lambda'_{wt}(\theta_t(z))}{\lambda_{wt}(\theta_t(z))}[W_{mt}(z) + W_{pt}(z) - U_t(z)]$$

for all firms. Note that this first-order condition, which determines $\theta_t(z)$ given the values $Y_t(z)$, $W_{mt}(z)$, $W_{pt}(z)$, and $U_t(z)$, is the key condition that guarantees a competitive search equilibrium is efficient. A simple way to see this result is to observe that if we multiply both sides of (27) by $\theta_t(z)$, and use $\eta_t(\theta_t(z)) =$

$-\theta_t(z)\lambda'_{ft}(\theta_t(z))/\lambda_{ft}(\theta_t(z))$ and $1-\eta_t(\theta_t(z)) = \theta_t(z)\lambda'_{wt}(\theta_t(z))/\lambda_{wt}(\theta_t(z))$, then this condition is equivalent to the Hosios condition for Nash bargaining, which in turn implies the efficiency of equilibrium.

E. Equilibrium: Definition and Characterization

A collection of state-contingent sequences $\{C_t, Q_{t,t+1}, S_t\}_{t=0}^{\infty}$ and $\{W_{mt}(z), W_{pt}(z), U_t(z), \mathcal{W}_t(z), Y_t(z),$- $V_t(z), \theta_t(z), e_t(z), u_t(z), v_t(z)\}_{t=0}^{\infty}$ is a *competitive search equilibrium* if: *i)* for each t, taking as given $W_{pt}(z)$, $U_t(z)$, $\mathcal{W}_t(z)$, $Y_t(z)$, $V_t(z)$, and $Q_{t,t+1}$, the wage offer $W_{mt}(z)$ and market tightness $\theta_t(z)$ solve the firm's problem (25), *ii)* $\{W_{pt}(z), U_t(z), \mathcal{W}_t(z), Y_t(z), V_t(z)\}_{t=0}^{\infty}$ satisfy the valuation equations (16), (17), (18), (20), and (21), *iii)* the law of motions for employment and unemployment satisfy (6) and (7), *iv)* the free-entry condition (22) holds, *v)* the resource constraint (8) holds, and *vi)* the pricing kernel $\{Q_{t,t+1}\}$ satisfies (12). We turn now to characterizing equilibrium. We first show that since market production, home production, and the cost of posting vacancies scale with z, all equilibrium value functions are linear in z. Thus, market tightness, job-finding rates, and job-filling rates are independent of z. In establishing this result, we let W_{mt} denote $W_t(1)$ and use similar notation for the remaining values.

Lemma 1 (Linearity of Competitive Search Equilibrium)**.** *In a competitive search equilibrium, labor market tightness $\theta_t(z)$, the job-finding rate $\lambda_{wt}(\theta_t(z))$, the job-filling rate $\lambda_{ft}(\theta_t(z))$, and the elasticity $\eta_t(\theta_t(z))$ are independent of z, and values are linear in z in that $W_{mt}(z) = W_{mt}z$, $W_{pt}(z) = W_{pt}z$, $U_t(z) = U_tz$, $\mathcal{W}_t(z) = \mathcal{W}_tz$, and $Y_t(z) = Y_tz$.*

This result implies that to solve for equilibrium values, we do not need to record the measures $e_t(z)$ and $u_t(z)$ but, rather, only the aggregate human capital of employed and unemployed workers given by $Z_{et} = \int_z ze_t(z)\,dz$ and $Z_{ut} = \int_z zu_t(z)\,dz$. Integrating (6) and (7) gives the transitions laws for the aggregate human capital of employed and unemployed workers,

$$(28)\quad Z_{et} = \phi(1-\sigma)(1+g_e)Z_{et-1} + \phi\lambda_{wt}(1+g_u)Z_{ut-1},$$

$$(29)\quad Z_{ut} = \phi\sigma(1+g_e)Z_{et-1} + \phi(1-\lambda_{wt})(1+g_u)Z_{ut-1} + 1 - \phi,$$

which can be used to express the aggregate resource constraint as

$$(30)\quad C_t \leq A_tZ_{et} + bA_tZ_{ut} - \kappa A_t\phi\theta_t(1+g_u)Z_{ut-1},$$

where we have used that aggregate vacancy costs satisfy $Z_{vt} = \int_z zv_t(z)\,dz = \phi\theta_t(1+g_u)Z_{ut-1}$. In light of Lemma 1, we denote the job-finding rate and the job-filling rate by λ_{wt} and λ_{ft}, respectively.

The next proposition establishes that, for given initial conditions for the aggregate human capital Z_{e-1} and Z_{u-1} of employed and unemployed consumers, the competitive search equilibrium allocations $\{C_t, Z_{et}, Z_{ut}, \theta_t\}$ solve the *planning problem*, namely, maximize (9) subject to (28)-(30). This proposition extends well-known results in the literature as surveyed in Wright et al. (forthcoming).

Proposition 1 (Efficiency of Competitive Search Equilibrium). *The competitive search equilibrium allocations solve the planning problem.*

F. Characterizing the Job-Finding Rate

Consider now the first-order conditions for the planning problem given by

$$\text{(31)}\quad \mu_{et} = A_t + \phi(1+g_e)\mathbb{E}_t Q_{t,t+1}[(1-\sigma)\mu_{et+1} + \sigma\mu_{ut+1}],$$

$$\text{(32)}\quad \mu_{ut} = bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}[\eta_{t+1}\lambda_{wt+1}\mu_{et+1} + \left(1-\eta_{t+1}\lambda_{wt+1}\right)\mu_{ut+1}],$$

$$\text{(33)}\quad \kappa A_t = (1-\eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}),$$

where μ_{et} and μ_{ut} are the multipliers associated with the transition laws for the aggregate human capital of employed and unemployed workers, (28) and (29), and so describe the shadow values of augmenting the stocks of human capital of employed and unemployed workers by one unit. The discount factors $\{Q_{t,t+1}\}$ are defined from the allocations by (12). Note that conditions (31) to (33) are similar to those that arise in standard search models. In particular, equation (31) is analogous to the sum of the value of an employed worker and the value of an employing firm, (32) is analogous to sum of the value of an unemployed worker and the value of an unmatched firm, and (33) is analogous the free-entry condition in those models. The key difference is that in our competitive search equilibrium, the planner takes into account the impact of vacancy creation on job-finding and job-filling rates and, hence, internalizes the search externality generated by firms posting vacancies to attract workers. We can rewrite (33) as

$$\text{(34)}\quad \log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right)\log\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right)$$

with $\chi = (1-\eta)\log\left[(1-\eta)\,B^{\frac{1}{1-\eta}}/\kappa\right]/\eta$ by using that the job-filling rate λ_{ft} and the job-finding rate λ_{wt} are determined by the Cobb-Douglas matching function $m(u,v) = Bu^{\eta}v^{1-\eta}$ we use in our quantitative analysis, which implies that $\lambda_{ft}^{1-\eta} = B\lambda_{wt}^{-\eta}$ since $\lambda_{ft}^{1-\eta} = (Bu^{\eta}v^{1-\eta}/v)^{1-\eta}$ and $\lambda_{wt}^{-\eta} = (Bu^{\eta}v^{1-\eta}/u)^{-\eta}$. Expression (34) makes it clear that the job-finding rate λ_{wt} is completely determined by the value $\mu_{et} - \mu_{ut}$ of hiring a worker scaled by aggregate productivity, A_t, up to constants.

Given $\{Q_{t,t+1}\}$, the multipliers μ_{et} and μ_{ut} are solutions to the dynamical system determined by (31) and (32). To develop intuition for the solution to this system, we consider an approximation to it in which we ignore the variation in the future job-finding rates by assuming that $\lambda_w(\theta_s) = \lambda_w(\theta)$, $s > t$, for a given θ. (In the Appendix, we show that the intuition we develop here and later continues to hold when we drop the assumption of constant λ_{wt+s} for all s. In our quantitative analysis, we solve this system through an accurate global nonlinear algorithm described later that involves no approximation.) Imposing the limiting condition that the discounted values of future multipliers converge to zero, we solve the dynamical system forward to obtain[8]

$$(35)\quad \begin{bmatrix}\mu_{et}\\ \mu_{ut}\end{bmatrix} = \sum_{n=0}^{\infty}\phi^n \begin{bmatrix}(1+g_e)(1-\sigma) & (1+g_e)\sigma\\ (1+g_u)\eta\lambda_w & (1+g_u)(1-\eta\lambda_w)\end{bmatrix}^n \begin{bmatrix}1\\ b\end{bmatrix}\mathbb{E}_t Q_{t,t+n}A_{t+n}.$$

It is apparent from (35) that the value $\mu_{et} - \mu_{ut}$ of hiring a worker on the right side of (34) depends on the present value of aggregate productivity, which can be expressed as the present value of the surplus flows from a match between a firm and a worker, namely,

$$(36)\quad \mu_{et} - \mu_{ut} = \sum_{n=0}^{\infty}\mathbb{E}_t Q_{t,t+n}v_{t+n}.$$

Here $v_{t+n} = (c_\ell\delta_\ell^n + c_s\delta_s^n)A_{t+n}$ is the *surplus flow* in period $t+n$ from a match formed in period t, which is proportional to aggregate productivity in $t+n$, and δ_ℓ and δ_s are the large and small eigenvalues (or *roots*) of the two-by-two matrix in the vector difference equation given by (35) with corresponding weights c_ℓ and c_s derived later. The present value of these flows on the right side of (36) decays over time, because an employed worker can lose a job and an unemployed worker can find one. Critically, as we elaborate later, the present value of these flows decays more slowly the larger is the growth of human capital when a consumer is employed and the larger is the decline of human capital when a consumer is unemployed. That is, the persistence that the presence of human capital imparts to surplus flows implies that these flows have long durations. This feature will prove critical in amplifying the impact of aggregate shocks on the labor market.

3. Quantification and Algorithm

We begin by describing how we choose parameters for our quantitative analysis and discuss the model's steady-state implications. The model is monthly and its parameters are listed in Table 1: seven parameters, $\{B, b, \sigma, \eta, \phi, \rho_s, g_e\}$, are assigned and the remaining six, $\{g_a, \sigma_a, \kappa, \beta, S, \alpha\}$, are chosen to match six moments from the data. We set B equal to the mean job-finding rate in the deterministic steady state. Following Ljungqvist and Sargent (2017), we fix the home production parameter b to 0.6 and the matching function elasticity η to 0.5. We choose the separation rate σ equal to 2.8% to match the Abowd-Zellner corrected estimate of the separation rate by Krusell et al. (2017) based on data from the Current Population Survey (CPS).[9] We set the survival probability ϕ to be consistent with an average working life of 30 years and the growth rate of human capital during employment, g_e, to 3.5% per year. Note that taking into account an aggregate productivity growth of 2.2% per year, this rate matches the average annual growth rate of real hourly wages documented by Rubinstein and Weiss (2006, Table 2b) based on the 1979-2000

[8] Namely, $\lim_{T\to\infty}\mathbb{E}_t\left[\Psi(\theta)^{T-t}Q_{t,T}\frac{A_T}{A_t}\right]\begin{bmatrix}\tilde{\mu}_{eT}\\ \tilde{\mu}_{uT}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$ where $\Psi(\theta) = \phi\begin{bmatrix}(1+g_e)(1-\sigma) & (1+g_e)\sigma\\ (1+g_u)\eta\lambda_w & (1+g_u)(1-\eta\lambda_w)\end{bmatrix}$.

[9] This statistic is lower than the 3.4% monthly separation rate used by Shimer (2005) due to our correction for potential misclassification. We also experimented with a recalibration in which we used the higher separation rate in Shimer (2005) and found very similar results. As it will become evident, employment responses in our model are mainly determined by the duration of surplus flows from a match rather than by the length of time a worker spends in any given match.

waves of the National Longitudinal Survey of Youth (NLSY) for workers with up to 25 years of labor market experience. To clarify that our results do not rely on the rate of depreciation of human capital during unemployment, we set g_u to zero in our baseline. We later explore the sensitivity of our findings to lower rates of human capital accumulation and higher rates of human capital depreciation. As we will discuss, our results hold for a wide range of values for g_e and g_u. In particular, a locus of pairs (g_e, g_u) exists with identical predictions for the job-finding rate.

