

Keynesian Business Cycle Model

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Macroeconomic Analysis

Recall Classical Model

- Two key equations
 - Euler equation (assuming $C_t = Y_t$)

$$1 + r_t = \frac{1}{\beta} \left(\frac{Y_{t+1}}{Y_t} \right)^\sigma$$

- Equation that determines output

$$Y_t = f(\text{productivity, taxes, preferences, population})$$

Classical Model

- Also known as *supply-side* or *real business cycle* model
 - Real (production-side) factors determine output Y_t
 - Demand factors, or changes in fiscal policy, determine prices r_t
- Changes in monetary policy have no effect on Y_t
- Moreover, no need for government to intervene since efficient

Keynesian Model

- Keynes agrees classical model works well in the long-run
- However, prices do not adjust rapidly in short-run
- So Y_t may deviate from its efficient level in the short-run
- Need government interventions to prevent recessions

Analogy: Madison Square Garden

- Capacity 20,000 seats
- Classical model:
 - ticket prices will adjust to meet demand
 - if demand low, prices fall
 - if demand high, prices increase
 - but always 20,000 in attendance
- Keynesian model:
 - prices are set at beginning of the year
 - if demand low, attendance $< 20,000$
 - if demand high, attendance $> 20,000$

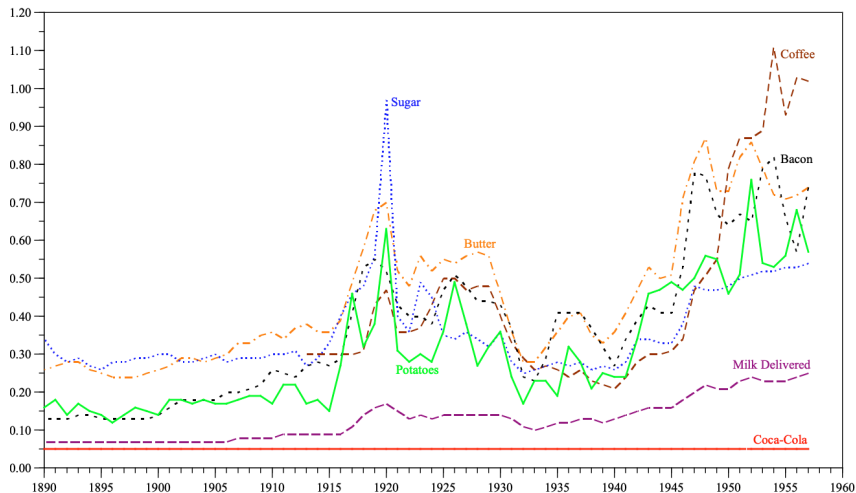
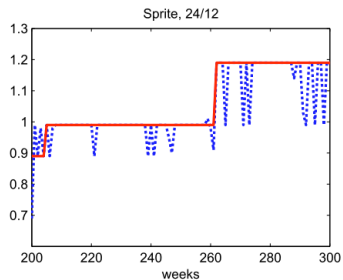
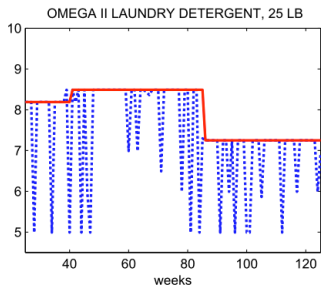
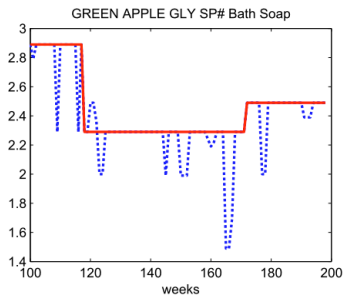
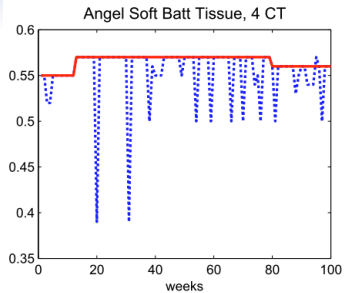


FIG. 1. Retail Price of 6 1/2 oz Coca-Cola vs Retail Prices of Other Foodstuff, 1890–1957

Source: *Historical Statistics of the United States: Colonial Times to 1970, 1989 Edition.*

Units: Coca-Cola (\$/6.5oz), Milk Delivered (\$/Qt), Coffee (\$/Lb), Butter (\$/Lb), Sugar (\$/Lb), Bacon (\$/Lb), and Potatoes (\$/10 Lb).



..... original price
—— regular price

Monopolistic Competition

- Firms do not choose prices under perfect competition
 - many firms, identical goods, cannot charge more than marginal cost
- So think of firms as each having monopoly power for a single good
- Overall consumption: many goods sold by different firms, $i = 1 \dots N$

$$c_t = \left(\sum_{i=1}^N c_{i,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} = \left(c_{1,t}^{\frac{\theta-1}{\theta}} + c_{2,t}^{\frac{\theta-1}{\theta}} + c_{3,t}^{\frac{\theta-1}{\theta}} + \dots + c_{N,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

- number of firms N very large, so each firm too small to affect W_t

Consumer's Problem

- Utility function

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{1}{2} h_t^2 \right)$$

subject to

$$c_t = \left(\sum_{i=1}^N c_{i,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

and budget constraint

$$\sum_{i=1}^N p_{i,t} c_{i,t} + b_{t+1} = W_t h_t + d_t + (1 + i_{t-1}) b_t$$

d_t are dividends from firms and transfers from government

Two-Stage Budgeting

- First, suppose consumer spends e_t dollars in total
- How to allocate this expenditure across various firms?

$$\max_{c_{i,t}} \left(\sum_{i=1}^N c_{i,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

subject to

$$\sum_{i=1}^N p_{i,t} c_{i,t} \leq e_t$$

- Lagrangean

$$\mathcal{L} = \left(\sum_{i=1}^N c_{i,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} + \lambda_t \left(e_t - \sum_{i=1}^N p_{i,t} c_{i,t} \right)$$

First Order Conditions

$$\mathcal{L} = \left(\sum_{i=1}^N c_{i,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} + \lambda_t \left(e_t - \sum_{i=1}^N p_{i,t} c_{i,t} \right)$$

- Differentiate w.r.t to some good $c_{i,t}$:

$$\left(\sum_{i=1}^N c_{i,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}-1} c_{i,t}^{-\frac{1}{\theta}} = \lambda_t p_{i,t}$$

- Let $P_t = \sum_{i=1}^N p_{i,t} \frac{c_{i,t}}{c_t}$ be aggregate price index
- Multiply both sides of $c_{i,t}$ FOC by $c_{i,t}$ and add up

$$\left(\sum_{i=1}^N c_{i,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}-1} \sum_{i=1}^N c_{i,t}^{\frac{\theta-1}{\theta}} = \lambda_t \sum_{i=1}^N p_{i,t} c_{i,t}$$

Demand for Each Good

$$\lambda_t = \frac{1}{P_t}$$

- So $c_{i,t}$ FOC simplifies to

$$\left(\frac{c_{i,t}}{c_t} \right)^{-\frac{1}{\theta}} = \frac{p_{i,t}}{P_t}$$

$$c_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{-\theta} c_t$$

- With price index

$$P_t = \left(\sum_{i=1}^N p_{i,t}^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

Second Stage

- Problem identical to what we had earlier

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{1}{2} h_t^2 \right)$$

subject to

$$P_t c_t + b_{t+1} = W_t h_t + d_t + (1 + i_{t-1}) b_t$$

so the same solutions as previous chapter

Firms' Problem: Flexible Prices

- Absent price adjustment frictions, choose $p_{i,t}$ each period to

$$\max p_{i,t} y_{i,t} - W_t l_{i,t}$$

subject to production function

$$y_{i,t} = A_t l_{i,t}$$

and demand function

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{-\theta} C_t$$

Solve Firm's Problem

$$\max_{p_{i,t}} p_{i,t} \left(\frac{p_{i,t}}{P_t} \right)^{-\theta} C_t - \frac{W_t}{A_t} \left(\frac{p_{i,t}}{P_t} \right)^{-\theta} c_t$$

- First-order condition

$$(1 - \theta) \left(\frac{p_{i,t}}{P_t} \right)^{-\theta} c_t + \theta \frac{W_t}{A_t} \left(\frac{p_{i,t}}{P_t} \right)^{-\theta} c_t \frac{1}{p_{i,t}} = 0$$

- Rearranging

$$p_{i,t} = \frac{\theta}{\theta - 1} \frac{W_t}{A_t}$$

- Charge markup over marginal cost W_t/A_t
 - markup higher if θ lower: goods less substitutable

Aggregation

- Since all firms identical,

$$P_t = N^{\frac{1}{1-\theta}} p_{i,t} = \frac{\theta}{\theta - 1} N^{\frac{1}{1-\theta}} \frac{W_t}{A_t}$$

