Online Appendix

Markups and Inequality

Not For Publication

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This appendix provides detailed derivations for our optimal regulation results in Section 3, describes the SCF data we used to parameterize the model, and reports the parameter values and moments for the two perturbations considered in the robustness section in the dynamic economy.

1 Optimal Regulation

We first analyze the case of complete information and then turn to the economy with private information.

1.1 Complete Information

The problem under complete information is described in equations (15)–(18) in the text. The Lagrangean is

$$\max_{q(z),c(z),W,Y} V^w(W) + \alpha\omega \int_0^\infty \frac{c\left(z\right)^{1-\theta}}{1-\theta} f\left(z\right) \mathrm{d}z + \lambda \left[Y - C^w\left(W\right) - \omega \int_0^\infty c\left(z\right) f\left(z\right) \mathrm{d}z\right]$$

$$+\kappa \left[\omega \int_0^\infty \Upsilon\left(q\left(z\right)\right) f\left(z\right) \mathrm{d}z - 1\right] + \nu \left[L\left(W\right) - Y^{\frac{1}{\eta}} \omega \int_0^\infty \left(\frac{q\left(z\right)}{z}\right)^{\frac{1}{\eta}} f\left(z\right) \mathrm{d}z\right]$$

The FOC with respect to $c\left(z\right)$ is

$$\alpha c\left(z\right)^{-\theta} = \lambda,$$

so all entrepreneurs receive the same level of consumption

$$c(z) = c^e.$$

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The FOC with respect to $q(z)$ is

$$\kappa \Upsilon'(q(z)) = \nu Y^{\frac{1}{\eta}} \frac{1}{\eta} \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}-1} \frac{1}{z},$$

or arranging terms,

$$\Upsilon'(q(z))\, q(z) = \frac{1}{\eta} \Lambda \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}},$$

where $\Lambda = \frac{\nu Y^{\frac{1}{\eta}}}{\kappa}$. To find Λ, we note that this expression implicitly determines the relative quantity choice as a function of productivity and Λ, $q(z;\Lambda)$. We can therefore find Λ by requiring that the Kimball aggregator is satisfied. That is, Λ is the implicit solution to

$$\omega \int_0^\infty \Upsilon(q(z;\Lambda))\, f(z)\, \mathrm{d}z = 1.$$

Once we have the relative quantity choice, we calculate Z from equation (7) in text and use the resource constraint to rewrite the regulator's problem as

$$\max_W V^w(W) + \alpha\omega \frac{\left[\frac{ZL(W)^\eta - C^w(W)}{\omega}\right]^{1-\theta}}{1-\theta}.$$

The FOC with respect to W is

$$\frac{\partial V^w(W)}{\partial W} = \alpha (c^e)^{-\theta} \left[\frac{\partial C^w(W)}{\partial W} - \eta Z L(W)^{\eta-1} \frac{\partial L^w(W)}{\partial W}\right].$$

When $\theta = 1$, as assumed in the paper, L^w is constant, so this expression simplifies to

$$(1-\omega)\frac{1}{W} = \alpha (c^e)^{-\theta} \frac{C^w(W)}{W},$$

which implies that

$$\frac{C^w(W)}{1-\omega} = \frac{c^e}{\alpha}.$$

1.2 Economy with Private Information

We now assume that the regulator does not observe productivity z. Without loss of generality, we invoke the revelation principle and constrain the regulator to choose functions $c(z)$ and $q(z)$ that ensure truth-telling.

Let $\tau(z)$ denote a subsidy received by a firm that claims to be of type z and sells $q(z)$ units of output as prescribed by the regulator. The producer's consumption, if it reports truth-fully, is

$$c(z) = DY\left[\Upsilon'(q(z))\,q(z) - \frac{WY^{\frac{1}{\eta}-1}}{D}\left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} + \tau(z)\right].$$