To pin down the persistence ρ_s of the log surplus consumption ratio s_t, we follow Mehra and Prescott (1985), Campbell and Cochrane (1999), and Wachter (2006) and interpret dividends as claims to aggregate consumption in the model and as claims to aggregate dividends in the data from the Center for Research in Security Prices (CRSP). Based on this strategy, we choose ρ_s to match the observed autocorrelation of log price-dividend ratios. We note for later that when we do so, the standard deviation of the log price-consumption ratio in the model is 82% of the standard deviation of the log price-dividend ratio in the CRSP data, namely 0.36 versus 0.44.

We turn now to the endogenously chosen parameters. We choose the parameters g_a and σ_a of the exogenous aggregate productivity process to match the mean and standard deviation of labor productivity growth from the Bureau of Labor Statistics (BLS) for the period between January 1947 and December 2007.[10] To pin down the vacancy posting cost κ, we normalize the value of market tightness θ to 1 in the steady state, as in Shimer (2005), and then choose κ to reproduce a mean unemployment rate of 5.9% based on data from the BLS between January 1948 and December 2007.

Consider next the preference parameters, $\{\beta, S, \alpha\}$. We choose the rate of time preference β and the mean S of the state S_t to match the mean and the standard deviation of the real risk-free rate r_{ft} measured as $i_t - \mathbb{E}_t\pi_{t+1}$, where i_t is the one-month Treasury bill rate and $\mathbb{E}_t\pi_{t+1}$ is expected inflation.[11] To see how the mean S of the process governing the state S_t can be chosen to generate only a modest volatility in the risk-free rate r_{ft}, defined by $\exp(r_{ft}) = 1/\mathbb{E}_t Q_{t,t+1}$, note that when consumption is conditionally lognormally distributed, as it is approximately the case in our model, the real risk-free rate is

$$(37)\quad r_{ft} = -\log(\beta) + \alpha\mathbb{E}_t\Delta c_{t+1} + \alpha\mathbb{E}_t\Delta s_{t+1} - \frac{\alpha^2[1+\lambda_a(s_t)]^2}{2}\sigma_t^2\left(\varepsilon_{ct+1}\right),$$

where $\sigma_t(\varepsilon_{ct+1})$ is the conditional standard deviation of the innovation to consumption growth. Thus, the impact of $\sigma_t(\varepsilon_{ct+1})$ on r_{ft} is affected by the level of S through $\lambda_a(s_t)$ by (11).

We choose the inverse elasticity of intertemporal substitution α in the model so that the mean maximum Sharpe ratio in the model matches the Sharpe ratio of the aggregate stock market return measured

[10] The actual variable is "Nonfarm Business Sector: Real Output Per Hour of All Persons." Note that we use data starting from 1947 to guarantee that the time series for productivity *growth* conforms to our time series for unemployment, which covers the period between 1948 and 2007 as in Shimer (2012).

[11] We compute the real rate r_{ft} as $i_t - \mathbb{E}_t\pi_{t+1}$, where we measure i_t using an updated version of the Fama and French's (1993) data for the thirty-day Treasury bill rate available from Kenneth French's website. We measure $\mathbb{E}_t\pi_{t+1}$ as the inflation predicted from a regression of monthly CPI inflation on twelve of its lags, as is common in the literature.

from the CRSP value-weighted stock index, which covers all firms continuously listed on NYSE, AMEX, and NASDAQ.[12] This strategy is similar to that used by Campbell and Cochrane (1999) and Wachter (2006) in a very related context.

Lastly, we briefly describe the global algorithm that we use to solve the model and provide more details in the Appendix. Asset prices in our model are highly nonlinear in the state (s_t, Z_{et}, Z_{ut}) so we found, as Wachter (2005) did, that only a global solution provides accurate approximations. Accordingly, we use Chebyshev polynomials for policy rules and evaluate expectations by a Gauss-Hermite quadrature with a sufficiently large number of nodes so that results are not sensitive to changes in the number of nodes. Specifically, we use Chebyshev polynomials of degree twenty in the surplus consumption state s_t and of degree five in each of the human capital states Z_{et} and Z_{ut}. We follow Wachter (2005) in allowing for a large fine grid over the surplus consumption space that, crucially, places many grid points close to zero.

4. Findings

Shimer (2012) has argued that a key issue confronting existing search models is that they generate much too little variation in the job-finding rate, which accounts for over two-thirds of the observed fluctuations in unemployment. Accordingly, our study is focused solely on a mechanism that increases the volatility of the job-finding rate. For this reason, we purposely abstract from fluctuations in the job-separation rate and compare, across the model and the data, statistics on the job-finding rate and a constant-separation unemployment rate implied by it as in Shimer (2012), which we discuss next. We then turn to the model's implications for the prices of stocks and bonds and their returns.

A. Job-Finding Rates and Unemployment

As Table 1 shows, our model produces a volatility of the job-finding rate of 6.60%, which is very similar to that in the data, 6.66%. The autocorrelation of the job-finding rate in the model, 0.98, is also close to that in the data, 0.94. Note, though, that even if our model exactly matched the observed time series for the job-finding rate, it would not be able to match the observed time series for the unemployment rate, because the separation rate varies in the data whereas it is constant in our model. To address this issue, we follow Shimer (2012) and construct a *constant-separation unemployment rate* series $\{\bar{u}_t\}$ from data on unemployment from the BLS between 1948 and 2007 with law of motion $\bar{u}_{t+1} = \sigma(1-\bar{u}_t)+(1-\lambda_{wt+1})\bar{u}_t$ and σ set as in our baseline (2.8%), which implies an average unemployment rate of 5.9%. See Shimer (2012) for details. For brevity, both in Table 1 and hereafter, we refer to this series as simply the *unemployment rate*.

Table 1 shows that our model successfully matches the volatility of this constant-separation unem-

[12] Note that we would have obtained similar results by using data from the Flow of Funds, since, as shown by Larrain and Yogo (2008), the returns measured from CRSP are highly correlated with the returns on the aggregate stock market measured from the Flow of Funds. In our sample, this correlation is of 0.97. Note also that in our model, the Sharpe ratio of a consumption claim, 0.446, is very close to the maximum Sharpe ratio, 0.449.

ployment rate in the data, 0.75%, and implies a serial correlation for it of 0.99 that is similar to that in data, 0.97. Table 1 also shows that our model reproduces well the highly negative correlation between job-finding and unemployment rates, which is -0.98 in the model and -0.96 in the data. This result is consistent with Shimer's (2005) emphasis that unemployment rises in recessions because the job-finding rate falls due to a decline in vacancy creation.

Based on all of these statistics, we conclude that our model solves the unemployment volatility puzzle.

B. Asset Prices: Stocks and Long-Term Bonds

We now discuss our model's implications for stock market returns and long-term bonds.

Stock Market Returns

In the data, flows of payments to equity or debt holders are mostly payments for physical and intangible capital, and depend on firm leverage. Our simple model without either physical or intangible capital features none of these payments and abstracts from leverage. Indeed, as the free-entry condition shows, equity flows in our model are simply payments for the up-front costs of posting job vacancies. For these reasons, we follow the simple approach in the asset pricing literature that dates back at least to Mehra and Prescott (1985) and interprets stocks as claims to streams of aggregate consumption—see, for instance, Campbell and Cochrane (1999) and Wachter (2006). Following this approach, we price claims to streams of aggregate consumption in the model and contrast them to stock prices in the data.

In Table 1, we compare the mean and standard deviation of the excess return, their ratio, and the mean and standard deviation of the (log) price-dividend ratio computed from the Flow of Funds to the corresponding statistics on consumption claims implied by our baseline model. As is apparent from the table, the two sets of statistics are indeed close. In this sense, our model has predictions for the stock market in line with the data.

Long-Term Bond Returns

One problem with the early habit model of, say, Jermann (1998), is that they implied excessively volatile interest rates. As Cochrane (2008, p. 295) explains, the next generation of habit models, such as Campbell and Cochrane (1999), were designed to overcome this limitation. As Table 1 shows, the standard deviation of the risk-free rate is the same in our model and in the data. Hence, our baseline model does not imply excessively volatile short-term interest rates.

We now turn to examining our model's implications for long-term interest rates and show that our model does not generate a volatility puzzle for such rates either. Two basic approaches have been followed in the literature to analyze long-term interest rates. The first approach is to use Treasury Inflation-Protected Securities, or TIPS, as a measure of real interest rates. The second approach is to augment the economic model of interest with an inflation process and assess the resulting model's implications for nominal interest rates. In terms of the data, which we take from Gurkaynak, Sack and Wright (2007, 2010), we note that reliable measures of nominal 20-year bonds are available only from 1981 and reliable measures of TIPS are available only from 1999.

Consider the first approach of directly constructing measures of real interest rates using TIPS. One issue with this approach, as discussed by D'Amico, Kim, and Wei (2018), is that in several well-known periods, these securities were quite illiquid and thus their yields were distorted by sizable liquidity premia. We address this issue by following the strategy by D'Amico, Kim and Wei (2018), who argued that a TIPS-specific liquidity premium can be filtered out from the TIPS yields by regressing TIPS yields on the first three principal components of nominal yields of up to 20-year maturity. To lengthen the sample, we backcast TIPS yields using this same regression on principal components of nominal yields estimated over the post-1999 period to obtain TIPS yields for the period between 1981 and 1999 during which nominal 20-year bonds were actively traded but TIPS were not, as discussed by Gurkaynak et al. (2007).

In Table 1, we report the mean and standard deviation of *real* yields on 20-year bonds constructed using this strategy as well as the corresponding statistics for real yields in our baseline model. The mean yield in the model, 3.75, is a bit lower than that in the data, 4.81, but the standard deviation of these yields in the model, 2.20, is quite close to that in the data, 2.00.

Consider next the second approach, which we implement following Wachter (2006). We first solve for all real variables in our model. We then append to the model a purely exogenous process for inflation with shocks that are correlated with those to the real side of the economy.[13] By construction, though, inflation has no effect on real variables. It is worth noting that, as in Wachter (2006), nominal bonds carry a positive risk premium over real bonds because the estimated process for inflation implies that inflation is high when surplus consumption is low, so that nominal bonds buy fewer goods when consumers have a high marginal utility of consumption. In practice, this inflation-risk premium on nominal bonds is very small, about 25 basis points. (See the Appendix for details.)

Given the estimated parameters for the inflation process, we can compare nominal yields in the model to those in the data. As reported in Table 1, the mean yield on 20-year bonds is very similar across the model and in the data, 7.73 versus 7.71, and the standard deviation of nominal yields in the model is also very close to that in the data, 2.28 versus 2.41.

In summary, our model does not display a risk-free rate puzzle at either short or long horizons.

5. Two Critical Ingredients: Time-Varying Risk and Human Capital

Here we demonstrate the critical roles played by time-varying risk and human capital accumulation for our results. Without either ingredient, our model would not generate volatile job-finding rates or unemployment. To illustrate the role of time-varying risk, we study a model with CRRA preferences and show that such a model implies no volatility for the job-finding rate or unemployment. To illustrate the

[13] Specifically, we follow Wachter (2006) in estimating an affine process for consumption and inflation with correlated errors and then using this estimated process to pin down the process for inflation.

role of human capital accumulation, we study a model with constant human capital and show that such a model generates negligible fluctuations in labor market variables.

A. Role of Time-Varying Risk

Here we investigate the importance of time-varying risk in our model by showing that as the degree of risk in the economy decreases, so does the volatility of the job-finding rate. We begin with a stark example in which we mute time-varying risk by eliminating the external habit in consumption so that our preferences reduce to preferences with constant relative risk aversion. We then show that as we reduce the degree of risk in the economy, the volatility of both the labor market and the stock market decreases.