- Markup reduces the real wage

$$\frac{W_t}{P_t} = \frac{\theta - 1}{\theta} N^{\frac{1}{\theta-1}} A_t$$

- And the labor share

$$\frac{W_t L_t}{P_t Y_t} = \frac{\theta - 1}{\theta}$$

Solution with Sticky Prices

- Only fraction $1 - \lambda$ of firms can change prices in period t
- Firms that change price at t chooses single price p_t to maximize
 - current and future profits
 - needs to forecast future inflation and marginal costs
- After some messy algebra, solution is

$$\log(P_t) - \log(P_{t-1}) = \pi_t = \pi_t^e + \kappa(\log Y_t - \log Y_t^n)$$

- π_t is inflation, π_t^e forecast of future inflation
- $\kappa = \frac{(1-\lambda)^2}{\lambda}$ depends on how sticky prices are
 - $\kappa = 0$: prices never change ($\lambda = 1$)
 - $\kappa = \infty$: prices change all the time ($\lambda = 0$ as in Classical model)

Phillips Curve

- Let $\hat{y}_t = \log Y_t - \log Y_t^n$: output gap

$$\pi_t = \pi_t^e + \kappa(\log Y_t - \log Y_t^n)$$

$$\pi_t = \pi_t^e + \kappa\hat{y}_t$$

- If \hat{y}_t high: production too high relative to natural rate
 - must hire overtime labor, use capital more intensively
 - increases marginal cost of production, so firms want higher prices

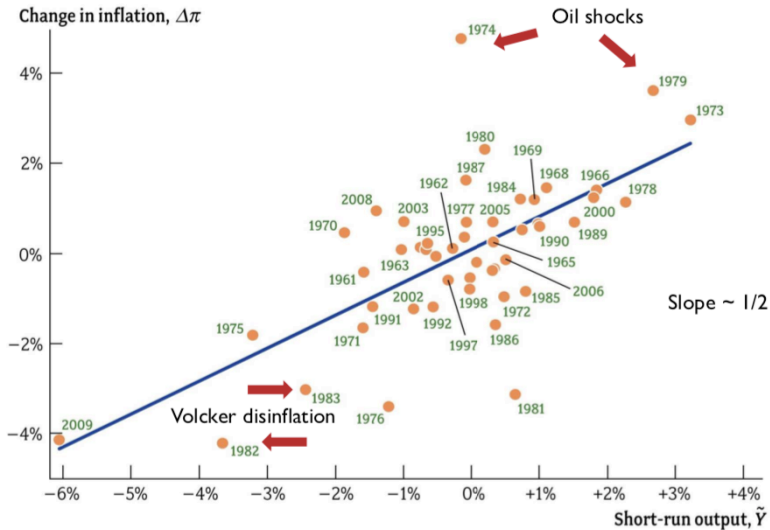
Adaptive Expectations

- Firms need to forecast next year's inflation π_t^e
- At the time they make forecast, only π_{t-1} known
- Assume random walk forecast

$$\pi_t^e = \pi_{t-1}$$

- So Phillips curve becomes

$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t$$



IS Curve

- IS stands for investment-savings (another name for Euler equation)

$$\frac{Y_{t+1}}{Y_t} = [\beta(1 + r_t)]^{\frac{1}{\sigma}},$$

$$\frac{Y_{t+1}^n}{Y_t^n} = [\beta(1 + r_t^n)]^{\frac{1}{\sigma}},$$

$$\frac{Y_{t+1}/Y_{t+1}^n}{Y_t/Y_t^n} = \left[\frac{1 + r_t}{1 + r_t^n} \right]^{\frac{1}{\sigma}},$$

$$\log Y_{t+1} - \log Y_{t+1}^n - (\log Y_t - \log Y_t^n) = \frac{1}{\sigma} (r_t - r_t^n)$$

$$\hat{y}_{t+1} - \hat{y}_t = \frac{1}{\sigma} (r_t - r_t^n),$$

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma} (r_t - r_t^n),$$

Key Equations

1. Inflation Equation (Phillips Curve)

$$\pi_t = \pi_t^e + \kappa \hat{y}_t = \pi_{t-1} + \kappa \hat{y}_t$$

- reflects pricing choices of individual firms (not all can change prices)
- π_t inflation rate, \hat{y}_t : output gap (log deviation of y_t from y_t^n)
- π_t^e is *expected* inflation, assume $\pi_t^e = \pi_{t-1}$
- κ determines how flexible prices are, $\kappa = \infty$ in Classical model

Key Equations

2. Euler Equation (IS Curve)

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma} (r_t - r_t^n)$$

- reflects consumer's savings decisions, desire to smooth consumption
- r_t real rate, r_t^n natural rate (real rate from Classical model)
- σ concavity of utility function, higher σ stronger desire to smooth

Natural Rate of Output and Interest

- Natural Rate: those from the Classical model

- output is supply-determined ($Ak^\alpha h^{1-\alpha}$)

$$Y_t^n = f(\text{productivity, taxes, preferences, demographics})$$

- real rate satisfies an Euler equation

$$1 + r_t^n = \frac{1}{\beta} \left(\frac{Y_{t+1}^n}{Y_t^n} \right)^\sigma$$

or in logs

$$r_t^n = -\log(\beta) + \sigma (\log Y_{t+1}^n - \log Y_t^n)$$

- more expected output growth, lower β increases natural rate

Monetary Policy

- Fed sets nominal i-rate i_t via open market operations
 - increase money supply to reduce i_t
 - reduce money supply to increase i_t
- Real rate $r_t = i_t - \pi_t^e = i_t - \pi_{t-1}$ pinned down as well
- So Fed controls r_t (in contrast to Classical model)
- Since $\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma} (r_t - r_t^n)$
 - Fed controls \hat{y}_t
 - and via Phillips curve, inflation: $\pi_t = \pi_{t-1} + \kappa \hat{y}_t$

Effects of Various Shocks to the Macroeconomy

1. Figure out what happens to natural rates of output and interest
2. Given an interest rate $r_t = i_t - \pi_t^e = i_t - \pi_{t-1}$, find \hat{y}_t

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma} (r_t - r_t^n)$$

3. Given \hat{y}_t , find inflation

$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t$$

Divine Coincidence

- In classical model, $\kappa = \infty$ so Phillips curve implies

$$\hat{y}_t = \frac{1}{\kappa} (\pi_t - \pi_{t-1}) = 0$$

- Suppose the Fed in Keynesian model sets $r_t = r_t^n$, $r_{t+1} = r_{t+1}^n \dots$ forever

$$i_t = r_t^n + \pi_{t-1}$$

- IS curve then implies that

$$\hat{y}_t = 0$$

- Fed can reproduce the natural rate of output (recall efficient)

- need $\hat{y}_{t+1} = 0$, i.e that Fed can credibly commit to $r_{t+1} = r_{t+1}^n \dots$ forever

Inflation

- If $\hat{y}_t = 0$, then inflation is constant

$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t = \pi_{t-1}$$

- If $\hat{y}_t > 0$ for only one period, then inflation permanently increases

$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t > \pi_{t-1}$$

- If $\hat{y}_t < 0$ for only one period, then inflation permanently falls

$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t < \pi_{t-1}$$

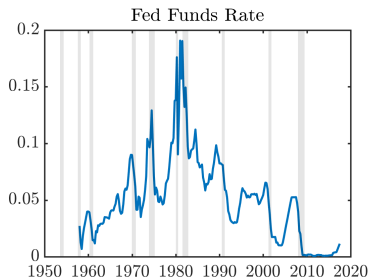
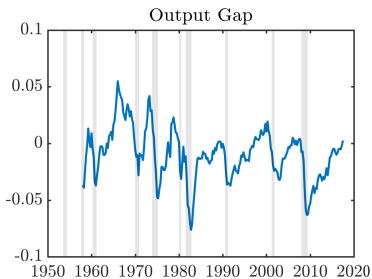
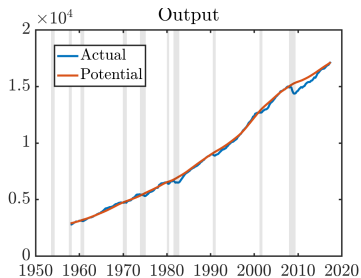
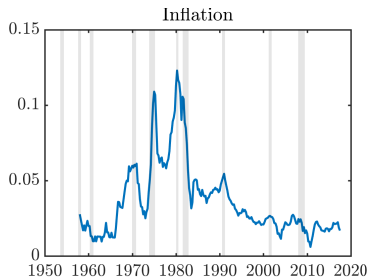
Example

- Suppose $\sigma = 1$, $\kappa = 0.5$, public expects $\hat{y}_{t+1} = 0$, $r_t^n = 2\%$, $\pi_{t-1} = 10\%$

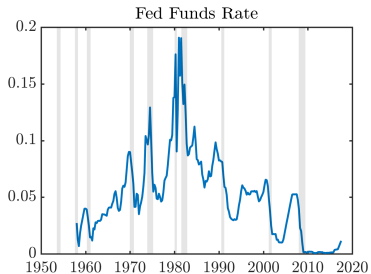
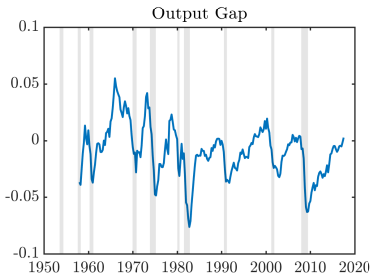
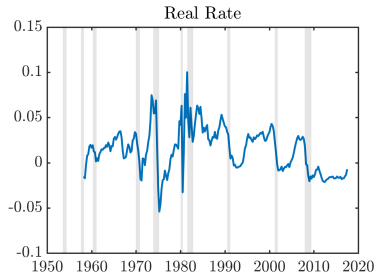
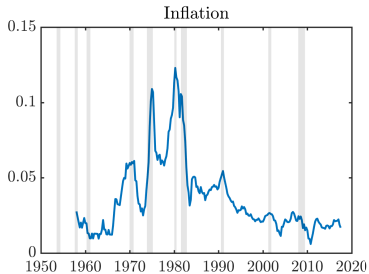
$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t = \pi_{t-1} + \frac{1}{2}(2\% - r_t)$$

- If $r_t = 2\%$, then $\hat{y}_t = 0$, $\pi_t = \pi_{t-1} + 0 = 10\%$
- Suppose $r_t = 6\%$, then $\hat{y}_t = -4\%$, $\pi_t = \pi_{t-1} + \frac{1}{2}(-4\%) = 8\%$
- Disinflation is costly

U.S. Experience



U.S. Experience



Volcker Disinflation

- Volcker raised i_t to reduce inflation. Since $r_t = i_t - \pi_{t-1}$ also raised r_t
- Higher r_t reduced output gap, $\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma} (r_t - r_t^n)$
- Negative output gap caused disinflation, $\pi_t - \pi_{t-1} = \kappa \hat{y}_t \leq 0$

Is the Fed Keeping Interest Rates Too Low?