If the producer claims instead to have productivity $\hat{z}$, it receives

$$c(\hat{z}, z) = DY\left[\Upsilon'(q(\hat{z}))\,q(\hat{z}) - \frac{WY^{\frac{1}{\eta}-1}}{D}\left(\frac{q(\hat{z})}{z}\right)^{\frac{1}{\eta}} + \tau(\hat{z})\right]$$

units of consumption. Following the first-order approach, the local incentive constraint is

$$\left.\frac{\partial \hat{c}(\hat{z}, z)}{\partial \hat{z}}\right|_{\hat{z}=z} = 0 = DY\left[\left(\Upsilon''(q(z))\,q(z) + \Upsilon'(q(z)) - \frac{1}{\eta}\frac{WY^{\frac{1}{\eta}-1}}{D}\left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}}\frac{1}{q(z)}\right)q'(z) + \tau'(z)\right].$$

Differentiating the expression for $c(z)$ with respect to z and imposing the local incentive constraint gives

$$c'(z) = WY^{\frac{1}{\eta}}\frac{1}{\eta}\left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}}\frac{1}{z}.$$

The Lagrangean is

$$\max_{c(z),q(z),W,Y} V^w(W) + \alpha\omega\int_0^\infty \frac{c(z)^{1-\theta}}{1-\theta}f(z)\,\mathrm{d}z +$$

$$\int_0^\infty \hat{\mu}(z)\left[c'(z) - \frac{1}{\eta}WY^{\frac{1}{\eta}}\left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}}\frac{1}{z}\right]\mathrm{d}z + \lambda\left[Y - C^w(w) - \omega\int_0^\infty c(z)\,f(z)\,\mathrm{d}z\right]$$

$$+\kappa\left[\omega\int_0^\infty \Upsilon(q(z))\,f(z)\,\mathrm{d}z - 1\right] + \hat{\nu}\left[L^w(W) - \omega Y^{\frac{1}{\eta}}\int_0^\infty\left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}}f(z)\,\mathrm{d}z\right],$$

where $\hat{\mu}(z)$ is the multiplier on the IC constraint and we now use $\hat{\nu}$ to denote the multiplier on the labor resource constraint.

Consider the term $\int_0^\infty \hat{\mu}(z)\,c'(z)\,\mathrm{d}z$. Integrating by parts and using the transversality conditions $\hat{\mu}(0) = \hat{\mu}(\infty) = 0$ gives

$$\int_0^\infty \hat{\mu}(z)\,c'(z)\,\mathrm{d}z = -\int_0^\infty \hat{\mu}'(z)\,c(z)\,\mathrm{d}z,$$

which allows us to rewrite the Lagrangean as

$$\max_{c(z),q(z),W,Y} V^w(W) + \alpha\omega \int_0^\infty \frac{c(z)^{1-\theta}}{1-\theta} f(z)\,\mathrm{d}z - \int_0^\infty \hat{\mu}'(z)\, c(z)\,\mathrm{d}z$$

$$-\frac{1}{\eta} W Y^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}\mathrm{d}z + \lambda \left[Y - C^w(w) - \omega \int_0^\infty c(z)\, f(z)\,\mathrm{d}z\right]$$

$$+\kappa \left[\omega \int_0^\infty \Upsilon(q(z))\, f(z)\,\mathrm{d}z - 1\right] + \hat{\nu} \left[L^w(W) - \omega Y^{\frac{1}{\eta}} \int_0^\infty \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} f(z)\,\mathrm{d}z\right].$$

The FOC with respect to $c(z)$ is

$$\alpha\omega c(z)^{-\theta} f(z) - \hat{\mu}'(z) - \lambda\omega f(z) = 0,$$

or

$$\hat{\mu}'(z) = \omega \left[\alpha c(z)^{-\theta} - \lambda\right] f(z).$$

The FOC with respect to $q(z)$ is

$$-\frac{1}{\eta} W Y^{\frac{1}{\eta}} \frac{1}{\eta} \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z} \frac{1}{q(z)} + \kappa\omega \Upsilon'(q(z))\, f(z) - \hat{\nu}\omega Y^{\frac{1}{\eta}} \frac{1}{\eta} \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{q(z)} f(z) = 0.$$

The FOC with respect to Y is

$$-\frac{1}{\eta}\frac{1}{\eta} W Y^{\frac{1}{\eta}-1} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}\mathrm{d}z + \lambda - \hat{\nu}\omega \frac{1}{\eta} Y^{\frac{1}{\eta}-1} \int_0^\infty \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} f(z)\,\mathrm{d}z = 0.$$