Constant Relative Risk Aversion Preferences

Consider a version of our model in which we set $S_t = 1$ so that our utility reduces to the CRRA form

$$(38)\quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\alpha}}{1-\alpha} \right).$$

For this specification of preferences, we can show that the fluctuations of the job-finding rate and unemployment are identically zero.

Proposition 2 (Constant Job-Finding Rate and Unemployment Under CRRA)**.** *Starting from the steady-state values of the total human capital of employed and unemployed workers, Z_e and Z_u, with preferences of the form in (38), both the job-finding rate and unemployment are constant.*

In interpreting this result, it is important to note that, by construction, we have abstracted from the standard mechanism of *differential productivity across sectors* of search models, which implies that a decrease in aggregate productivity A_t reduces a consumer's productivity in market production but leaves a consumer's productivity in home production and the cost of posting vacancies unaffected. Hence, by this mechanism alone, a reduction in aggregate productivity leads to an increase in unemployment. In our model, instead, an increase in A_t increases equally a worker's productivity in market and home production and the cost of posting vacancies. In particular, a consumer with human capital z produces $A_t z$ when employed and $bA_t z$ when unemployed, and it costs a firm $\kappa A_t z$ to post a vacancy for such a consumer. Therefore, the only effect of a change in aggregate productivity in our model is that it changes the expected discounted value of the surplus from a firm-worker match scaled by current productivity, as the right side of (34) shows, on which λ_{wt} depends. With CRRA preferences and random-walk productivity shocks, this expected value is constant and so are the job-finding rate and unemployment.

To see why, note that by substituting for the surplus flow $v_{t+n} = (c_\ell \delta_\ell^n + c_s \delta_s^n) A_{t+n}$ and the pricing kernel $Q_{t,t+n}$ for the CRRA pricing kernel $(C_{t+n}/C_t)^{-\alpha}$ into (36), we can rewrite the right side of (34) using

$$(39)\quad \frac{\mu_{et} - \mu_{ut}}{A_t} = \sum_{n=0}^{\infty} \beta^n (c_\ell \delta_\ell^n + c_s \delta_s^n) \mathbb{E}_t \left\{ \left(\frac{A_{t+n}}{A_t} \right)^{1-\alpha} \left(\frac{\tilde{C}_{t+n}}{\tilde{C}_t} \right)^{-\alpha} \right\},$$

where $\tilde{C}_t = C_t/A_t$ is scaled consumption. With these preferences and random walk productivity, consumption moves proportionately to productivity so that scaled consumption $\tilde{C}_t$ is constant in A_t. Moreover, as productivity follows a random walk, its expected growth rate, $\mathbb{E}_t(A_{t+n}/A_t)^{1-\alpha}$, does not vary with A_t. Hence, the right side of this expression is constant, which implies that both the job-finding rate and unemployment are invariant to changes in A_t.

Relation between Labor Market Volatility and Stock Market Volatility

We consider now two experiments in which we reduce the degree of risk in the economy and show that when we do so, both labor market and stock market volatility fall. In this sense, our model generates volatility in the job-finding rate and unemployment only if it also generates volatility in stock prices.

Recall that time-varying risk arises in our model because of risk aversion, which is measured by the coefficient α/S, and because of fluctuations in surplus consumption, S_t. Correspondingly, in the first experiment, we reduce the value of the risk aversion coefficient α/S. In panel (a) of Figure 1, we graph the resulting volatility of the job-finding rate, the volatility of the stock market as measured by the standard deviation of the log price-consumption ratio, and the equity premium as measured by the difference between the mean return on the consumption claim and the risk-free rate. All three statistics are reported in percentage of their level in the baseline. It is apparent from the figure that as we reduce the value of the risk aversion coefficient, the risk in the economy, as reflected in the volatility of the stock market and the equity premium, falls and along with it, the volatility of the job-finding rate.

In the second experiment, we consider the role of the consumption habit in generating risk. To isolate its importance, we vary the standard deviation of the surplus consumption process holding fixed the primitive volatility of the productivity process at its baseline value. Formally, recall that the standard deviation σ_a affects both the process for productivity,

$$(40) \quad \log(A_{t+1}) = g_a + \log(A_t) + \sigma_a \varepsilon_{at+1},$$

and the process for surplus consumption,

$$(41) \quad s_{t+1} = (1-\rho_s)\, s + \rho_s s_t + \lambda_a(s_t)\tilde{\sigma}\varepsilon_{at+1},$$

since $\tilde{\sigma} = \sigma_a$ in the baseline. In this second experiment, we reduce the fluctuations in surplus consumption s_{t+1} by lowering $\tilde{\sigma}$ while keeping the volatility σ_a of the productivity process in (40) unchanged. In panel (b) of Figure 1, we graph the same three variables implied by our model as in panel (a), as we change $\tilde{\sigma}$. As is apparent from the figure, in this case too, as we reduce risk by reducing the variability of the surplus consumption ratio, the equity premium falls together with the volatility of both stock and labor markets.

These two experiments thus make it clear that our model with human capital produces labor market volatility through the same forces by which it produces stock market volatility.

B. Role of Human Capital

Here we consider the role of human capital in generating fluctuations in the job-finding rate and unemployment by examining the implications of alternative values for the rate of human capital accumulation on the job and depreciation off the job. In the next section, we develop further intuition for our findings by characterizing the elasticity of the job-finding rate with respect to the exogenous state of the economy s_t and by analytically showing how this elasticity is affected by human capital acquisition.

In Table 2, we compare our baseline model to one in which we set $g_e = g_u = 0$. In this latter model, which we refer to as *DMP with baseline preferences*, as well as in other variants that we will consider in later sections, we maintain the same parametrization as in the baseline model with the exception of the vacancy cost parameter κ, which is chosen in each instance to ensure that the model reproduces the mean unemployment rate in the data reported in panel B of Table 1. As Table 2 shows, the volatility of job-finding rate in the DMP with baseline preferences drops to about 2% of that in the data (0.15/6.66). Thus, absent human capital, the unemployment rate barely moves. In the last column of Table 2, we consider the *baseline model with* $g_e = g_u = 3.5\%$ so that human capital grows at the same rate regardless of whether a consumer is employed or unemployed. In this case too, the volatility of the job-finding rate is quite low, again approximately 2% of that in the data (0.15/6.66). These results illustrate that it is not the presence of human capital per se that is important for our results, but, rather, the differential growth of human capital on and off the job, which makes hiring a worker an investment with long-duration payoffs.

The two panels of Figure 2 plot the impulse responses of the job-finding rate and unemployment to a one-percent decrease in aggregate productivity, starting from the mean of the state variables S_t, Z_{et}, and Z_{ut}, for two versions of our model: the baseline model and the DMP with baseline preferences.[14] Clearly, the responses of both the job-finding rate and unemployment are much larger in the presence of human capital than in the absence of it.

So far we have considered an extreme scenario in which all of the duration in surplus flows is due to the accumulation of human capital on the job, captured by g_e, by setting the depreciation of human capital off the job, captured by g_u, to zero. A variety of studies, though, have documented that the wage losses following a spell of nonemployment can be substantial. In light of this evidence, we now show that we can decrease the rate of human capital accumulation on the job and, in a manner consistent with the evidence on the wage losses due to nonemployment, correspondingly increase the rate of human capital depreciation off the job, and obtain nearly identical results to those implied by our baseline parametrization.

For instance, a conservative estimate of the degree of human capital depreciation off the job is $g_u = -5.7\%$, which matches the average wage loss after up to one year of nonemployment for workers with

[14] Note that since the model is nonlinear, the response of a variable to a shock depends on the levels of the state variables and the size of the shock. As is standard, we compute the impulse response for, say, the job-finding rate at $t+n$ as $E_t(\lambda_{wt+n}|\varepsilon_t = \Delta, S_t, Z_{ut}, Z_{et}) - E_t(\lambda_{wt+n}|\varepsilon_t = 0, S_t, Z_{ut}, Z_{et})$ with S_t, Z_{ut}, and Z_{et} all set to their means.

fewer than 35 years of labor market experience in the Panel Study of Income Dynamics (PSID).[15] If we set $g_u = -5.7\%$ per year, then a value of g_e of just 2.11% per year is sufficient to generate the same volatility of the job-finding rate and unemployment as under our baseline parametrization. More generally, in Figure 3, we graph loci of values for (g_e, g_u) that give rise to a volatility of the job-finding rate that is equal to a given percentage of the volatility generated by our model under our baseline parametrization. We trace these loci by varying g_e and g_u while keeping all other parameters fixed at their baseline values except for κ, which, as discussed, we adjust to keep the mean unemployment rate unchanged.

Consider first the locus labeled by 100%, which is associated with the same volatility of the job-finding rate as in our baseline model. Clearly, our amplification results hold for very modest rates of human capital accumulation on the job and depreciation off the job relative to standard estimates in the literature. Consider next the locus labeled by 120%. This locus shows that moving from our baseline values of $(3.5, 0)$ for (g_e, g_u) to the values of $(3.5, -13.3)$, which imply the same accumulation rate of human capital on the job but a higher depreciation rate off the job, increases the volatility of the job-finding rate by 20%. Alternatively, moving to the values of $(4.0, -10.5)$ for (g_e, g_u) by increasing both the rate of human capital accumulation on the job and depreciation off the job also increases the volatility of the job-finding rate by 20%.

Indeed, even if we lower the rate of human capital accumulation on the job by a factor of 7 so that $g_e = 0.5\%$, our model still produces 80% of the baseline volatility of the job-finding rate as long as $g_u = -4.5\%$, which is a lower depreciation rate than our conservative estimate of $g_u = -5.7\%$. Moreover, for this much smaller value of g_e than in our baseline, the model still generates 50% of the baseline volatility of the job-finding rate if g_u just equals -1.4, which corresponds to a very low degree of human capital depreciation relative to our conservative estimate.

We therefore conclude that our findings on the volatility of the job-finding rate are not knife-edged results that hold only for a specific parameterization. Rather, they are quite robust to reasonable perturbations of the parameters describing the evolution of human capital.

6. Inspecting the Mechanism

Here we inspect our mechanism by deriving a closed-form solution for the job-finding rate and its dependence on the exogenous state of the economy s_t, based on the simple approximation for the multipliers μ_{et} and μ_{ut} in (35). We also identify a sufficient statistic for the volatility of the job-finding rate that will turn out to be common across all preference structures we consider.

To this purpose, recall that surplus flows follow a second-order difference equation whose solution is such that n-th flow is

(42) $$v_{t+n} = \left(c_\ell \delta_\ell^n + c_s \delta_s^n\right) A_{t+n}$$

[15] We computed this value of g_u using the same sample used by Buchinsky, Fougère, Kramarz, and Tchernis (2010).

and rewrite the expected present discounted value of these flows as $\mathbb{E}_t Q_{t,t+n} v_{t+n} = (c_\ell \delta_\ell^n + c_s \delta_s^n) P_{nt}$, where $P_{nt} \equiv \mathbb{E}_t Q_{t,t+n} A_{t+n}$ is the price of an asset that pays a one-time dividend of A_{t+n} in period $t+n$. We refer to this asset as a claim to productivity in n periods or simply a *productivity strip*. Consider now the solution for $\mu_{et} - \mu_{ut}$ from the system in (35). To keep the algebra simple, we set the survival probability ϕ to one and maintain that g_u is zero. Thus, the large root $\delta_\ell > 1$ and the small root $\delta_s < 1$ of this solution are given by

$$\delta_\ell = 1 + \frac{1}{2}\left[\sqrt{(1-\lambda)^2 + 4\eta\lambda_w g_e} - \sqrt{(1-\lambda)^2}\right] \text{ and } \delta_s = \lambda - \frac{1}{2}\left[\sqrt{(1-\lambda)^2 + 4\eta\lambda_w g_e} - \sqrt{(1-\lambda)^2}\right],$$

and the corresponding weights on these roots are $c_\ell = [(1-b)(\lambda - \delta_s) + b g_e] / (\delta_\ell - \delta_s)$ and $c_s = 1 - b - c_\ell$ with $\lambda \equiv (1-\sigma)(1+g_e) - \eta\lambda_w < 1$.[16] Note that these roots and weights do not depend on either the utility function or the productivity process. Combining these formulae with (34) and (36), we then have:

Proposition 3 (Job-Finding Rate)**.** *The job-finding rate approximately satisfies*

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right) \log\left[\sum_{n=0}^{\infty} (c_\ell \delta_\ell^n + c_s \delta_s^n) \frac{P_{nt}}{A_t}\right], \tag{43}$$

where χ is a constant and δ_ℓ, δ_s, c_ℓ, and c_s are given above.