Neel Kashkari

@neelkashkari

Follow



Need to look at "neutral interest rate" = the rate that balances savings and investment in the economy. Rates have been falling all around the world since 1980. Not driven by short-term central bank movements, but by broader macro forces. We manage around that trend. [#AskNeel](#)

Stalingrad & Poorski @Stalingrad_Poor

@neelkashkari Why is the Fed Funds still at crisis levels 8 years after the crisis?
[#AskNeel](#)

Is the Fed Keeping Interest Rates Too Low?

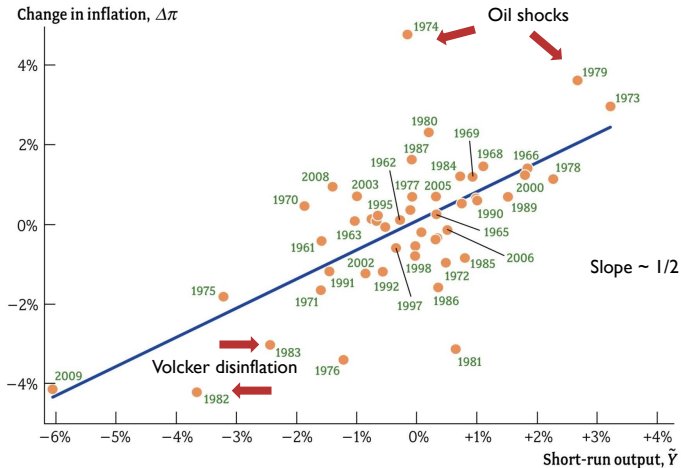
- Many argued Fed has kept r too low for too long, raising asset prices
- Good policy requires keeping $r_t = r_t^n$
- If $r_t < r_t^n$, increase inflation and output gap
- Hard to argue we had $r_t < r_t^n$ until this year
 - π_t and \hat{y}_t were not unusually high
 - low r_t^n due to expectations of low growth
- This year inflation unusually high so perhaps time to increase r_t

Shocks to Inflation Equation

- Fed's role is boring in what we did so far: keep $r_t = r_t^n$
 - ensures constant inflation and no output gap
- Sometimes inflation increases even though output gap falls, e.g. 70s
- Capture with shocks o_t to Phillips curve:

$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t + o_t$$

The Phillips curve in the data



Inflation Shocks

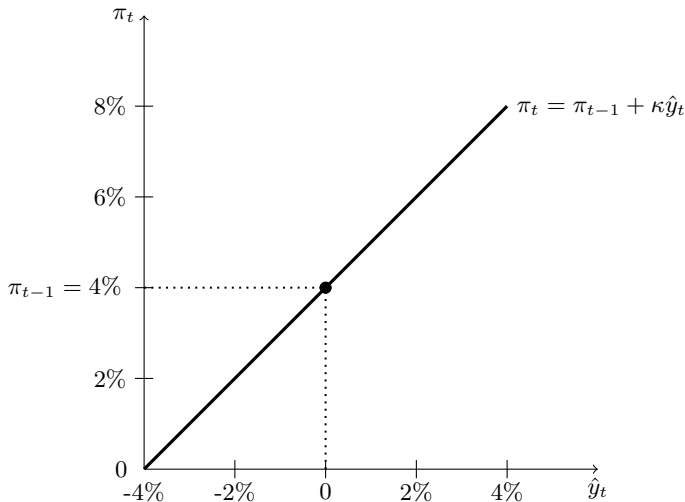
- Tradeoff between output and inflation stabilization

$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t + o_t$$

- suppose Fed has an inflation target of $\bar{\pi} = 2\%$
 - suppose also $\pi_{t-1} = 2\%$ so Fed achieved target last period
 - cannot keep $\hat{y}_t = 0$ and $\pi_t = 2$ if $o_t \neq 0$
- Tradeoff:
 - set $\hat{y}_t = 0$ (by keeping $r_t = r_t^n$) and $\pi_t = \pi_{t-1} + o_t$
 - or set $\pi_t = \pi_{t-1} = 2\%$ and $\hat{y}_t = -\frac{1}{\kappa} o_t$
 - or anything in between

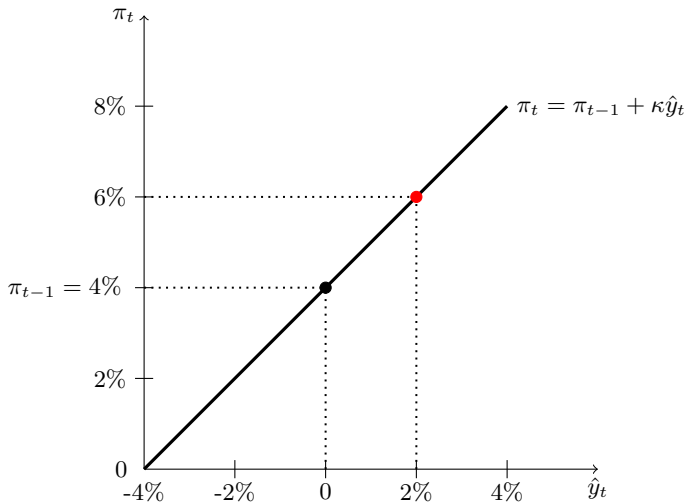
Example: $\kappa = 1$, $\pi_{t-1} = \bar{\pi} = 4\%$, $o_t = 0$

- Fed can choose any point along Phillips curve, $(0, 4\%)$ best



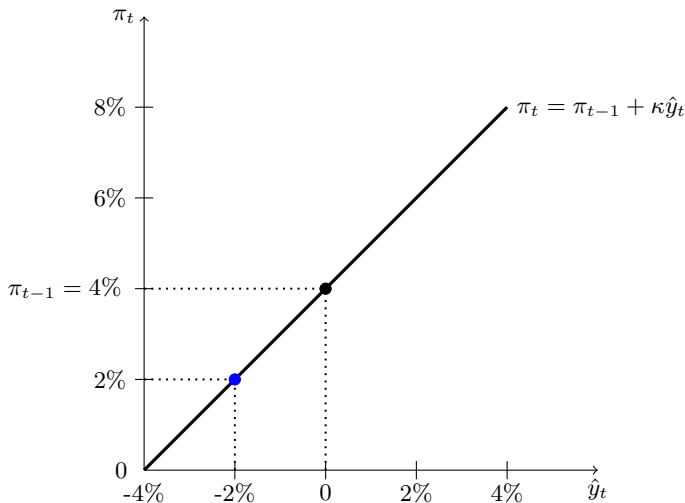
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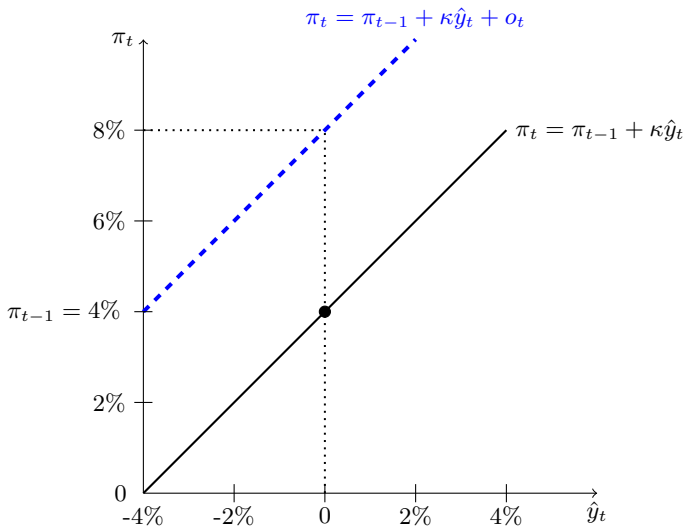
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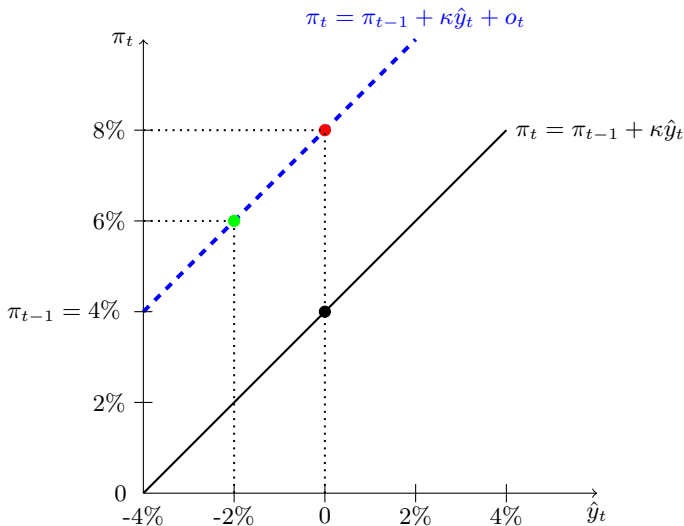
Suppose $o_t = 4\%$