The FOC with respect to W is

$$\frac{\partial V^w(W)}{\partial W} - \lambda \frac{\partial C^w(W)}{\partial W} + \hat{\nu} \frac{\partial L^w(W)}{\partial W} - \frac{1}{\eta} Y^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}\mathrm{d}z = 0.$$

To derive equation (28) in the text, we note that, since $\hat{\mu}(\infty) = 0$,

$$\hat{\mu}(z) = -\int_z^\infty \hat{\mu}'(x)\, dx = \omega \int_z^\infty \left[\lambda - \alpha c(x)^{-\theta}\right] f(x)\,\mathrm{d}x.$$

Moreover, since $\hat{\mu}(0) = 0$ we have

$$\hat{\mu}(0) = \omega \int_0^\infty \left[\lambda - \alpha c(z)^{-\theta}\right] f(z)\,\mathrm{d}z = 0,$$

which implies that

$$\lambda = \alpha \int_0^\infty c(z)^{-\theta} f(z)\,\mathrm{d}z.$$

Consider next the Y FOC:

$$\hat{\nu}\omega \frac{1}{\eta} Y^{\frac{1}{\eta}-1} \int_0^\infty \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} f(z)\,\mathrm{d}z = \lambda - \frac{1}{\eta}\frac{1}{\eta} W Y^{\frac{1}{\eta}-1} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}\mathrm{d}z.$$

Since $Z^{-\frac{1}{\eta}} = \omega \int_0^\infty \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} f(z)\,\mathrm{d}z$ we can simplify this expression to

$$\hat{\nu}\frac{1}{\eta} Y^{\frac{1}{\eta}-1} Z^{-\frac{1}{\eta}} = \lambda - \frac{1}{\eta}\frac{1}{\eta} W Y^{\frac{1}{\eta}-1} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}\mathrm{d}z,$$

so the multiplier on the labor resource constraint is

$$\hat{\nu} = \eta Y^{1-\frac{1}{\eta}} Z^{\frac{1}{\eta}} \lambda - \frac{1}{\eta} W Z^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}\mathrm{d}z.$$

Since

$$\eta Y^{1-\frac{1}{\eta}} Z^{\frac{1}{\eta}} = \eta \frac{Y}{L},$$

the marginal rate of substitution between employment and consumption is equal to

$$\nu = \frac{\hat{\nu}}{\lambda} = \eta \frac{Y}{L} - \frac{1}{\lambda}\frac{1}{\eta} W Z^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}\mathrm{d}z.$$

Let

$$\mu(z) = \frac{\hat{\mu}(z)}{\lambda\omega(1-F(z))} = 1 - \frac{1}{\lambda}\frac{1}{1-F(z)} \int_z^\infty \alpha c(x)^{-\theta} f(x)\,\mathrm{d}x$$

and recall that $c'(z) = W Y^{\frac{1}{\eta}} \frac{1}{\eta} \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}$. We can therefore write

$$\nu = \eta \frac{Y}{L} - \left(\frac{Z}{Y}\right)^{\frac{1}{\eta}} \omega \int_0^\infty \frac{\hat{\mu}(z)}{\omega\lambda(1-F(z))} W Y^{\frac{1}{\eta}} \frac{1}{\eta} \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z}(1-F(z))\,\mathrm{d}z$$

or

$$\nu = \eta \frac{Y}{L} - \frac{1}{L}\omega \int_0^\infty \mu(z) c'(z)(1-F(z))\,\mathrm{d}z.$$

We can finally rewrite the $q(z)$ FOC as

$$\kappa \Upsilon'(q(z))\,q(z) = \hat{\nu}\frac{1}{\eta} Y^{\frac{1}{\eta}} \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} + \frac{1}{\eta} W Y^{\frac{1}{\eta}} \frac{\hat{\mu}(z)}{\omega f(z)} \frac{1}{\eta} \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z},$$

which implies

$$\Upsilon'\left(q\left(z\right)\right)q\left(z\right)=\left(1+\mu\left(z\right)\frac{\frac{1}{\eta}\frac{W}{z}\left(1-F\left(z\right)\right)}{\nu f\left(z\right)}\right)\frac{\hat{\nu}}{\kappa}\frac{1}{\eta}Y^{\frac{1}{\eta}}\left(\frac{q\left(z\right)}{z}\right)^{\frac{1}{\eta}}.$$