This proposition shows that the job-finding rate is a weighted average of the prices of claims to future productivity. Hence, movements in the job-finding rate are *only* due to movements in the prices of these claims. Note that this result applies as stated to all preferences we examine. In particular, since the weights $(c_\ell \delta_\ell^n + c_s \delta_s^n)$ are determined solely by the labor market side of the economy and remain fixed as we vary preferences, this formula for the job-finding rate has this same form for all the preferences we consider and differs across them only in terms of the expression for P_{nt}/A_t, which we characterize next for our baseline preferences and in the Appendix for alternative preferences.

We simplify the calculation of the price P_{nt} in (43) by approximating the growth rate of consumption by the growth rate of productivity. Under this approximation, the pricing kernel becomes $Q_{t,t+1} = \beta[S_{t+1}A_{t+1}/(S_t A_t)]^{-\alpha}$. In the next lemma, we derive a risk-adjusted log-linear approximation to $P_{nt}/A_t \equiv \mathbb{E}_t Q_{t,t+n} A_{t+n}/A_t$, which is the price of a claim to the growth rate of productivity A_{t+n}/A_t in $t+n$. To see why we focus on P_{nt}/A_t rather than P_{nt}, observe that since A_t is governed by a random walk process with drift, P_{nt} grows over time and, hence, is nonstationary, whereas the *scaled price* P_{nt}/A_t is stationary.

Lemma 2 (Price of Productivity Claim)**.** *The price P_{nt} of a claim to productivity in n periods approximately satisfies*

$$\log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n(s_t - s), \tag{44}$$

[16] In the general case with ϕ and g_u, $c_\ell = [(\phi\lambda - \delta_s)(1-b) + \phi(g_e - g_u)b]/(\delta_\ell - \delta_s)$, $c_s = 1 - b - c_\ell$, and $\delta_{\ell,s} = \phi(1 + g_u + \lambda)/2 \pm \phi[(1 + g_u - \lambda)^2 + 4\eta\lambda_w(1+g_u)(g_e - g_u)]^{1/2}/2$.

where $a_0 = b_0 = 0$, $a_n = \log(\beta) + (1-\alpha)g_a + a_{n-1} + [1 - b_{n-1} - (\alpha - b_{n-1})/S]^2 \sigma_a^2/2$, *and*

$$b_n = \alpha(1-\rho_s) + \rho_s b_{n-1} + \left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\sigma_a^2. \tag{45}$$

The constant a_n in (44) equals the log of the discount factor β^n up to an adjustment for productivity growth and risk, as captured by g_a and σ_a, and decreases with n as long as the drift rate g_a is not too large. The elasticity b_n of the (scaled) price P_{nt}/A_t with respect to the exogenous state s_t, instead, captures how this price moves with s_t, and increases monotonically from 0 to α provided that $1-\rho_s+(1-\alpha/S)\sigma_a^2/S > 0$, which is easily satisfied by our baseline model.

By (44), since the elasticity b_n increases with the maturity n of a claim, the longer is the maturity of a claim, the more sensitive is the price at horizon n to the exogenous state s_t and so the lower is the price of a long-maturity claim relative to that of a short-maturity one when the state is low ($s_t < s$). To understand why the elasticity b_n increases with n, consider first an economy without risk ($\sigma_a = 0$) for which the expression for b_n in (45) reduces to $b_n = \alpha(1-\rho_s)+\rho_s b_{n-1}$. In this case, then, b_n equals $\alpha(1-\rho_s^n)$ and so increases with n since ρ_s^n decreases with n. This result is due to intertemporal substitution motives. Intuitively, when the exogenous state s_t is below its mean s, since it is expected to revert to s, consumers value current consumption more and so are willing to pay relatively *less* for a claim in the far future, when the state is expected to be much closer to its mean, and relatively *more* for a claim in the near future, when the state is expected to be close to s_t. The third term in (45) is simply an adjustment factor for risk. In the presence of risk, the elasticity b_n still increases with the maturity n albeit at a lower rate, because, all else equal, a precautionary saving motive makes consumers more willing to save, which attenuates the intertemporal substitution motive just discussed.

For our purposes, the key implication of the elasticity b_n increasing with n is that the response of the job-finding rate to a given shock to s_t is larger, the larger are the weights on long-maturity claims to productivity in the surplus flows $v_{t+n} = (c_\ell \delta_\ell^n + c_s \delta_s^n)A_{t+n}$, $n \geq 1$. Since our baseline asset-pricing preferences imply that the prices of long-maturity claims are much more sensitive to such shocks than those of short-maturity ones, our model can then generate large movements in any given asset's price only if the asset features large weights on long-maturity claims. (As we discuss in the Appendix, most existing state-of-the-art asset pricing models share this same property.) It turns out that in the presence of human capital accumulation, the surplus flows from a match are characterized by large weights on long-maturity claims to productivity, as we will discuss. Hence, with our preferences, the present value of surplus flows is sensitive to shocks to s_t. It is precisely through this feature that the combination of our asset-pricing preferences and human capital accumulation generates volatile job-finding rates and unemployment. We formalize these intuitions in the following proposition, where $\sigma(s_t)$ denotes the standard deviation of s_t.[17]

[17] For this result, we compute the approximate solution to our model with θ set at its value at the *risky steady state*, defined as the steady state reached when agents use the decision rules computed in our stochastic equilibrium but with all future shocks set to zero. This value of θ is very close to that in the deterministic steady state. A simple log-linear approximation

Proposition 4 (Sufficient Statistic for Job-Finding Rate Volatility)**.** *Under the approximation in Lemma 2, the response of the job-finding rate to a change in s_t evaluated at a risky steady state is given by*

$$\frac{d\log(\lambda_{wt})}{ds_t} = \left(\frac{1-\eta}{\eta}\right)\sum_{n=0}^{\infty}\omega_n b_n \quad \text{with} \quad \omega_n = \frac{e^{a_n}(c_\ell\delta_\ell^n + c_s\delta_s^n)}{\sum_{n=0}^{\infty} e^{a_n}(c_\ell\delta_\ell^n + c_s\delta_s^n)}, \tag{46}$$

where a_n and b_n are given in Lemma 2 and the standard deviation of the job-finding rate $\sigma(\lambda_{wt})$ satisfies

$$\sigma(\lambda_{wt}) = \frac{d\log(\lambda_{wt})}{ds_t}\sigma(s_t). \tag{47}$$

As argued, since the elasticities $\{b_n\}$ of claims to productivity increase with the horizon of a claim, a change in the exogenous state s_t leads to a large change in the job-finding rate only if the weights ω_n on long-maturity claims to productivity are large. In panel (a) of Figure 4, we graph the global solution for the prices of productivity strips against the exogenous state s_t when the endogenous states (Z_{et}, Z_{ut}) are set to their mean values—namely, we use neither the approximation that $\Delta c_{t+1} \approx \Delta a_{t+1}$ nor the risk-adjusted log-linear approximation of Lemma 2. Note that the prices of longer-maturity strips are indeed much more sensitive to changes in the state s_t than those of shorter-maturity ones. Moreover, as the figure makes clear, log prices are indeed approximately linear in the state s_t. For comparison, we plot in panel (b) the corresponding solution when we use both of the approximations of Lemma 2—namely, that $\Delta c_{t+1} = \Delta a_{t+1}$ and that $\log(P_{nt}/A_t)$ is linear in the state s_t. As is apparent from the two panels, the global and approximate solutions are quite close.

In panel (a) of Figure 5, we graph the impulse responses of these strips to a one-percent decrease in aggregate productivity based on the global solution to our model. Clearly, the prices of short-horizon strips fall little whereas the prices of long-horizon strips fall greatly after this shock. Thus, together with Proposition 4, these figures illustrate that our model generates large variations in the job-finding rate only when the weights $\{\omega_n\}$ are sufficiently large for large n. We now turn to showing that without human capital accumulation, these weights decay very quickly. More generally, the larger is the difference $g_e - g_u$ in the rate of human capital accumulation during employment and during unemployment, the slower these weights decay, and the greater is the response of the job-finding rate to changes in aggregate productivity.

DMP Model with Baseline Preferences. Now, consider the DMP model with our baseline preferences and $g_e = g_u = 0$. In this case, the constant c_ℓ on the large root is zero and the small root, referred to as the *DMP root*, is given $\delta_{DMP} = 1 - \sigma - \eta\lambda_w$, where σ is the separation rate, η is the elasticity of the matching function with respect to the measure of unemployed workers, and λ_w is the job-finding rate. Thus, in the DMP version of our model, surplus flows in the n-th period of a match follow a first-order difference equation with the surplus flow at n proportional to $\delta_{DMP}^n A_{t+n}$. The weight in the corresponding

around the deterministic steady state, however, is not close to the *risk-adjusted* log-linear approximation that allows for terms in σ_a in the formulae for a_n and b_n and, hence, in our sufficient statistic. See Coeurdacier, Rey, and Winant (2011) and Lopez, Lopez-Salido, and Vazquez-Grande (2017) for references, and the Appendix for more details.

expression for $d\log(\lambda_{wt})/ds_t$ is $\omega_n = e^{a_n}\delta_{DMP}^n / \sum_{n=0}^{\infty} e^{a_n}\delta_{DMP}^n$. For standard parametrizations, the DMP root is substantially smaller than one so that surplus flows decay quickly over time. Specifically, with $\delta_{DMP} = 1 - \sigma - \eta\lambda_w$, $\sigma = 2.8\%$, $\eta = 0.5$, and $\lambda_w = 46\%$, which is the mean job-finding rate in the data, it follows that $\delta_{DMP} = 74.2\%$, which amounts to a decay rate of over 25% per month. Hence, $(\delta_{DMP})^{24}$ is 0.08% after only two years. Accordingly, the weights on productivity strips with long maturity are essentially zero. These observations intuitively explain why the DMP model gives rise to an unemployment volatility puzzle.

Baseline Model. By our above formula for the roots of the solution to the system in (35), the large root δ_ℓ is bigger than one and the weight c_ℓ on this root is positive with human capital accumulation so that the discounted value of surplus flows decays slowly over time. In turn, this fact implies that the formula for the job-finding rate in (43) assigns sizable weights to productivity strips with long maturities, which are very sensitive to the exogenous state s_t and, hence, lead to large fluctuations in the job-finding rate in response to productivity shocks. In panel (b) of Figure 5, we report the cumulative weights implied by the DMP model with baseline preferences and those implied by our baseline model without any approximation. Clearly, the weights in the DMP model with baseline preferences decay very quickly relative to those in our baseline model. For a sense of magnitudes, we compute the (Macaulay) *duration* of these weights as $\sum_{n=0}^{\infty} \omega_n n$. The duration of the weights $\{\omega_n\}$ is 3.6 months in the DMP model with baseline preferences and 11 years in the baseline model under our parametrization. The first expression in (46), however, implies that a more relevant measure of the duration of these weights is the elasticity of the job-finding rate with respect to the exogenous state s_t, which is given by $\sum_{n=0}^{\infty} \omega_n b_n$ in our baseline with $\eta = 0.5$.[18] For the DMP model with baseline preferences, this elasticity is 0.03 and for our baseline model, it is 0.89.

Implications for Wages. Here we discuss additional implications of our model for wages. Note that our competitive search equilibrium determines the present value of wages paid to a worker over the course of a match with a firm, but not the flow wage received by a worker each period. More generally, in any model with complete markets and commitment by both firms and workers to a state-contingent employment contract, many alternative sequences of flow wages give rise to the same present value of wages. Hence, in this precise sense, our model does not have specific predictions for flow wages.