- Phillips curve shifts up, $(0, 4\%)$ no longer attainable



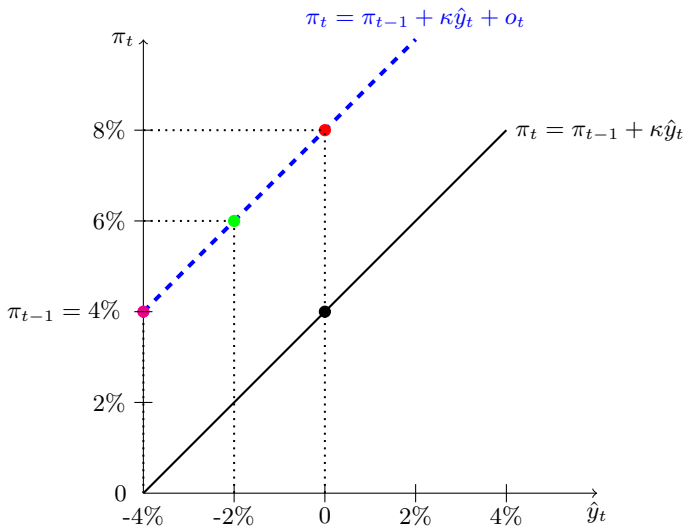
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Suppose $o_t = 4\%$

- Phillips curve shifts up, $(0, 4\%)$ no longer attainable



Response to oil shock

- Depends on Fed's preference for stabilizing output vs inflation
 - if strongly dislike inflation: reduce \hat{y}_t to combat inflation
 - if less worried about inflation: keep $\hat{y}_t \approx 0$
- Absent a rule for how Fed reacts, cannot predict how y_t and π_t respond
- Let us assume a rule: Taylor rule
 - Fed increase r (i) whenever inflation or future output gap too high
 - If strongly dislike inflation, increase r a lot whenever $\pi_t > \bar{\pi}$

Taylor rule

$$i_t = 2\% + \pi_t + 0.5\hat{y}_t + 0.5(\pi_t - 2\%)$$

look like. One policy rule that captures the spirit of the recent research and which is quite straightforward is:

$$r = p + .5y + .5(p - 2) + 2 \quad (1)$$

where

- r is the federal funds rate,
- p is the rate of inflation over the previous four quarters
- y is the percent deviation of real GDP from a target.

That is,

- $y = 100(Y - Y^*)/Y^*$ where
- Y is real GDP, and
- Y^* is trend real GDP (equals 2.2 percent per year from 1984.1 through 1992.3).

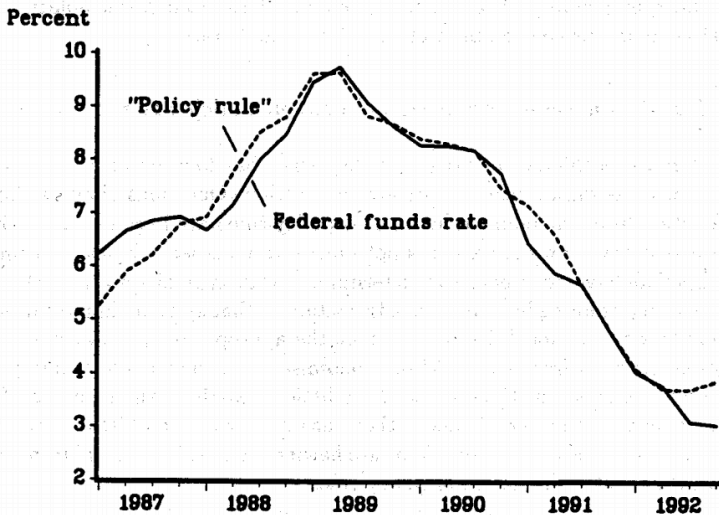


Figure 1. Federal funds rate and example policy rule.

Billions of
1987 dollars

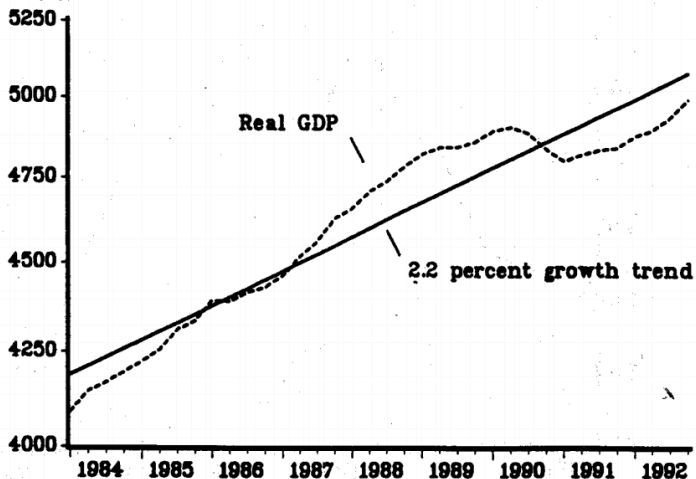


Figure 2. Real GDP and 2.2 percent growth trend.

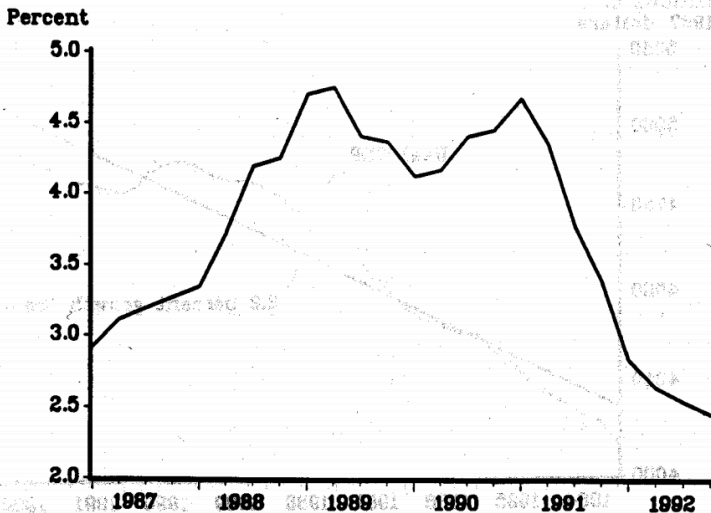
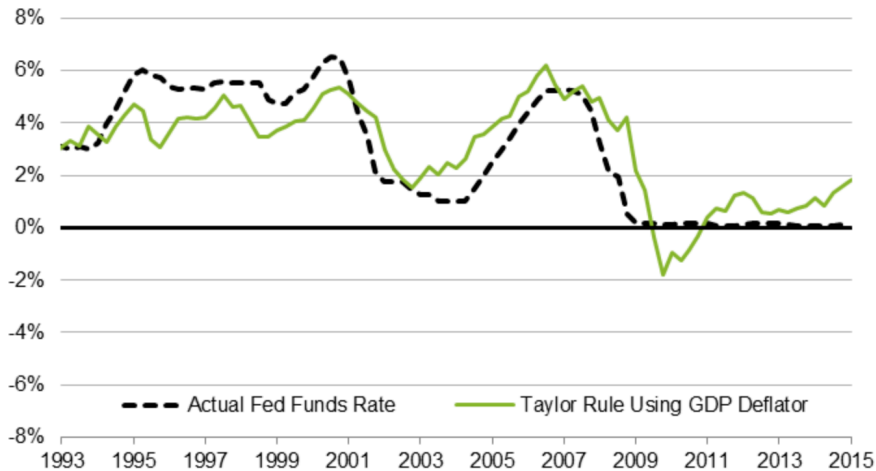


Figure 3. Inflation during previous 4 quarters (GDP deflator).