To find κ, we multiply the $q\left(z\right)$ FOC by $q\left(z\right)$ and integrate across all producers:

$$-\frac{1}{\eta}WY^{\frac{1}{\eta}}\frac{1}{\eta}\int_{0}^{\infty}\hat{\mu}\left(z\right)\left(\frac{q\left(z\right)}{z}\right)^{\frac{1}{\eta}}\frac{1}{z}\mathrm{d}z+\kappa\omega\int_{0}^{\infty}\Upsilon'\left(q\left(z\right)\right)q\left(z\right)f\left(z\right)\mathrm{d}z-\hat{\nu}\omega Y^{\frac{1}{\eta}}\frac{1}{\eta}\int_{0}^{\infty}\left(\frac{q\left(z\right)}{z}\right)^{\frac{1}{\eta}}f\left(z\right)\mathrm{d}z=0.$$

Similarly, multiplying the Y FOC by Y gives

$$-\frac{1}{\eta}\frac{1}{\eta}WY^{\frac{1}{\eta}}\int_{0}^{\infty}\hat{\mu}\left(z\right)\left(\frac{q\left(z\right)}{z}\right)^{\frac{1}{\eta}}\frac{1}{z}\mathrm{d}z+\lambda Y-\hat{\nu}\omega\frac{1}{\eta}Y^{\frac{1}{\eta}}\int_{0}^{\infty}\left(\frac{q\left(z\right)}{z}\right)^{\frac{1}{\eta}}f\left(z\right)\mathrm{d}z=0.$$

Combining these two expressions and using the definition of the demand index,

$$D=\frac{1}{\omega\int_{0}^{\infty}\Upsilon'\left(q\left(z\right)\right)q\left(z\right)f\left(z\right)\mathrm{d}z},$$

implies

$$\kappa=\lambda YD,$$

which gives equation (28) for $q\left(z\right)$ in the text:

$$\Upsilon'\left(q\left(z\right)\right)q\left(z\right)=\left(1+\mu\left(z\right)\frac{\frac{1}{\eta}\frac{W}{z}\left(1-F\left(z\right)\right)}{\nu f\left(z\right)}\right)\frac{1}{\eta}\frac{\nu Y^{\frac{1}{\eta}-1}}{D}\left(\frac{q\left(z\right)}{z}\right)^{\frac{1}{\eta}}.$$

Having solved for $q\left(z\right)$, we use the ICC to find the consumption of each entrepreneur,

$$c\left(z\right)=c\left(0\right)+\frac{1}{\eta}WY^{\frac{1}{\eta}}\int_{0}^{z}\left(\frac{q\left(x\right)}{x}\right)^{\frac{1}{\eta}}\frac{1}{x}\mathrm{d}x,$$

where the lump-sum transfer $c\left(0\right)$ adjusts to ensure revenue neutrality

$$C^{w}\left(W\right)+\omega\int c\left(z\right)f\left(z\right)\mathrm{d}z=Y.$$

To find the optimal choice of W, we note that aggregating workers' optimal consumption and hours choices gives

$$C\left(W\right)=\left(1-\omega\right)\left(\int e^{\frac{1+\gamma}{\gamma+\theta}}\mathrm{d}H\left(e\right)\right)W^{\frac{1+\gamma}{\gamma+\theta}},$$

$$L(W) = (1-\omega)\left(\int_0^\infty e^{\frac{1+\gamma}{\gamma+\theta}} \mathrm{d}H(e)\right) W^{\frac{1-\theta}{\gamma+\theta}},$$

and the overall welfare of workers is

$$V^w(W) = (1-\omega)\frac{\gamma+\theta}{(1-\theta)(1+\gamma)} W^{\frac{(1-\theta)(1+\gamma)}{\gamma+\theta}} \int e^{\frac{(1-\theta)(1+\gamma)}{\gamma+\theta}} \mathrm{d}H(e).$$

The FOC for W can therefore be written

$$\frac{\partial V^w(W)}{\partial W} - \lambda \frac{\partial C^w(W)}{\partial W} + \hat{\nu}\frac{\partial L^w(W)}{\partial W} - \frac{1}{W}\omega\lambda \int_0^\infty \mu(z)\, c'(z)(1-F(z))\, \mathrm{d}z = 0.$$