Given this indeterminacy, we follow the approach popularized by Barlevy (2008) and Bagger et al. (2014), who assume that when a match is formed in period t, a firm commits to pay a worker each period a share ϱ_t of the period output for the duration of the match. Thus, $w_{t,\tau} = \varrho_t A_\tau z_\tau$ is the wage in period $\tau \geq t$. Accordingly, we determine flow wages in our model as follows. For any present value of wages

[18] The term $\sum_{n=0}^{\infty} \omega_n b_n$ is a measure of duration different from the standard Macaulay one, where instead of weighting the horizon length n by the fraction ω_n of the present value of surplus flows accruing at that horizon at the risky steady state, we weight the elasticity of the price of a claim to productivity at horizon n to the state, namely, b_n by the fraction ω_n.

$W_{mt}(z_t) = W_{mt}z_t$ implied by our model for a match between a firm and a worker with human capital z_t that starts at t, we choose ϱ_t so that the present value of the wages $w_{t,t} = \varrho_t A_t z_t$, $w_{t,t+1} = \varrho_t A_{t+1} z_{t+1}$, and so on, calculated using our stochastic discount factor, exactly equals $W_{mt}(z_t)$.

Using this approach, we examine our baseline model's implications for wages. We first discuss additional evidence on wage growth in support of our parametrization of the human capital process. We then argue that our model is robust to the critique by Kudlyak (2014) of the degree of rigidity of the wage process implied by prominent solutions to the unemployment volatility puzzle. Specifically, we find that our model is consistent with the estimated degree of cyclicality of wages by Kudlyak (2014) and, hence, does not rely on counterfactually rigid wages.

Consider first wage growth, which is the moment we use to pin down the growth of human capital on the job in our model. As noted, we have set the growth rate of human capital, g_e, so as to match *longitudinal* wage growth with experience. We now argue that under the parametrization discussed earlier, the wage process implied by our model also matches the evidence on *cross-sectional* wage growth with experience documented by Elsby and Shapiro (2012). These authors report that the difference between the log real wages of workers with 30 years of experience and those with 1 year of experience is 1.2 in the data.[19] Our baseline model is consistent with this untargeted statistic as it implies a difference of 1.0.

Consider next wage rigidity. As Becker (1962) emphasized, in general, the present discounted value of the wages paid to a worker over the course of an employment relationship is allocative for employment, not the flow wage. Kudlyak (2014) further proved that for a large class of search models, the appropriate allocative wage is the difference in the present values of wages across two matches that start in two consecutive periods as captured by the *user cost of labor*. Intuitively, in a search model, hiring a worker is akin to acquiring a long-term asset subject to adjustment costs. Thus, by measuring the rental price of the services of a worker potentially employed for many years, the user cost of labor is a more suitable measure of the cost of hiring a worker than the flow wage.

Both Kudlyak (2014) and Basu and House (2016) measure the cyclicality of the user cost of labor as the semi-elasticity of the user cost to unemployment. Based on NLSY data, these authors estimate the user cost of labor as $UC_t \equiv PV_t - \beta(1-\sigma)PV_{t+1}$, where PV_t is an empirical measure of the present value of wages from a match that begins at t defined as $PV_t = w_{t,t} + \sum_{\tau=t+1}^{T}[\beta(1-\sigma)]^{\tau-t}w_{t,\tau}$, where $w_{t,\tau}$ is the wage in period $\tau \geq t$ and $\beta(1-\sigma)$ is a fixed discount factor that takes into account the real interest rate and the job separation rate. (See Kudlyak 2014 and Basu and House 2016 for details.) Intuitively, the user cost measures the shadow wage that would make a risk-neutral firm indifferent between hiring a worker today, and creating a match that survives to tomorrow with probability $1-\sigma$, or hiring a worker tomorrow. Importantly, the user cost of labor at t does not just capture the flow wage of new hires at t but also the difference in the present value of wages from $t+1$ on between a worker hired at t and a worker

[19] We consider the census years of 1980, 1990, and 2000 for consistency with the panel horizon of the data of Rubinstein and Weiss (2006), who use the 1979-2000 waves of NLSY in their analysis of wage growth.

hired at $t+1$. Hence, the user cost incorporates any potential extra cost or benefit of committing at t to a (possibly state-contingent) sequence of wage payments from $t+1$ on, relative to waiting and hiring an identical worker at $t+1$ at the present value of wages prevailing at $t+1$. Thus, if recessions are times of *scarring* in that workers hired in downturns not only obtain a lower wage when hired but also in any subsequent period relative to workers hired in upturns, then it is clear that the user cost of labor can be much more cyclical than the flow wage.

Kudlyak (2014) and Basu and House (2016) indeed estimate a semi-elasticity of the user cost of labor to unemployment of -5.2% and -5.8%, respectively, so that a one percentage point increase in the unemployment rate is associated with an approximately 6% decrease in the user cost of labor. Hence, the user cost is quite procyclical. In computing the user cost of labor in our model, we treat the empirical measure of the user cost in Kudlyak (2014) as simply a particular statistic of the allocative wage that takes as inputs a sequence of flow wages $\{w_{t,\tau}\}$ and the fixed discount factor $\beta(1-\sigma)$, according to the above formulae for UC_t and PV_t. Based on the flow wages constructed as described, our model implies a cyclicality of the user cost of labor of -6.4%. Hence, the user cost of labor implied by our baseline model, although untargeted, is in line with the data.[20] Therefore, our mechanism for unemployment volatility does not rely on a counterfactual degree of wage rigidity.

7. Toward a Real and Financial Business Cycle Model

The influential early business cycle work by Merz (1995) and Andolfatto (1996) integrated search theory into real business cycle models. Although ambitious, those contributions did not attempt to make their models consistent with any asset pricing patterns. Since those early contributions, the subsequent literature has mostly shied away from doing so and, instead, focused on models without physical capital. Here we embed our mechanism into a real business cycle model with physical capital, retaining our baseline preferences. We thus construct a simple real and financial business cycle model that solves the unemployment volatility puzzle and is in line with key patterns of job-finding rates, unemployment, output, consumption, investment, and asset prices observed in the data. Note that in contrast to the classic separation result between the real and financial sides of an economy in Tallarini (2000), here we show that the presence of time-varying risk greatly amplifies the fluctuations of real variables.

Consider then the following extension of our baseline model. We assume that capital augments the production of goods in the market so that a consumer with human capital z paired with $K_t(z)$ units of physical capital produces $(A_t z)^{1-\gamma} K_t(z)^\gamma$ when employed. We allow for costs of adjustment of the aggregate capital stock. We maintain the same specifications of the technology for producing goods at

[20] Intuitively, with human capital accumulation, the wage of a worker with human capital z_t is $w_{t,\tau}(z_t) = (1+g_e)^\tau w_{t,\tau} z_t$ in any period τ so that the present value of wages at t used to calculate the user cost of labor at t for such a worker is $PV_t(z_t) = z_t\{w_{t,t} + \sum_{\tau=t+1}^{T} [\beta(1-\sigma)(1+g_e)]^{\tau-t} w_{t,\tau}\}$. Then, for any given sequences of wages $\{w_{t,\tau}\}_\tau$ and $\{w_{t+1,\tau}\}_\tau$ for a match starting at t and $t+1$, respectively, the difference in these present values increases with g_e. In particular, $(1+g_e)^{\tau-t}$ "up-weights" the future terms in the relevant present value differences, thus magnifying the cyclicality of the user cost of labor.

home and vacancies as in our baseline model.

We characterize the competitive search allocations in this economy by solving the planning problem. It is immediate that the economy aggregates in a similar fashion as does the economy of our baseline model. The aggregate resource constraint can then be written as

$$C_t + I_t \leq (A_t Z_{et})^{1-\gamma} K_t^{\gamma} + b_h A_t Z_{ut} - \kappa A_t Z_{vt},$$

where $K_t = \int_z K_t(z) e_t(z) dz$ is the physical capital used by employed consumers. As before, aggregate vacancy costs are given by $Z_{vt} = \int_z z v_t(z) dz = \phi \theta_t (1 + g_u) Z_{ut-1}$. The aggregate capital stock follows the accumulation law $K_{t+1} = (1-\delta) K_t + \Phi(I_t/K_t) K_t$, where $\Phi(I_t/K_t) = \delta\{[I_t/(\delta K_t)]^{1-1/\xi} - 1\}/(1-1/\xi)$ as in Jermann (1998). We set γ equal to 0.26, δ equal to 0.10/12, and choose the curvature parameter ξ of the adjustment cost function so that the model produces a standard deviation of investment growth relative to consumption growth equal to that in the data. Observe that in our baseline model, the home production parameter b can be thought of as measuring the (expected) ratio of the amount of goods produced at home to those produced in the market so that $b = \mathbb{E}[b_h A_t z_t / A_t z_t]$ and thus $b = b_h$. In the model with physical capital, b similarly measures the (expected) ratio of the amount of home-produced goods to market-produced goods, but now $b = \mathbb{E}[b_h A_t z_t / (A_t z_t)^{1-\gamma} k_t^{\gamma}(z_t)]$. To keep this model parallel to our baseline, we set $b = 0.6$ here as well.[21]

As shown in Table 3, this augmented model gives rise to a volatility of the job-finding rate and unemployment that match those in the data: the volatility of the job-finding rate is 6.66% in both the model and the data and that of unemployment is 0.77% in the model and 0.75% in the data. The model also well replicates key observed features of stock and bond returns. In particular, the model does not exhibit the excess volatility of long-term bond returns of the early habit model of Jermann (1998). For example, for nominal long-term bonds, the model implies mean returns (7.77% in the model versus 7.71% in the data) and volatility (2.39% in the model versus 2.41% in the data) similar to those in the data.

8. Extension to a Life-Cycle Model

Our baseline model is a perpetual youth model in which all consumers face the same probability of survival and are characterized by the same human capital process on and off the job. In the data, though, the growth rate of human capital on the job tends to be higher for younger workers than for mature workers; see Rubinstein and Weiss (2006) for a comprehensive review of the evidence on wage growth. At the same time, as we document later, the volatility of unemployment for young workers is higher than that for mature workers. One question, then, is whether an extension of our model in which we explicitly

[21] With b_h equal to 0.6, the ratio of home to market production is only 0.2 and the volatility of the job-finding rate falls to 4.41%. In the baseline model too, if we set the home production parameter to 0.2, then the volatility of the job-finding rate is only 4.62%. Alternatively, if we let physical capital be used in home production as well, so that the ratio of home to market production, $\mathbb{E}[(b_h A_t z_t)^{1-\gamma} k_{ut}^{\gamma}(z_t) \ / (A_t z_t)^{1-\gamma} k_{et}^{\gamma}(z_t)]$, is 0.6, then $b_h = 0.6$, where k_{ut} and k_{et}, respectively, are the physical capital used in home and market production. In this case, the volatility of the job-finding rate is 6.45%.

account for differences in the human capital process between young and mature workers can reproduce the observed volatility of the job-finding rate and unemployment for these two groups. Intuitively, our mechanism can account for these aspects of the data as it implies that the greater is the rate of human capital accumulation, the higher is the volatility of the job-finding rate and unemployment.

To address this question formally, we augment our model with the following simple life-cycle structure. We assume that consumers are born young and that at the end of each period, young consumers remain young with probability ϕ_y and become mature consumers with probability $1-\phi_y$. Mature consumers survive with probability ϕ_m and die with probability $1-\phi_m$. Every period, dying mature consumers are replaced by an equal measure γ of unemployed newborn consumers with human capital z drawn from a distribution $\nu(z)$ with mean 1. We assume that the measure of newborns each period is $\gamma = (1-\phi_y)(1-\phi_m)/[(1-\phi_y)+(1-\phi_m)]$ and the initial measures of young and mature workers are $\gamma/(1-\phi_y)$ and $\gamma/(1-\phi_m)$ so that the measures of young and mature workers are constant over time and sum to one.