Figure 1: The Original Taylor Rule, 1993-Present



BROOKINGS

Introduce Taylor Rule in Keynesian Model

- Instead of ad-hoc rule, use a more convenient one:

$$r_t = r_t^n + \sigma \hat{y}_{t+1} + \sigma m (\pi_t - \bar{\pi})$$

- Raise r one-for-one with natural rate
- Also respond to increase in future output gap
- And deviations of inflation from target $\bar{\pi}$
- Higher m the more Fed dislikes missing inflation target

Combine with IS curve

- Taylor rule:

$$r_t - r_t^n = \sigma \hat{y}_{t+1} + \sigma m(\pi_t - \bar{\pi})$$

- Recall Euler equation (IS curve):

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}(r_t - r_t^n)$$

- Combine:

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}(\sigma \hat{y}_{t+1} + \sigma m(\pi_t - \bar{\pi}))$$

$$\hat{y}_t = -m(\pi_t - \bar{\pi})$$

Two equations

- Euler equation + Taylor rule:

$$\hat{y}_t = -m(\pi_t - \bar{\pi})$$

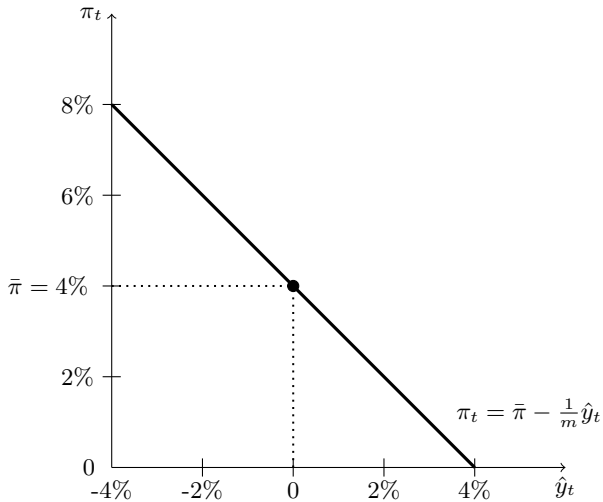
- often referred to as *Aggregate Demand*

- Phillips curve

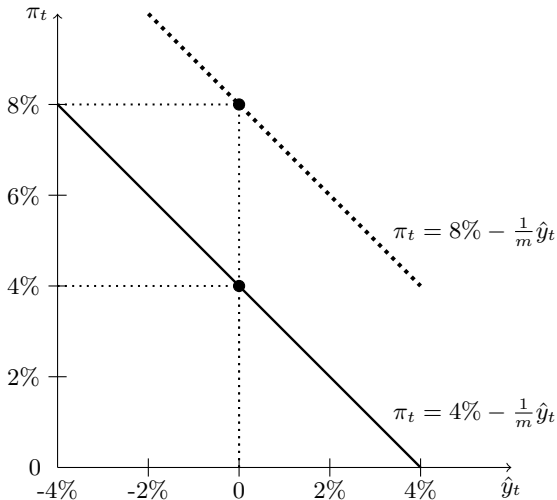
$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t + o_t$$

- often referred to as *Aggregate Supply*

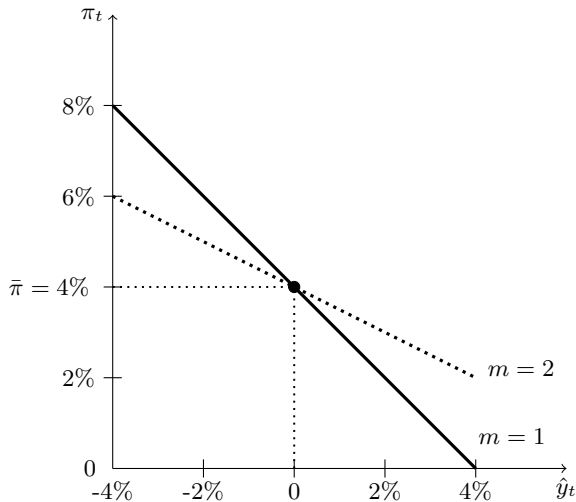
Aggregate Demand



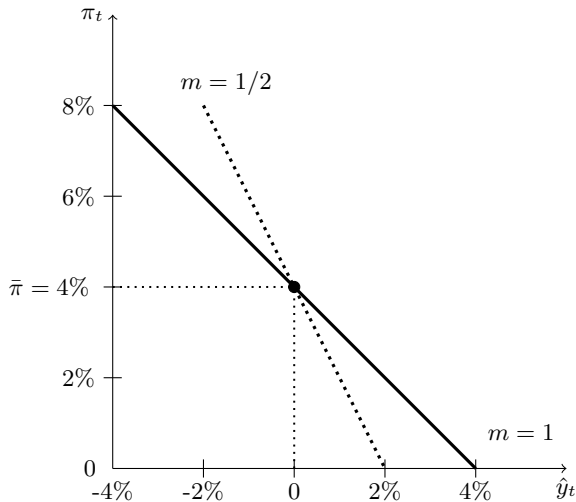
Changes in Inflation Target Shift AD Curve



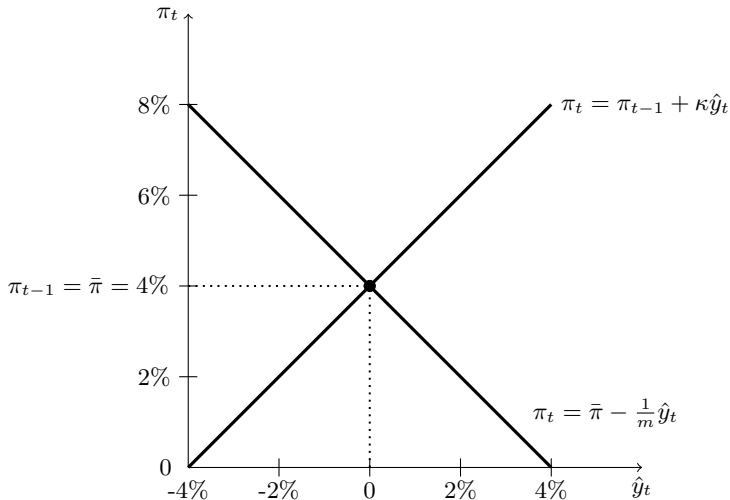
Higher m makes AD flatter (hawk)



Lower m makes AD steeper (dove)



Equilibrium output and inflation



Equilibrium Output and Inflation

$$\pi_t = \pi_{t-1} + \kappa \hat{y}_t + o_t$$

$$\hat{y}_t = -m(\pi_t - \bar{\pi})$$

- To find equilibrium, plug AD into AS

$$\pi_t = \pi_{t-1} - \kappa m(\pi_t - \bar{\pi}) + o_t$$

- Solve for π_t :

$$(1 + \kappa m)\pi_t = \pi_{t-1} + \kappa m\bar{\pi} + o_t$$

$$\pi_t = \frac{1}{1 + \kappa m}\pi_{t-1} + \frac{\kappa m}{1 + \kappa m}\bar{\pi} + \frac{1}{1 + \kappa m}o_t$$

- And then find \hat{y}_t from AD

$$\hat{y}_t = -m(\pi_t - \bar{\pi})$$

Steady State

- In steady state $o_t = 0$ and $\pi_t = \pi_{t-1}$
- Solve for π_t :

$$\pi_t = \frac{1}{1 + \kappa m} \pi_t + \frac{\kappa m}{1 + \kappa m} \bar{\pi}$$

$$\frac{\kappa m}{1 + \kappa m} \pi_t = \frac{\kappa m}{1 + \kappa m} \bar{\pi}$$

$$\pi_t = \bar{\pi}$$

- From AD,

$$\hat{y}_t = -m(\pi_t - \bar{\pi}) = 0$$

- Study next effect of oil shocks and changes in $\bar{\pi}$

Example: $\kappa = m = 1$, $\bar{\pi} = 4\%$, $\pi_{2016} = 4\%$, $o_{2017} = 4\%$

$$\pi_t = \frac{1}{2}\pi_{t-1} + \frac{1}{2}\bar{\pi} + \frac{1}{2}o_t, \text{ and } \hat{y}_t = -(\pi_t - 4\%)$$

- $\pi_{2017} = \frac{1}{2}4\% + \frac{1}{2}4\% + \frac{1}{2}4\% = \mathbf{6.0\%}, \quad \hat{y}_{2017} = -(6\% - 4\%) = \mathbf{-2\%}$

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- $\pi_{2018} = \frac{1}{2}\mathbf{6\%} + \frac{1}{2}4\% + \frac{1}{2}0\% = \mathbf{5.0\%}$, $\hat{y}_{2018} = -(5\% - 4\%) = \mathbf{-1\%}$

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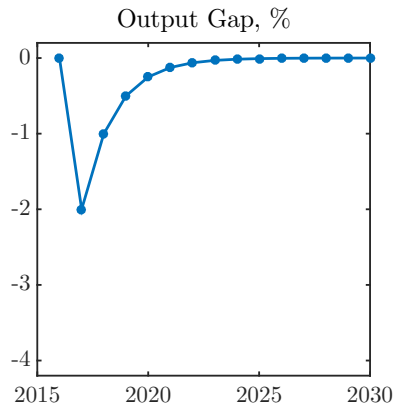
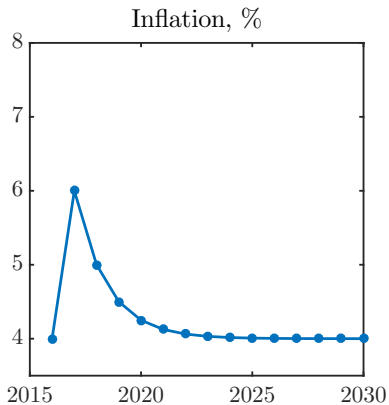
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- $\pi_{2019} = \frac{1}{2}\mathbf{5\%} + \frac{1}{2}4\% + \frac{1}{2}0\% = \mathbf{4.5\%}$, $\hat{y}_{2019} = -(4.5\% - 4\%) = \mathbf{-0.5\%}$