When $\theta = 1$, we have

$$V^w(W) = (1-\omega)\left(\log W + \int_0^\infty \log e \,\mathrm{d}H(e) - \frac{1}{1+\gamma}\right),$$

$$C^w(W) = (1-\omega)\left(\int_0^\infty e\,\mathrm{d}H(e)\right) W,$$

and

$$L^w(W) = (1-\omega)\left(\int_0^\infty e\,\mathrm{d}H(e)\right),$$

so the FOC with respect to W reduces to

$$\frac{1-\omega}{W} - \lambda\frac{C^w(W)}{W} - \frac{1}{W}\omega\lambda \int_0^\infty \mu(z)\, c'(z)(1-F(z))\, \mathrm{d}z = 0,$$

or

$$\frac{C^w(W)}{1-\omega} = \frac{1}{\lambda} - \frac{\omega}{1-\omega}\int_0^\infty \mu(z)\, c'(z)(1-F(z))\, \mathrm{d}z.$$

Since

$$\lambda = \alpha \int_0^\infty c(z)^{-1} f(z)\, \mathrm{d}z,$$

this reduces to equation (29) in text.

2 Survey of Consumer Finances

The Survey of Consumer Finances (SCF) is a survey conducted by the National Opinion Research Center at the University of Chicago. This survey is well suited for characterizing the earnings, income, and wealth concentration at the top because it over-samples rich households. The unit of observation we use is the household. Each wave of the survey samples more than 6,000 households and is representative of the US economy.

Sample Selection. As is standard in the literature, we exclude households with negative income. In addition, we focus on a sample of households in which the household head is between 22 and 79 years old.

Wealth. Our measure of household wealth is the variable constructed by the Federal Reserve for its Bulletin article which accompanies each wave of the SCF. Wealth is defined as total net worth, which equals assets minus debt. Assets includes both financial and non-financial assets. Financial assets include checking and savings accounts, stocks held directly and indirectly, bonds, etc. Non-financial assets, among others, include the value of houses and other real state, the value of farm and private businesses owned by the household.[3] Debt include both housing debt (e.g. mortgages), debt from lines of credit or credit cards, installment loans, etc.

Income. Our measure of income includes all sources of income excluding government transfers (e.g. social security and unemployment benefits) and excluding other (non classified) sources of income. Thus, we include wage income, income from businesses, income from interests and dividends, from capital gains, rent income and income from pensions and annuities.

Definition of entrepreneurs. In contrast to Cagetti and De Nardi (2006), we consider a broader measure of entrepreneurship that includes all households in which the household head owns a business, excluding those who own C-corporations.[4]

3 Robustness Economies

We show that our results are also robust to alternative parameterizations of the processes for entrepreneurial and labor market ability. In particular, we allow for a fat-tailed distribution of ability to better match top income and wealth inequality and consider an alternative parameterization that targets statistics reported by DeBacker et al. (2020) using IRS data on labor and business income.

3.1 Super-Star Ability State

As is well known, matching top wealth and income inequality in an incomplete markets economy like ours requires departures from a Gaussian distribution of ability. Following

[3]The value of houses, real state and businesses is self-reported. E.g. with respect to housing the survey asks: "What is the current value of this (home and land/apartment/property)?". For businesses the survey asks: "What is the net worth of (your share of) this business?"

[4]The exact question in the survey is: (does the household head) "own privately-held businesses?" The SCF reports the legal status of up to two businesses own by the household. We identify households as owners of C-corporations if at least one of their businesses is reported to be a C-corporation.

Castaneda et al. (2003), we consider an extension with a super-star state that allows the model to match the top income and wealth shares.

An agent can be in either a normal or a super-star state. In the normal state labor market ability follows an AR(1) process as earlier. In the super-star state, labor market ability is relatively high, $\bar{e}$ times higher than the average. We assume that agents transit from the normal to the super-star state with probability p_e and remain in the super-star state with probability q_e. When agents return to the normal state, they draw a new ability level from the ergodic distribution associated with the AR(1) process. An analogous process for entrepreneurial ability is characterized by parameters ρ_z, σ_z, p_z, q_z and $\bar{z}$.