We allow the rate of human capital accumulation on and off the job to differ across young and mature consumers. In particular, when a consumer of age (or type) $i \in \{y, m\}$, where y denotes a young and m denotes a mature consumer, is employed, human capital grows according to

$$(48)\quad z_{t+1} = (1+g_{ei})z_t.$$

When a consumer of age i is unemployed, human capital evolves according to

$$(49)\quad z_{t+1} = (1+g_{ui})z_t.$$

We let the cost of posting vacancies $\kappa_i A_t z$ and the job separation rate σ_i vary with a consumer's age i. Each family now consists of a continuum of young and mature consumers. The definition of a competitive search equilibrium is the natural generalization of that in our baseline model, except that now a labor market is defined by the triple $(z, i, W_{mit}(z))$ of a given skill level, age, and wage offer for a worker of that skill level and age.

It is immediate to show that the analogues of Lemma 1 and Proposition 1 hold. Here, though, we need to record the aggregate human capital of young and mature consumers by their employment status, namely, (Z_{eyt}, Z_{uyt}) and (Z_{emt}, Z_{umt}), as part of the state. The aggregate resource constraint is

$$(50)\quad C_t \leq A_t(Z_{eyt}+Z_{emt}) + bA_t(Z_{uyt}+Z_{umt}) - \kappa_y A_t \int_z z v_{yt}(z)dz - \kappa_m A_t \int_z z v_{mt}(z)dz,$$

where the first two terms on the right side of (50) are the total market and home output of young and mature consumers, and the last two terms are the posting costs for vacancies aimed at young and mature consumers. Note that the third term in (50), namely, the vacancy costs for recruiting young consumers, can be rewritten as

$$(51)\quad \kappa_y A_t \int_z z\theta_{yt} u_{byt}(z)dz = \kappa_y A_t \phi_y \theta_{yt}(1+g_{uy})Z_{uyt-1}$$

using that $v_{yt}(z) = \theta_{yt}u_{byt}(z)$ and $u_{byt}(z) = \phi_y u_{yt-1}\left(z/(1+g_{uy})\right)/(1+g_{uy})$. Similarly, the last term, namely the vacancy costs for recruiting mature consumers, can be expressed as

$$(52)\quad \kappa_m A_t \int_z z\theta_{mt}u_{bmt}(z)dz = \kappa_m A_t \theta_{mt}[\phi_m(1+g_{um})Z_{umt-1} + (1-\phi_y)(1+g_{uy})Z_{uyt-1}]$$

using that $v_{mt}(z) = \theta_{mt}u_{bmt}(z)$ and

$$(53)\quad u_{bmt}(z) = \frac{\phi_m}{1+g_{um}}u_{mt-1}\left(\frac{z}{1+g_{um}}\right) + \frac{1-\phi_y}{1+g_{uy}}u_{yt-1}\left(\frac{z}{1+g_{uy}}\right).$$

As (53) indicates, mature consumers with human capital z at the beginning of period t, u_{bmt}, consist of mature consumers at the end of period $t-1$, u_{mt-1}, who survived to period t and whose human capital grew from $z/(1+g_{um})$ to z between $t-1$ and t, together with young consumers at the end of period $t-1$, u_{yt-1}, who transited to mature age and whose human capital grew from $z/(1+g_{uy})$ to z between $t-1$ and t. Substituting (51) and (52) into (50) gives the resource constraint in terms of the aggregate states.

By a similar logic, we can generalize the transition laws for the aggregate human capital of employed and unemployed consumers from our baseline model as follows. In particular, the transition law for the aggregate human capital of employed and unemployed young consumers are

$$Z_{eyt} = \phi_y(1-\sigma)(1+g_{ey})Z_{eyt-1} + \phi_y\lambda_{wt}(\theta_{yt})(1+g_{uy})Z_{uyt-1}$$

and

$$Z_{uyt} = \phi_y\sigma(1+g_{ey})Z_{eyt-1} + \phi_y[1-\lambda_{wt}(\theta_{yt})](1+g_{uy})Z_{uyt-1} + \gamma,$$

where γ is the measure of newborn consumers who start with a mean human capital of 1. The transition laws for mature consumers' aggregate human capital are similar but also include terms that account for the human capital of young consumers who become mature at the end of period $t-1$,

$$\begin{aligned} Z_{emt} = &(1-\sigma)[\phi_m(1+g_{em})Z_{emt-1} + (1-\phi_y)(1+g_{ey})Z_{eyt-1}] \\ &+ \lambda_{wt}(\theta_{mt})[\phi_m(1+g_{um})Z_{umt-1} + (1-\phi_y)(1+g_{uy})Z_{uyt-1}], \end{aligned}$$

and

$$\begin{aligned} Z_{umt} = &\sigma[\phi_m(1+g_{em})Z_{emt-1} + (1-\phi_y)(1+g_{ey})Z_{eyt-1}] \\ &+ [1-\lambda_{wt}(\theta_{mt})][\phi_m(1+g_{um})Z_{umt-1} + (1-\phi_y)(1+g_{uy})Z_{uyt-1}]. \end{aligned}$$

Our strategy for choosing parameter values is nearly identical to that in the baseline model, so we remark here only on the main differences. We choose the probability ϕ_y that a young consumer remains young and the probability ϕ_m that a mature consumer survives in the market so that the mean duration of life as a young consumer is 10 years and the mean duration of life as a mature consumer is 30 years. We interpret each model year as corresponding to one year of potential labor market experience, defined, as standard, as an individual's age minus education minus six. Correspondingly, we pin down the key new parameters $\{g_{ei}\}$ and $\{g_{ui}\}$, respectively, based on wage growth on the job and wage changes before and after a spell of nonemployment for individuals with less and more than 10 years of labor market experience.

Specifically, we choose g_{ey} and g_{em} so as to reproduce, respectively, an average growth of real hourly wages, net of the growth rate of aggregate productivity, of 4.86% over the first 10 years of labor market experience and of 3.22% over the remaining 30 years, based on the estimates of Rubinstein and Weiss (2006); see the Appendix for details. Similarly, we determine g_{uy} and g_{um} so as to match the average percentage difference between the first wage in the first employment spell *after* a nonemployment spell and the last wage in the last employment spell *before* a nonemployment spell for all workers in the two age groups who experience a complete spell of nonemployment. Using data from the PSID at monthly frequency available between 1988 and 1997, we estimate that such an average percentage wage difference after a spell of nonemployment is 1.43% for individuals with less than 10 years of labor market experience and −12.26% for individuals with more than 10 years of labor market experience, net of the growth rate of aggregate productivity.

Consider now the job-finding and unemployment rates for young and mature consumers. Since unemployment rates from the BLS are available disaggregated by age groups rather than by experience, we measure the job-finding rate for workers below and above the cutoff of 30 years of age, since 30 years is approximately the age of an individual with the mean (and median) number of years of education and 10 years of labor market experience. The mean and standard deviation of the quarterly job-finding rate over the period between June 1976 and December 2007 are 49% and 4.94% for individuals with less than 30 years of age, and 36% and 4.75% for individuals with more than 30 years of age.[22] We then use these estimated job-finding rates to construct age-specific constant-separation unemployment rate series, in which the age-specific separation rates σ_y and σ_m are chosen so that the mean of each constructed unemployment rate series reproduces the mean unemployment rate of the relevant age group in the data. As the mean unemployment rate of young workers is much higher than that of mature workers (10.5% for the young versus 5.55% for the mature), the implied separation rate for young workers is also higher (5.7% for the young versus 2.1% for the mature). Given these parameters, we choose the cost of vacancy creation κ_i for the two age groups so that the model replicates the mean unemployment rate of the two groups in the data. Finally, we recomputed our asset pricing moments for this same period between June 1976 and December 2007.

As Table 4 shows, the volatility of the job-finding rate and unemployment in the data are both higher for the young than for the mature. The volatility of the job finding rate is 4.94% for the young and 4.75% for the mature, and the volatility of unemployment is 0.93% for the young and 0.61% for the mature. Our life-cycle model reproduces this same pattern: the volatility of the job-finding rate is 5.50% for the young and 4.48% for the mature, and the volatility of unemployment is 1.10% for the young and 0.68% for the

[22] We restrict attention to the period after June 1976 because the BLS reports short-term unemployment by age, which is necessary to compute the job-finding rate as we did in our baseline following Shimer (2012), only from that date onward.

mature. In sum, in the data younger workers' unemployment is 1.5 times more volatile than that of older workers. Our model reproduces this pattern well and predicts that younger workers' unemployment is 1.6 times more volatile than that of older workers.

In our simple exercise, we have focused on examining whether our basic mechanism can account for the differential volatility of unemployment of young and mature consumers in light of differences in the human capital accumulation patterns of the two groups. To do so, we highlighted the role of human capital accumulation on and off the job and kept other parameters, such as home production, identical across the two groups. If we further allowed, say, mature consumers to be less attached to the labor market than young ones so as to capture the shifting preference for leisure with age, then the model could reproduce the data even more closely.

9. Results for Alternative Preferences

Here we show that the results for alternative preferences are similar to those for our baseline preferences. In the Appendix, we provide details and also show that the asset pricing implications of these alternative preferences in our production economy are nearly identical to their pure exchange counterparts. (See also Tables A.1-A.5 and Figure A.2 in the Appendix.)

Campbell-Cochrane Preferences with External Habit. We adapt the setup of Campbell and Cochrane (1999) with external habit designed for a pure exchange economy, discussed earlier, to our production economy. The only difference from the original specification in Campbell and Cochrane (1999) is that we replace their sensitivity function with

$$(54)\quad \lambda_t(\bar{s}_t) = \frac{\sigma(\varepsilon_{ct+1})}{\sigma_t(\varepsilon_{ct+1})}\frac{1}{\bar{S}}\left[1 - 2\left(\bar{s}_t - \bar{s}\right)\right]^{1/2} - 1.$$

Here $\sigma(\varepsilon_{ct+1})$ and $\sigma_t(\varepsilon_{ct+1})$ are, respectively, the unconditional and conditional standard deviations of the innovation to aggregate consumption growth, $\varepsilon_{ct+1} = \Delta\bar{c}_{t+1} - \mathbb{E}_t\Delta\bar{c}_{t+1}$. The term $\sigma(\varepsilon_{ct+1})/\sigma_t(\varepsilon_{ct+1})$ in (54) adjusts for the time-varying conditional volatility of consumption in our production economy relative to the pure exchange economy of Campbell and Cochrane (1999). Table 5 shows that this model produces nearly identical results to those produced by our baseline model.

Epstein-Zin Preferences with Long-Run Risk. We consider a model with Epstein-Zin preferences, a slow-moving predictable component in productivity as in Bansal and Yaron (2004), and discount rate shocks as in Albuquerque et al. (2016) and Schorfheide et al. (2018). In this case, preferences are given by

$$V_t = \left[(1-\beta)S_t C_t^{1-\rho} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}.$$

Productivity growth now has a long-run risk component x_t in that $\Delta a_{t+1} = g_a + x_t + \sigma_a\varepsilon_{at+1}$ and $x_{t+1} = \rho_x x_t + \phi_x\sigma_a\varepsilon_{xt+1}$, where the shocks ε_{at} and ε_{xt} are standard normal i.i.d. and orthogonal to each other.