Example: $\kappa = m = 1$, $\bar{\pi} = 4\%$, $\pi_{2016} = 4\%$, $o_{2017} = 4\%$

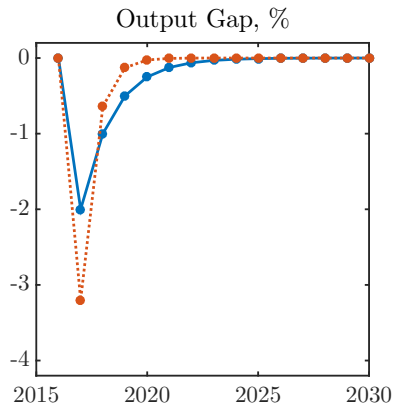
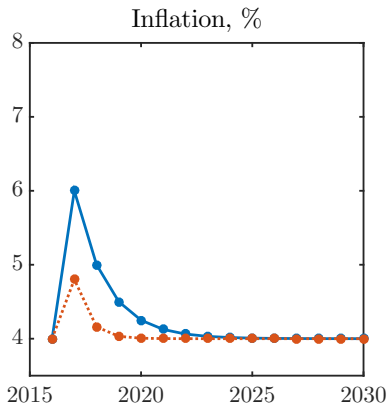
$$\pi_t = \frac{1}{2}\pi_{t-1} + \frac{1}{2}\bar{\pi} + \frac{1}{2}o_t, \text{ and } \hat{y}_t = -(\pi_t - 4\%)$$

- $\pi_{2017} = \frac{1}{2}4\% + \frac{1}{2}4\% + \frac{1}{2}4\% = \mathbf{6.0\%}$, $\hat{y}_{2017} = -(6\% - 4\%) = \mathbf{-2\%}$
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- $\pi_{2019} = \frac{1}{2}\mathbf{5\%} + \frac{1}{2}4\% + \frac{1}{2}0\% = \mathbf{4.5\%}$, $\hat{y}_{2019} = -(4.5\% - 4\%) = \mathbf{-0.5\%}$
- Output and inflation return to steady state slowly

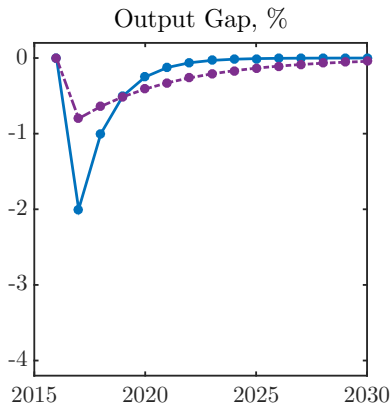
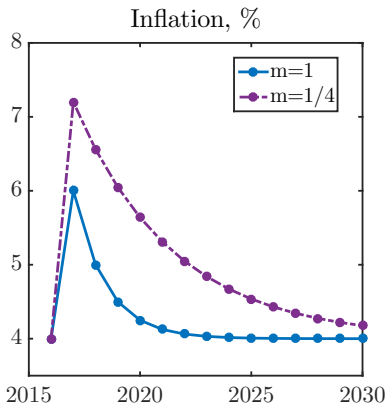
Transition Dynamics