To calibrate the additional parameters describing the super-star state, we augment the original set of moments we target with statistics describing the wealth and income shares of the top 1% of households, as well as the top 1% of workers and entrepreneurs in isolation. Table 1 reports the calibrated parameter values in this economy. Table 2 shows that the model reproduces the targeted moments well. For example, the top 1% of households hold 35% of all wealth in the data, 37% in this calibration, and 28% in our baseline model without a super-star state.

The second column of Table 3 reports the effects of implementing the optimal product market regulation in this version of the model. The regulator now sets the lump-sum transfer to firms equal to zero, while the values of τ_1 and τ_2 are nearly the same as in the baseline parameterization. Once again, the regulator subsidizes larger firms and increases their market share by 0.09. Wages, output and productivity increase slightly more, as does overall welfare, which increases by 2.3% compared to 2.2% in the baseline model. Workers benefit more and experience a welfare gain of 3.7% compared to 3%. Entrepreneurs lose much more now and experience a welfare loss of 7.6% compared to 3.7% in the baseline model.

3.2 Matching Moments on Labor and Business Income from IRS

Our baseline parameterization targets moments describing wealth and income inequality in the 2013 SCF. We now consider an alternative that targets statistics describing the persistence and volatility of labor and business income computed by DeBacker et al. (2020) using a large panel of income tax returns for the 1987-2009 period. These researchers estimate error-component models to describe the processes for labor and business income. They do so by first applying an inverse hyperbolic sine transformation to business income and a logarithmic transformation to labor income. They then remove the component of income accounted for by observable characteristics and fit a process characterized by a fixed effect and an AR(1) component to the residuals. The first column of Table 5 reports the implied persistence, unconditional standard deviation and the standard deviation of changes of transformed income implied by their estimates.

We apply identical transformations to data on labor and business income from our model and find that while our baseline parameterization reproduces the persistence of business income, it generates a standard deviation that is twice as high. We also find that our baseline model predicts too much persistence in labor income (a serial correlation of 0.99 compared to 0.91 in the IRS data) and 75% larger unconditional dispersion. We conjecture that these discrepancies are accounted for by several factors. First, our baseline model abstracts from additional sources of dispersion in wealth, such as heterogeneity in rates of return, that would arise, for example, in the presence of financial frictions. Second, our baseline model targets inequality moments for 2013, while the DeBacker et al. (2020) estimates use data that go back as far as 1987 when inequality was much lower. Third, our baseline model targets broad measures of inequality, while DeBacker et al. (2020) remove the component accounted for by observable characteristics.

We argue, however, that our results are robust to the parameters describing the process for labor and entrepreneurial ability. To that end, we calibrate a version of our model to match the moments implied by the DeBacker et al. (2020) estimates. In addition to these moments, we target a broader measure of entrepreneurship which includes all households who file some source of business income, namely 25%, the number they report for 2009, the latest year in their sample. We also target that 10% of all income is business income, as DeBacker et al. (2020) report.

Table 4 shows the calibrated parameters in this economy. Notice that the persistence of both labor and entrepreneurial ability is substantially lower than in our baseline model. Table 5 shows that our alternative calibration reproduces all these targets well, but, as Table 6 shows, it fails to match the large degree of wealth and income inequality in the SCF data. We also note that this parameterization implies a much lower markup (1.14 compared to 1.22 in our baseline) and losses from misallocation (0.28% compared to 0.72% in our baseline), owing to the lower dispersion in productivity.

The third column of Table 3 reports the effects of implementing the optimal product market regulation in this version of the model. The regulator now imposes a lump-sum tax on producers which amounts to 1.25% of per-capita GDP, thus forcing the least productive 1% of firms to shut down. The regulator sets $\tau_1 = 0.87$ and $\tau_2 = 0.006$, once again subsidizing larger firms, even more than required to restore allocative efficiency, thereby increasing concentration. The model's implications for wages, output and productivity are similar to our baseline. Overall welfare increases by less now (0.8% compared to 2.2% in the baseline), but once again workers greatly benefit from optimal regulation. Their welfare increases by 2.6%, very similar to the 3% increase in the baseline model.

We therefore conclude that our results are robust to changes in the process for labor and entrepreneurial ability.