The growth rate of the discount factor shock, $\Delta s_t = \Delta \log(S_t)$, follows an autoregressive process given by $\Delta s_{t+1} = \rho_s \Delta s_t + \phi_s \sigma_a \varepsilon_{st+1}$, where the innovation ε_{st} is standard normal i.i.d. and orthogonal to ε_{at} and ε_{xt}. The pricing kernel is

$$(55) \quad Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left(\frac{S_{t+1}}{S_t}\right) \left[\frac{V_{t+1}}{\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}}}\right]^{\rho-\alpha}.$$

We set the mean and the standard deviation of the aggregate productivity process to replicate those in the data. We choose the persistence ρ_x of the long-run risk state x_t so that the model generates the same standard deviation of the log price-consumption ratio as that generated by the baseline model. We assume that the parameter ϕ_x of the long-run risk state x_t is such that its volatility $\sigma_x^2 = \phi_x^2 \sigma_a^2/(1-\rho_x^2)$ accounts for the same share of the volatility of productivity growth, $\sigma_x^2/(\sigma_a^2 + \sigma_x^2) = 0.0445$, as that used by Bansal and Yaron (2004). We select the parameter ϕ_s of the process for the discount factor shock to match the standard deviation of the risk-free rate. As for ρ_s, notice that the role of the surplus consumption ratio in our baseline model is similar to that of the discount factor shock in this model. Because of this feature, we set the persistence ρ_s of the process for the discount factor shock in the same way as we set the persistence of the process for the surplus consumption ratio in the baseline model.

We choose a risk-aversion coefficient α of 4.3 to match a maximum Sharpe ratio of 0.45 and set the elasticity of intertemporal substitution to 10 ($\rho = 0.1$). To understand this choice, note first that with an elasticity of intertemporal substitution equal to one, it is easy to use logic similar to that underlying Proposition 2 to show that the volatility of the job-finding rate is exactly zero. As noted by Kilic and Wachter (2018) in a related context, a large elasticity of intertemporal substitution is consistent with the available evidence of a low elasticity of intertemporal substitution of consumption, which reflects the weak correlation between consumption growth and interest rates. Indeed, when we estimate the contemporaneous elasticity of consumption growth with respect to interest rates based on data simulated from our model, using powers of the states s_t and x_t and lagged consumption growth as instruments, we find a coefficient of around 0.2, which is consistent with estimates in the literature (see, for instance, Hall 1988 and Beeler and Campbell 2012).

As Table 5 shows, this model can produce around 92% (0.69/0.75) of the observed volatility of unemployment and has reasonable asset pricing implications.

Epstein-Zin Preferences with Variable Disaster Risk. We adopt a discrete-time version of the model of Wachter (2013) with Epstein-Zin preferences and a slow-moving probability of rare disasters. In this case, preferences are specified as

$$(56) \quad V_t = \left[(1-\beta) C_t^{1-\rho} + \beta \left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}.$$

The process for productivity growth now includes a discrete-valued jump component j_{t+1} and is given by $\Delta a_{t+1} = g_a + \sigma_a \varepsilon_{at+1} - \theta j_{t+1}$, where the disaster component j_{t+1} is a Poisson random variable with intensity s_t, which evolves according to the process $s_{t+1} = (1-\rho_s)s + \rho_s s_t + \sqrt{s_t}\sigma_s \varepsilon_{st+1}$.

We choose the mean and the standard deviation of the aggregate productivity process to match those in the data and a mean disaster intensity s of 3.55% per year as in Wachter (2013). We select the volatility σ_s of the disaster intensity to reproduce the standard deviation of the risk-free rate in the data, and the risk aversion coefficient to target a maximum Sharpe ratio of 0.45. We choose the persistence ρ_s of the disaster intensity to generate the same standard deviation of the log price-consumption ratio as in our baseline model. Like Wachter (2013), we set the disaster impact θ to 0.26 and the elasticity of intertemporal substitution to 10 ($\rho = 0.1$).[23] As Table 5 shows, the model produces a volatility of unemployment of 0.77% that is similar to that in the data, 0.75%, and has also asset pricing implications broadly in line with the data.

An Affine Discount Factor. Our results also hold for reduced-form discount factors of the type considered by Ang and Piazzesi (2003),[24]

$$\log(Q_{t,t+1}) = -(\mu_0 - \mu_1 s_t) - \frac{1}{2}(\gamma_0 - \gamma_1 s_t)^2 \sigma_a^2 - (\gamma_0 - \gamma_1 s_t)\sigma_a \varepsilon_{at+1}.$$

Here we assume that the exogenous state s_t evolves according to $s_{t+1} = \rho_s s_t + \sigma_a \varepsilon_{at+1}$ and is driven by fluctuations in productivity, ε_{at+1}. Productivity growth follows a random walk process as in our baseline. We keep the parameters for the mean and standard deviation of the aggregate productivity process, g_a and σ_a, as in the baseline model and choose the four parameters $(\mu_0, \mu_1, \gamma_0, \gamma_1)$ to reproduce the mean and standard deviation of the risk-free rate, the maximum Sharpe ratio, and the volatility of the excess return. We select the persistence ρ_s of the exogenous state to generate the same standard deviation of the log price-consumption ratio as in our baseline. Table 5 shows that this model produces 97% (0.73/0.75) of the volatility of unemployment in the data and also has reasonable asset pricing implications.

10. Conclusion

We propose a new mechanism that allows search models to reproduce the observed fluctuations in the job-finding rate and unemployment at business cycle frequencies. Our model solves the unemployment volatility puzzle of Shimer (2005) and is immune to the critiques of existing mechanisms that address it, namely, those by Chodorow-Reich and Karabarbounis (2016) on the cyclicality of the opportunity cost of employment, by Kudlyak (2014) and Basu and House (2016) on the cyclicality of wages, and by Borovicka and Borovickova (2019) on the asset pricing implications of these mechanisms.

[23] We estimate the elasticity of consumption growth to interest rates on data simulated from our model using powers of s_t and lagged consumption growth as instruments. We estimate this elasticity to be between 0.01 and 0.5 despite the assumption that $\rho = 0.1$.

[24] Here we simply posit a discount factor that is not derived from marginal utility and so we define a competitive search equilibrium given $\{Q_{t,t+1}\}$. Hence, we drop condition $vi)$ in the definition of equilibrium.

To this purpose, we augment the textbook search model with two features: preferences from the macro-finance literature that match the observed variation in asset prices and human capital accumulation on the job that is consistent with longitudinal and cross-sectional evidence on wage growth with experience. In such a framework, firms' investment in hiring workers becomes a risky activity with long-duration surplus flows from a match between a firm and a worker. Hence, shocks to either aggregate productivity or, directly, to discount factors make the present value of these surplus flows fluctuate sharply over the cycle. In turn, fluctuations in the present value of these surplus flows imply that investments in hiring workers are highly cyclical and, hence, that job-finding rates and unemployment are as volatile as in the data. We show that both new features we introduce play a critical role. That is, if we abstract from either preferences that generate time-varying risk or human capital accumulation, the model generates only negligible movements in unemployment. We show that the same intuition applies once we augment the model with physical capital or account for heterogeneity in the human capital process across young and mature workers. Overall, our results show that re-integrating search and business cycle theory is both a tractable and promising avenue of future research.

We have purposely kept our model simple so as to focus on our new mechanism. We have therefore abstracted from a host of potentially relevant sources of heterogeneity, for instance, in returns to experience across jobs, discount rates, households' exposure to time-varying risk, and labor market frictions. Including these multiple dimensions of heterogeneity and assessing their importance for observed labor market fluctuations would be a useful and interesting extension of the model for future research.

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Table 1: Parametrization and Results for Baseline Model

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
κ, hiring cost	0.975	Mean unemployment rate	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (%p.a.)	0.92	0.92
S, mean of state S_t	0.2066	S.d. risk-free rate (%p.a.)	2.31	2.31
α, inverse EIS	5.0	Maximum Sharpe ratio (p.a.)	0.45	0.45
Assigned		*Labor Market Results*		
B, efficiency of matching technology	0.46	Mean job-finding rate	0.46	0.46
b, home production parameter	0.6	S.d. job-finding rate	6.66	6.60
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.98
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.75
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, persistence of state	0.9944	Correlation unemployment, job-finding rate	-0.96	-0.98
g_e, human capital growth when employed (%p.a.)	3.5	Elasticity user cost labor to u (Kudlyak)	-5.2	-6.4
		Asset Market Results		
		Mean excess return (%p.a.)	6.96	6.30
		S.d. excess return (%p.a.)	15.6	14.1
		Mean excess return / s.d. excess return (p.a.)	0.45	0.45
		Mean log price-dividend ratio	3.51	3.36
		S.d. log price-dividend ratio	0.44	0.36
		Mean 20-year real yield (%p.a.)	4.81	3.75
		S.d. 20-year real yield (%p.a.)	2.00	2.20
		Mean 20-year nominal yield (%p.a.)	7.71	7.73
		S.d. 20-year nominal yield (%p.a.)	2.41	2.28

Table 2: Role of Human Capital Accumulation

	Data	Baseline	DMP with Baseline Preferences $g_e = 0$ and $g_u = 0$	Baseline Model with $g_e = 3.5\%$ and $g_u = 3.5\%$
S.d. job-finding rate	6.66	6.60	0.15	0.15
Autocorr. job-finding rate	0.94	0.98	0.99	0.99
S.d. unemployment rate	0.75	0.75	0.02	0.02
Autocorr. unemployment rate	0.97	0.99	0.99	0.99
Correlation u, job-finding rate	-0.96	-0.98	-0.98	-0.98

Table 3: Parametrization and Results for Model with Baseline Preferences and Physical Capital

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	1.36	Mean productivity growth (%p.a.)	1.36	1.36
σ_a, s.d. productivity growth (%p.a.)	1.79	S.d. productivity growth (%p.a.)	1.79	1.79
κ, hiring cost	1.8	Mean unemployment rate	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (%p.a.)	0.92	0.92
S, mean of state S_t	0.2894	S.d. risk-free rate (%p.a.)	2.31	2.31
α, inverse EIS	7.25	Maximum Sharpe ratio (p.a.)	0.45	0.45
γ, curvature of production function	0.26	Mean labor share of output	0.70	0.70
ξ, curvature of adjustment cost	0.26	Ratio s.d. invest. to consumption growth	4.5	4.5
Assigned		*Results*		
B, efficiency of matching technology	0.46	Mean job-finding rate	0.46	0.46
b, home production parameter	0.6	S.d. job-finding rate	6.66	6.66
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.77
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, persistence of state	0.9944	Correlation unemployment, job-finding rate	-0.96	-0.97
δ, physical capital depreciation rate	0.1/12			
g_e, human capital growth when employed (%p.a.)	3.5	*Asset Market Results*		
		Mean excess return (%p.a.)	6.96	5.48
		S.d. excess return (%p.a.)	15.6	12.1
		Mean excess return / s.d. excess return (p.a.)	0.45	0.45
		Mean log price-dividend ratio	3.51	3.24
		S.d. log price-dividend ratio	0.44	0.33
		Mean 20-year real yield (%p.a.)	4.81	3.79
		S.d. 20-year real yield (%p.a.)	2.00	2.34
		Mean 20-year nominal yield (%p.a.)	7.71	7.77
		S.d. 20-year nominal yield (%p.a.)	2.41	2.39

Table 4: Parametrization and Results for the Lifecycle Model with Baseline Preferences

Panel A: Parameters

Endogenously Chosen	
g_a, mean productivity growth (%p.a.)	1.95
σ_a, s.d. productivity growth (%p.a.)	1.41
κ_y, hiring cost, young	0.96
κ_m, hiring cost, mature	4.37
β, time preference factor	0.9978
S, mean of state S_t	0.1226
α, inverse EIS	4.0
Assigned	
B, efficiency of matching technology	0.43
b, home production	0.6
σ_y, probability of separation, young	0.057
σ_m, probability of separation, mature	0.021
η, matching function elasticity	0.5
ϕ_y, survival probability, young	0.9917
ϕ_m, survival probability, mature	0.9972
ρ_s, persistence of state	0.9944
g_{ey}, HK growth on job, young (%p.a.)	4.86
g_{uy}, HK growth off job, mature (%p.a.)	1.43
g_{em}, HK growth on job, young (%p.a.)	3.20
g_{um}, HK growth off job, mature (%p.a.)	-12.3