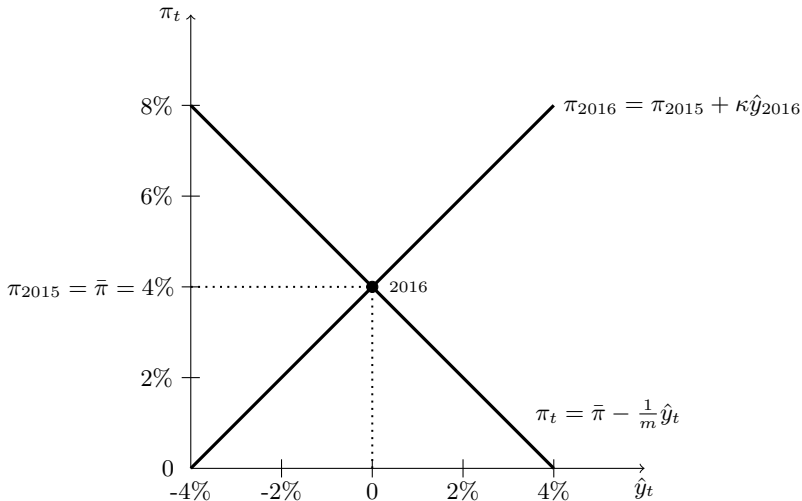
Transition Dynamics: $m = 4$



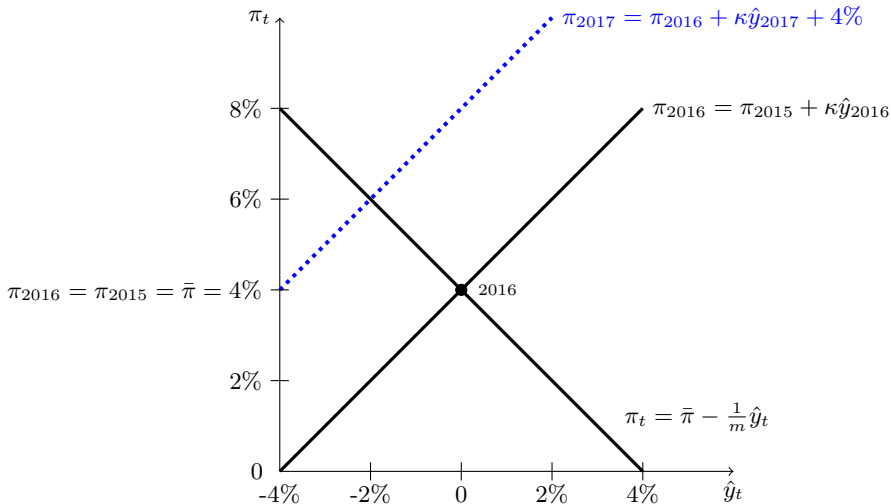
Transition Dynamics: $m = 1/4$



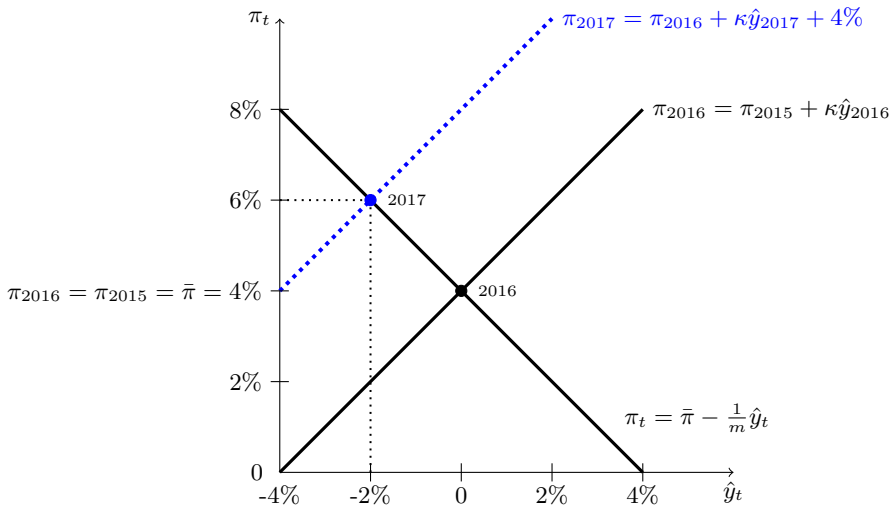
Transition Dynamics on AD/AS diagram



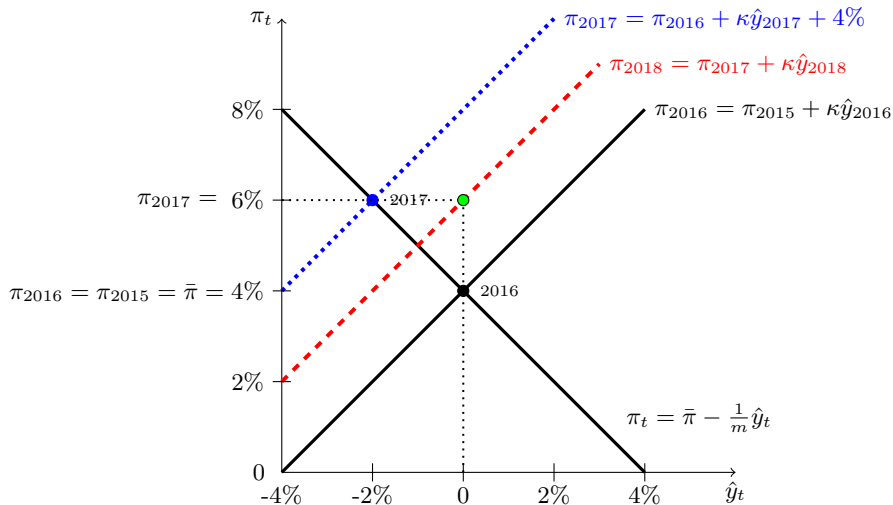
Transition Dynamics on AD/AS diagram



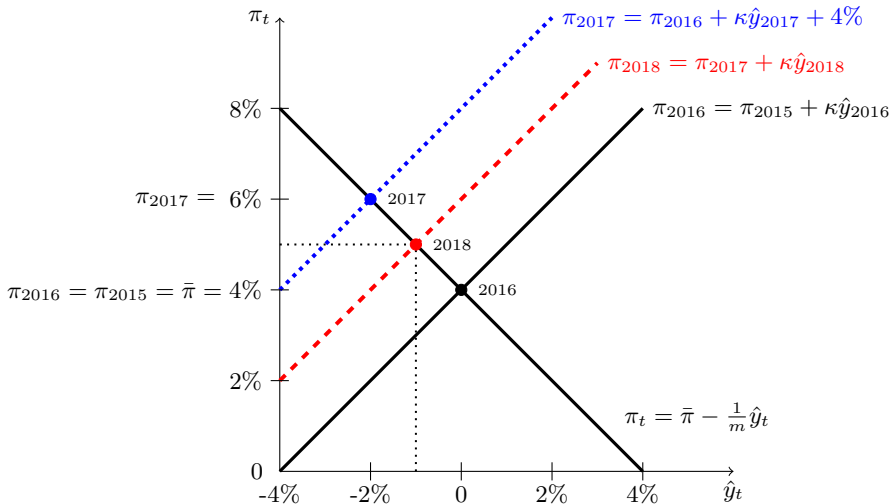
Transition Dynamics on AD/AS diagram



Transition Dynamics on AD/AS diagram



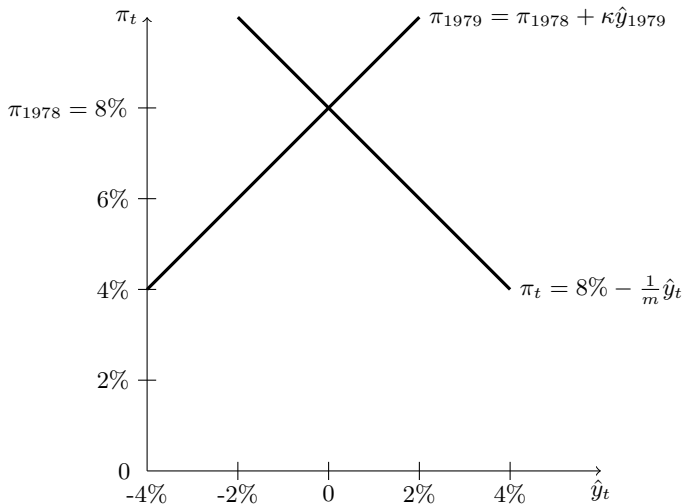
Transition Dynamics on AD/AS diagram



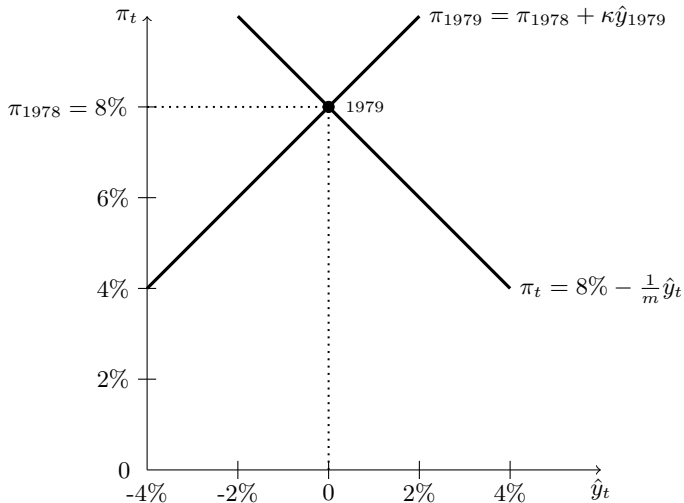
Effect of a Disinflation

- Suppose $\bar{\pi} = 8\%$ up to 1979
- Volcker comes in 1980 and changes $\bar{\pi} = 4\%$
- How does output and inflation react in the next few years?

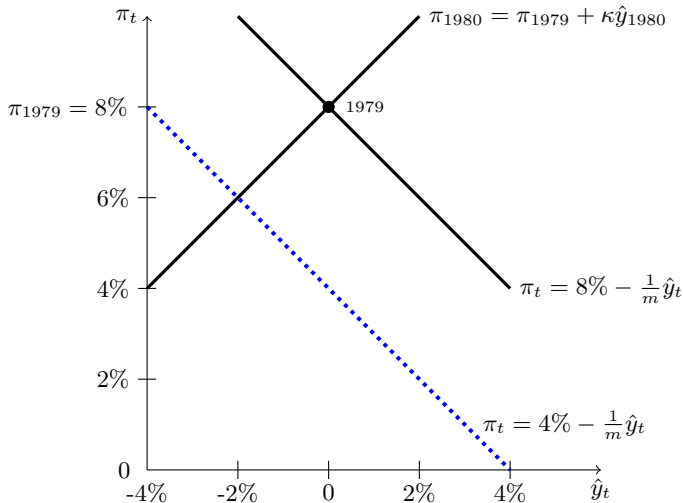
Disinflation on AD/AS diagram



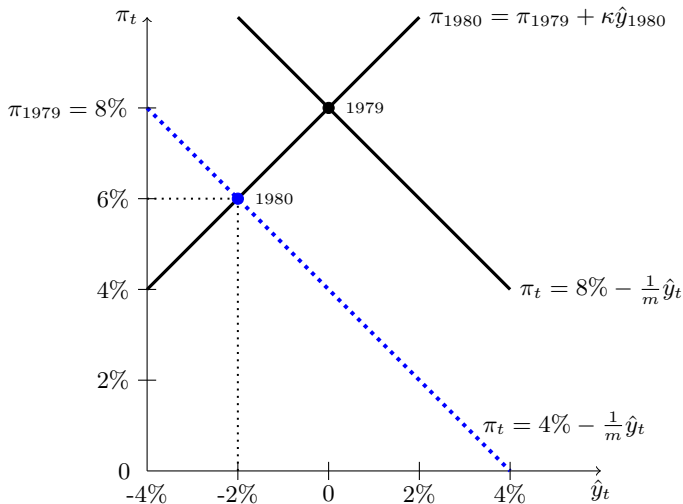
Disinflation on AD/AS diagram



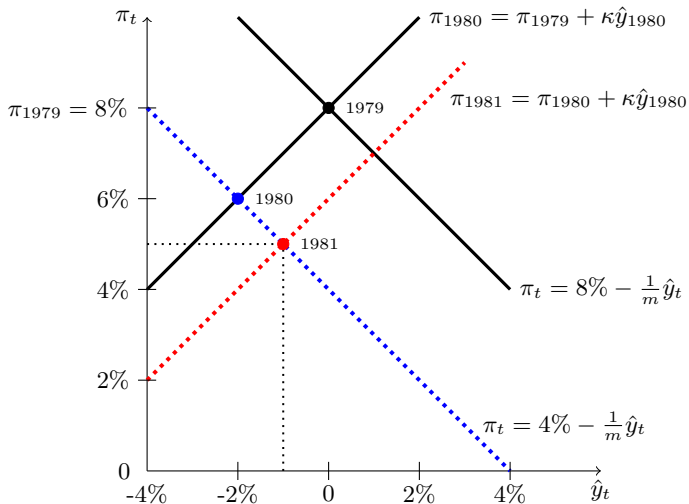
Disinflation on AD/AS diagram



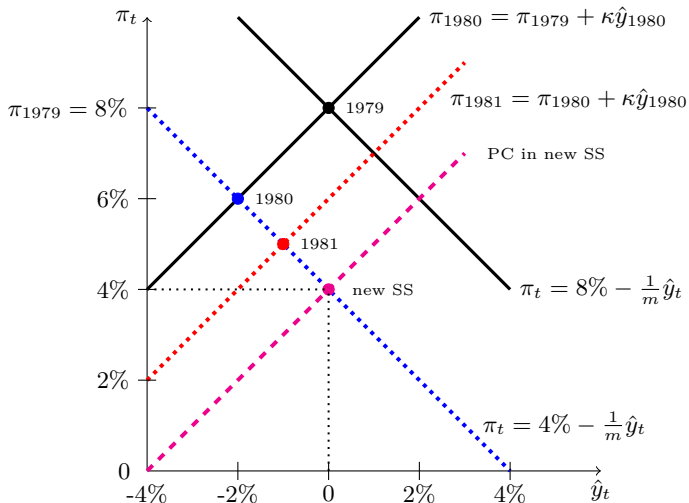
Disinflation on AD/AS diagram



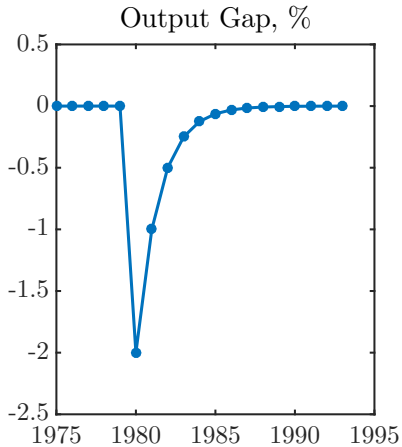
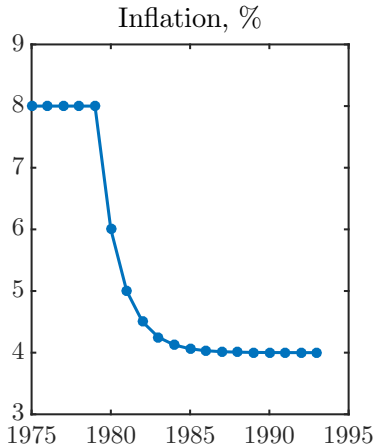
Disinflation on AD/AS diagram