Table 1: Parameter Values in Economy with Super-Star State

β	0.967	discount factor
ρ_e	0.986	AR(1) e
σ_e	0.144	std. dev. e shocks
p_e	2.2e-6	prob. enter super-star e state
q_e	0.986	prob. stay super-star e state
$\bar{e}$	6.008	log ability super-star e state, rel. mean
ρ_z	0.972	AR(1) z
σ_z	0.141	std. dev. z shocks
p_z	2.9e-6	prob. enter super-star z state
q_z	0.980	prob. stay super-star z state
$\bar{z}$	2.930	log ability super-star z state, rel. mean
σ	12.30	demand elasticity at $q = 1$
μ_c	1.340	mean productivity corporations
$\bar{\mathcal{K}}$	0.068	fixed entry cost / GDP

Table 2: Moments Used to Calibrate Economy with Super-Star State

	Data	Model		Data	Model
Wealth to income ratio	6.57	6.61	Wealth share top 1%	0.35	0.37
Wealth share of entrepr.	0.46	0.46	Income share top 1%	0.22	0.23
Income share of entrepr.	0.31	0.31	Wealth share top 1% entrepr.	0.24	0.23
Gini wealth, all hhs	0.85	0.86	Income share top 1% entrepr.	0.23	0.23
Gini income, all hhs	0.64	0.63	Wealth share top 1% workers	0.31	0.30
Gini wealth, entrepr.	0.78	0.78	Income share top 1% workers	0.16	0.16
Gini income, entrepr.	0.68	0.66	Fraction of corporate firms	0.05	0.05
Gini wealth, workers	0.83	0.84	Sales share corporate firms	0.63	0.63
Gini income, workers	0.59	0.57			

Table 3: Optimal Product Market Intervention, Robustness

	Baseline	Super-star state	IRS moments
$\hat{S}(0)$, rel. per-capita GDP, %	2.22	0.00	-1.25
τ_1	0.795	0.791	0.871
τ_2	0.009	0.010	0.006
Change in sales share top 1% firms	0.09	0.09	0.06
Change in steady-state wage, %	3.17	3.87	2.81
Change in steady-state output, %	1.03	1.48	1.36
Change in aggregate productivity, %	0.67	0.71	0.25
Change in welfare, cev, %	2.17	2.30	0.83
Change in welfare workers, cev, %	2.97	3.69	2.58
Change in welfare entrepreneurs, cev, %	-3.72	-7.56	-4.26

Table 4: Parameter Values in Economy Calibrated to IRS Data

β	0.971	discount factor
ρ_e	0.908	AR(1) e
σ_e	0.217	std. dev. e shocks
ρ_z	0.960	AR(1) z
σ_z	0.091	std. dev. z shocks
σ	14.20	demand elasticity at $q = 1$
μ_c	1.054	mean productivity corporations
$\bar{\mathcal{K}}$	0.059	fixed entry cost / GDP

Table 5: Calibration to IRS Data

	Data	Model
Fraction with business income	0.25	0.25
Share business income in all income	0.08	0.10
Business Income Process		
Standard deviation	2.11	2.12
Autocorrelation	0.96	0.96
Standard deviation of changes	0.60	0.60
Labor Income Process		
Standard deviation	0.71	0.69
Autocorrelation	0.91	0.90
Standard deviation of changes	0.30	0.30

Table 6: Untargeted Moments in Economy Calibrated to IRS Data

	Data	Model		Data	Model
Wealth to income ratio	6.57	6.63	Wealth share top 1%	0.35	0.08
Wealth share of entrepr.	0.46	0.35	Income share top 1%	0.22	0.05
Income share of entrepr.	0.31	0.35	Wealth share top 1% entrepr.	0.24	0.08
Gini wealth, all hhs	0.85	0.57	Income share top 1% entrepr.	0.23	0.05
Gini income, all hhs	0.64	0.36	Wealth share top 1% workers	0.31	0.05
Gini wealth, entrepr.	0.78	0.60	Income share top 1% workers	0.16	0.04
Gini income, entrepr.	0.68	0.36	Fraction of corporate firms	0.05	0.05
Gini wealth, workers	0.83	0.54	Sales share corporate firms	0.63	0.65
Gini income, workers	0.59	0.33			

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