Panel B: Moments

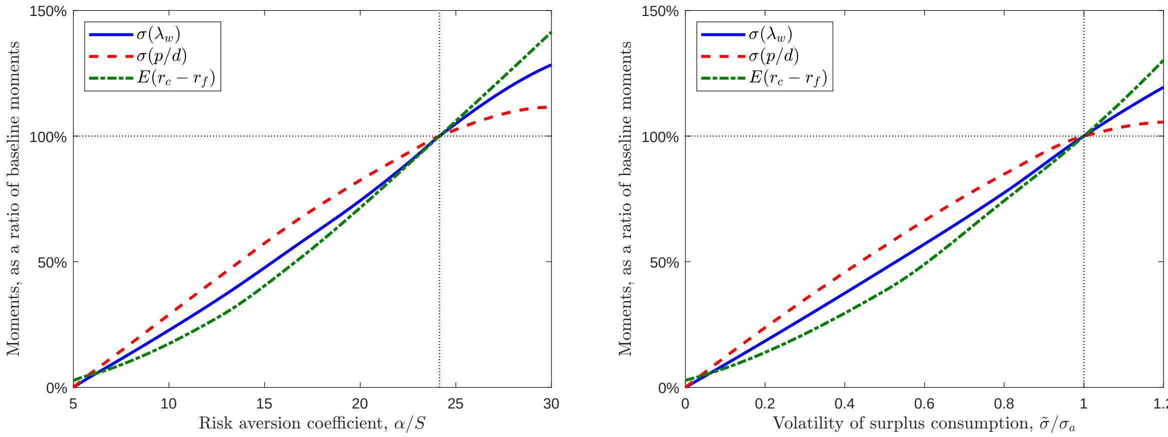
Targeted	Data	Model
Mean productivity growth (%p.a.)	1.95	1.95
S.d. productivity growth (%p.a.)	1.41	1.41
Mean unemployment rate, young	10.5	10.5
Mean unemployment rate, mature	5.55	5.55
Mean risk-free rate (%p.a.)	1.63	1.63
S.d. risk-free rate (%p.a.)	1.91	1.91
Maximum Sharpe ratio (p.a.)	0.44	0.44
Labor Market Results		
Mean job-finding rate, young	0.49	0.49
Mean job-finding rate, mature	0.36	0.36
S.d. job-finding rate, young	4.94	5.50
S.d. job-finding rate, mature	4.75	4.48
Autocorr. job-finding rate, young	0.93	0.99
Autocorr. job-finding rate, mature	0.91	0.99
S.d. unemployment rate, young	0.93	1.10
S.d. unemployment rate, mature	0.61	0.68
Autocorr. unemp. rate, young	0.97	0.99
Autocorr. unemp. rate, mature	0.97	0.99
Corr. unemp., job-find. rate, young	-0.97	-0.99
Corr. unemp., job-find. rate, mature	-0.93	-0.99
Asset Market Results		
Mean excess return (%p.a.)	5.83	4.25
S.d. excess return (%p.a.)	13.2	9.72
Mean excess return / s.d. excess return (p.a.)	0.44	0.44
Mean log price-dividend ratio	3.70	3.44
S.d. log price-dividend ratio	0.49	0.30
Mean 20-year real yield (%p.a.)	4.81	3.31
S.d. 20-year real yield (%p.a.)	2.00	1.39
Mean 20-year nominal yield (%p.a.)	7.71	7.57
S.d. 20-year nominal yield (%p.a.)	2.41	1.48

Table 5: Results for Other Preferences

	Data	Baseline	Alternative Preferences			
			CC	EZ w/ LRR	EZ w/ Disasters	Affine SDF
Labor Market Results						
S.d. job-finding rate	6.66	6.60	6.69	6.36	5.66	7.52
Autocorr. job-finding rate	0.94	0.98	0.99	0.99	0.99	0.99
S.d. unemployment rate	0.75	0.75	0.75	0.69	0.77	0.73
Autocorr. unemployment rate	0.97	0.99	0.99	0.99	0.99	0.99
Correlation unemployment, job-finding rate	-0.96	-0.98	-0.98	-0.98	-0.98	-0.97
Asset Market Results						
Mean excess return (%p.a.)	6.96	6.30	6.38	4.61	4.80	6.96
S.d. excess return (%p.a.)	15.6	14.1	15.2	10.3	10.7	15.6
Mean excess return / s.d. excess return (p.a.)	0.45	0.45	0.45	0.45	0.45	0.45
Mean log price-dividend ratio	3.51	3.36	3.37	3.77	3.24	3.24
S.d. log price-dividend ratio	0.44	0.36	0.36	0.36	0.36	0.36
Mean 20-year real yield (%p.a.)	4.81	3.76	3.84	2.80	-1.38	4.36
S.d. 20-year real yield (%p.a.)	2.00	2.20	2.28	1.25	2.19	2.11
Mean 20-year nominal yield (%p.a.)	7.71	7.73	7.81	6.48	2.28	8.43
S.d. 20-year nominal yield (%p.a.)	2.41	2.28	2.37	1.27	2.20	2.24

Figure 1: Sensitivity of Key Moments to Preference Parameters in Baseline Model

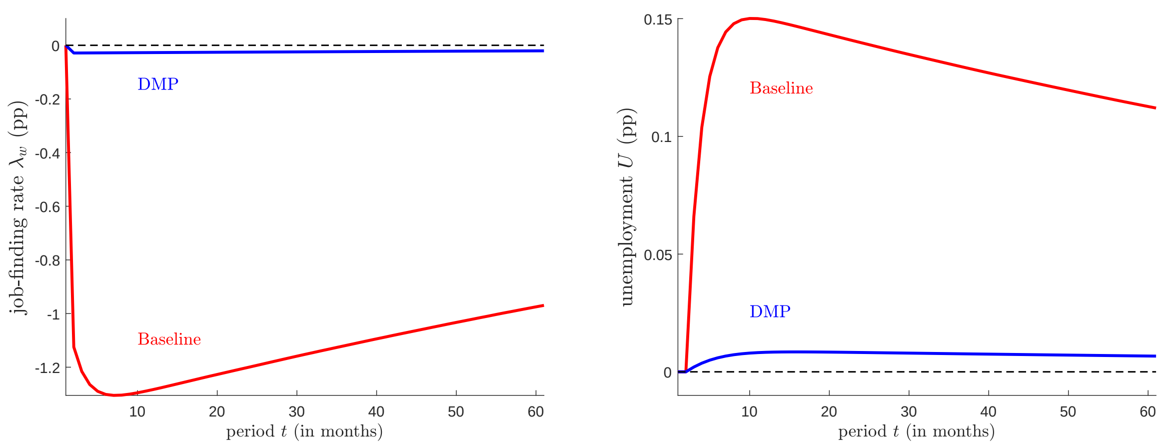
(a) Risk Aversion Coefficient (b) Volatility of Surplus Consumption



Note: λ_w denotes the job-finding rate, p/d the log price-dividend ratio of the consumption claim, and $r_c - r_f$ the excess return on the consumption claim over the risk-free rate.

Figure 2: Responses to Productivity Shock in Baseline Model

(a) Job-Finding Rate (b) Unemployment



Note: Impulse responses of the job-finding rate and unemployment to a -1% productivity shock. Generalized impulse response functions are based on 10,000 simulations.

Figure 3: Loci of (g_e, g_u) Leading to Different Fractions of Job-Finding Rate Volatility in Baseline Model

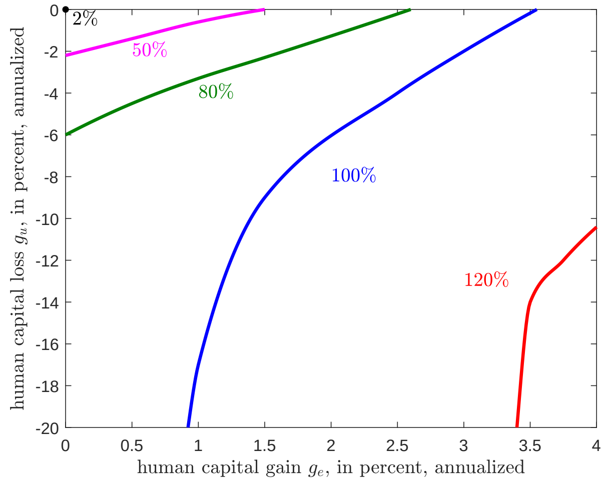


Figure 4: Prices of Productivity Strips in Baseline Model: Global and Approximate Solutions

(a) Global Solution

(b) Approximate Solution

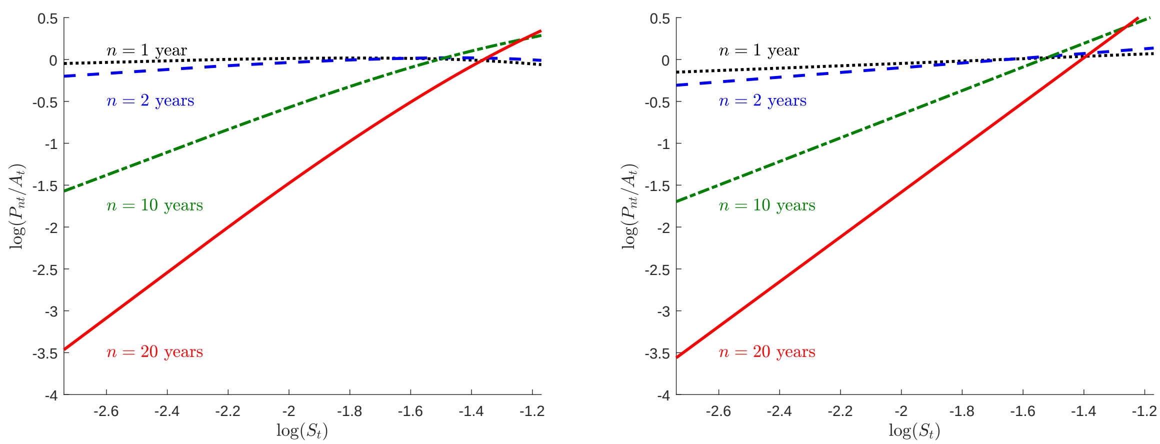
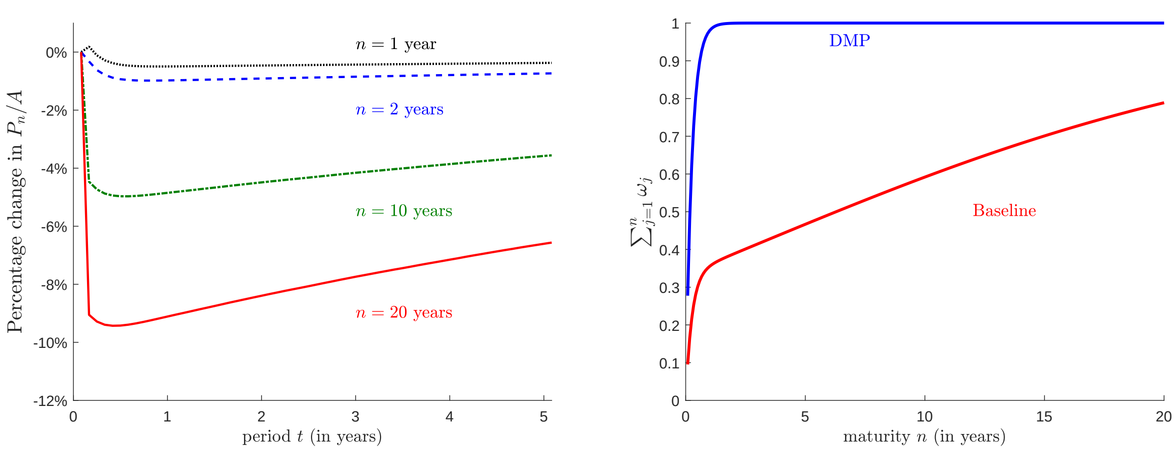


Figure 5: Responses to Productivity Shock in Baseline Model and Durations

(a) Prices of Productivity Strips by Maturity

(b) Cumulative Weights by Maturity



Note: Impulse responses of the prices of productivity strips to a -1% productivity shock (left panel) and duration of surplus flows in the baseline model and DMP model with baseline preferences (right panel). Generalized impulse response functions are based on 10,000 simulations.