Disinflation on AD/AS diagram



Transition Dynamics



Monetary Policy Shocks

- So far we assumed

$$r_t = r_t^n + \sigma \hat{y}_{t+1} + \sigma m(\pi_t - \bar{\pi})$$

- In reality Fed does not observe r_t^n or \hat{y}_{t+1} perfectly so makes mistakes
- Or if r_t^n falls a lot, cannot reduce r_t due to *zero lower bound*
- Let $r_t^* = r_t^n + \sigma \hat{y}_{t+1} + \sigma m(\pi_t - \bar{\pi})$: "correct" interest rate
- Assume instead of setting $r_t = r_t^*$ Fed instead sets

$$r_t = r_t^* - \sigma a_t$$

- Here a_t determines the size of the mistake; if $a_t > 0$ Fed sets r too low

Derive new AD curve

- We now have

$$r_t = r_t^n + \sigma \hat{y}_{t+1} + \sigma m(\pi_t - \bar{\pi}) - \sigma a_t$$

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}(r_t - r_t^n)$$

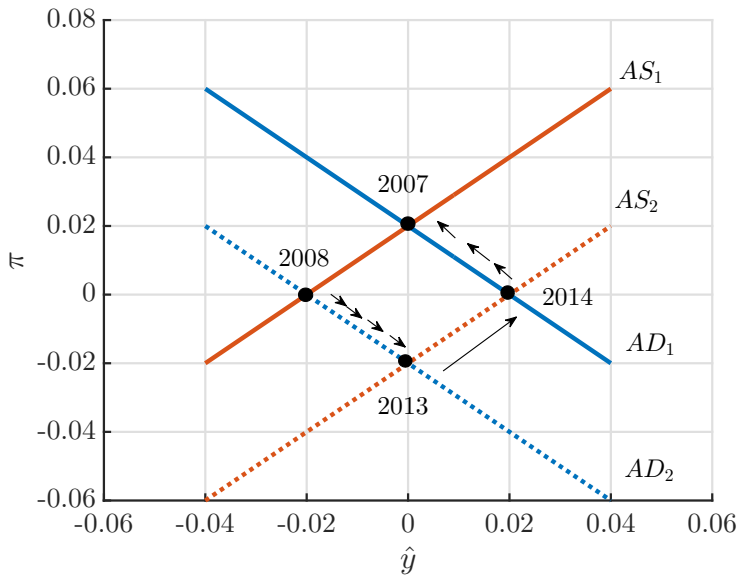
- Gives

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}(\sigma \hat{y}_{t+1} + \sigma m(\pi_t - \bar{\pi}) - \sigma a_t)$$

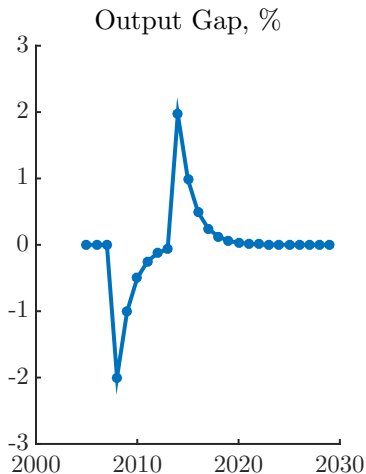
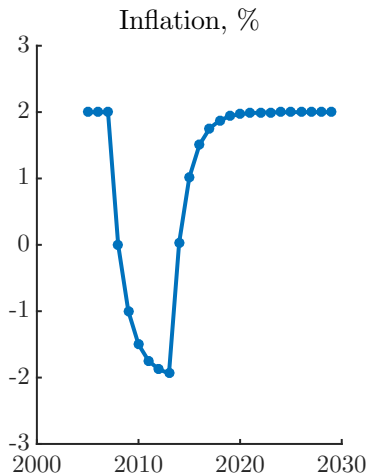
$$\hat{y}_t = a_t - m(\pi_t - \bar{\pi})$$

- a_t also referred to as *aggregate demand shock*
 - a lot like changes in inflation target, except that usually transitory

AD shock (hits in 2008, reverts 2014)



Transition Dynamics after AD shock



Rational Expectations

- So far assumed *adaptive expectations*: $\pi_t^e = \pi_{t-1}$
- Assume instead *rational expectations*:
 - agents know the model and compute model-consistent forecasts

$$\pi_t = \pi_t^e + \kappa \hat{y}_t + o_t$$

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma} (r_t - r_t^n)$$

$$r_t - r_t^n = \sigma \hat{y}_{t+1} + \sigma m (\pi_t - \bar{\pi})$$

$$\hat{y}_t = -m (\pi_t - \bar{\pi})$$

Rational Expectations

- So system is

$$\pi_t = \pi_t^e + \kappa \hat{y}_t + o_t$$

$$\hat{y}_t = -m(\pi_t - \bar{\pi})$$

- Rational expectations:

$$\pi_t^e = \mathbb{E}_t \pi_{t+1}$$

- \mathbb{E}_t : mathematical expectations operator using info up to t

$$\pi_t = \frac{1}{1 + \kappa m} \mathbb{E}_t \pi_{t+1} + \frac{\kappa m}{1 + \kappa m} \bar{\pi} + \frac{1}{1 + \kappa m} o_t$$

- Assume supply shocks o_t follow AR(1) process: $\mathbb{E}_t o_{t+1} = \rho o_t$
 - ρ determines how rapidly shock decays

Solve the Model

- Guess and verify

$$\pi_t = \gamma_0 + \gamma_1 o_t$$

- γ_0 and γ_1 unknown coefficients that need to be determined

- Forecast of inflation

$$\mathbb{E}_t \pi_{t+1} = \gamma_0 + \gamma_1 \mathbb{E}_t o_{t+1} = \gamma_0 + \gamma_1 \rho o_t$$

- Check guess is correct and use method of undetermined coefficients:

$$\pi_t = \frac{1}{1 + \kappa m} \mathbb{E}_t \pi_{t+1} + \frac{\kappa m}{1 + \kappa m} \bar{\pi} + \frac{1}{1 + \kappa m} o_t$$

$$\gamma_0 + \gamma_1 o_t = \frac{1}{1 + \kappa m} (\gamma_0 + \gamma_1 \rho o_t) + \frac{\kappa m}{1 + \kappa m} \bar{\pi} + \frac{1}{1 + \kappa m} o_t$$

Method of Undetermined Coefficients

$$\gamma_0 + \gamma_1 o_t = \frac{1}{1 + \kappa m} (\gamma_0 + \gamma_1 \rho o_t) + \frac{\kappa m}{1 + \kappa m} \bar{\pi} + \frac{1}{1 + \kappa m} o_t$$

- This equation must hold for any o_t so match up coefficients:

$$\gamma_0 = \frac{1}{1 + \kappa m} \gamma_0 + \frac{\kappa m}{1 + \kappa m} \bar{\pi}$$

$$\gamma_1 = \frac{1}{1 + \kappa m} \gamma_1 \rho + \frac{1}{1 + \kappa m}$$

Method of Undetermined Coefficients

- First equation implies:

$$\gamma_0 = \bar{\pi}$$

- Second equation implies

$$\gamma_1 = \frac{1}{1 + \kappa m - \rho}$$

- So equilibrium inflation and output

$$\pi_t = \bar{\pi} + \frac{1}{1 + \kappa m - \rho} o_t$$

$$\hat{y}_t = -\frac{m}{1 + \kappa m - \rho} o_t$$

- Does not depend on past inflation so disinflations costless

Optimal Monetary Policy

- Assume rational expectations and $\rho = 0$ so $\mathbb{E}_t \pi_{t+1} = \bar{\pi}$

$$\pi_t = \bar{\pi} + \kappa \hat{y}_t + o_t$$

- Euler equation ($\mathbb{E}_t \hat{y}_{t+1} = 0$)

$$\hat{y}_t = -\frac{1}{\sigma} (r_t - r_t^n)$$

- Fed objective

$$(\hat{y}_t - 0)^2 + \alpha (\pi_t - \bar{\pi})^2$$

- What is optimal policy (r_t) given o_t and r_t^n ?
 - since r_t determines \hat{y}_t and π_t , can directly choose these

Fed Problem

- Substitute Phillips curve into Fed objective

$$\max_{\hat{y}_t} \hat{y}_t^2 + \alpha (\kappa \hat{y}_t + o_t)^2$$

- FOC:

$$\hat{y}_t + \alpha \kappa (\kappa \hat{y}_t + o_t)$$

- Solution:

$$\hat{y}_t = -\frac{\alpha \kappa}{1 + \alpha \kappa^2} o_t$$

$$\hat{\pi}_t = \bar{\pi} + \frac{1}{1 + \alpha \kappa^2} o_t \rightarrow m = \alpha \kappa$$

$$r_t = r_t^n + \frac{\sigma \alpha \kappa}{1 + \alpha \kappa^2} o_t$$