Household Leverage and the Recession

Appendix

Not for Publication

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1 Equilibrium Equations

Here we present the set of equations that determine the equilibrium in our baseline model.

Variables: $h_t(s)$, $x_t(s)$, $b_t(s)$, $c_{it}(s)$, $a_t(s)$, $\mu_t(s)$, $\xi_{it}(s)$, $\lambda_t(s)$, $l_t(s)$, $e_t(s)$, $y_t(s)$, $w_t^*(s)$, $w_t(s)$, $n_t(s)$, $n_t^X(s)$, $n_t^N(s)$, $y_t^X(s)$, $y_t^N(s)$, $\bar{y}_t^X(s)$, $\bar{y}_t^N(s)$, $p_t(s)$, $p_t^N(s)$, $p_t^X(s)$, $\hat{p}_t^N(s)$, $\hat{p}_t^X(s)$, p_t^T, p_t, q_t, π_t, i_t, y_t, $\hat{y}_t$.

Housing FOC:

$$e_t(s)\lambda_t(s) - \beta_t(s)\mathbb{E}_t\lambda_{t+1}(s)e_{t+1}(s) = \beta_t(s)\mathbb{E}_t\frac{\eta_{t+1}^h(s)}{h_{t+1}(s)} + \mu_t(s)m_t(s)e_t(s). \tag{1}$$

Transfers FOC:

$$\lambda_t(s) = \beta_t(s)(1+i_t)\mathbb{E}_t\lambda_{t+1}(s) + \int_0^1 \xi_{it}(s)\mathrm{d}i. \tag{2}$$

Debt FOC:

$$\lambda_t(s) = \beta_t(s)\mathbb{E}_t\lambda_{t+1}(s)R_{t+1} + \mu_t(s) - \gamma\beta_t(s)\mathbb{E}_t\mu_{t+1}(s)\frac{q_{t+1}}{q_t}. \tag{3}$$

Consumption FOC:

$$p_t(s)c_{it}(s) = \min\left[\frac{v_{it}(s)}{\beta_t(s)(1+i_t)\mathbb{E}_t\lambda_{t+1}(s)}, \quad x_t(s)\right]. \tag{4}$$

Savings allocation:

$$a_{t+1}(s) = \left(x_t(s) - p_t(s)\int_0^1 c_{it}(s)\ \mathrm{d}i\right). \tag{5}$$

Borrowing constraint:

$$q_t l_t(s) \leq m_t(s)e_t(s)h_{t+1}(s). \tag{6}$$

Liquidity constraint:

$$p_t(s)c_{it}(s) \leq x_t(s). \tag{7}$$

Budget constraint:

$$x_t(s) + e_t(s)(h_{t+1}(s) - h_t(s)) \leq w_t(s)n_t(s) + q_t l_t(s) - b_t(s) + (1+i_{t-1})a_t(s) + T_t(s). \tag{8}$$

New loans:

$$l_t(s) + \gamma b_t(s) = b_{t+1}(s). \tag{9}$$

Housing market clearing:

$$h_t(s) = \bar{h}. \tag{10}$$

Resource constraint:

$$y_t(s) = \int c_{it}(s)\ \mathrm{d}i. \tag{11}$$

Wage Phillips curve:

$$\log(w_t^*(s)) = \beta\theta_w\mathbb{E}_t\log(w_{t+1}^*(s)) + \frac{1-\theta_w\beta}{1+\psi\nu}(-\log(\lambda_t(s)) + \psi\nu\log(w_t(s)) + \eta_t^n + \nu\log(n_t(s))) + \theta_w\beta\mathbb{E}_t\pi_{t+1}. \tag{12}$$

Real wages:

$$\log(w_t(s)) = \theta_w(\log(w_{t-1}(s)) - \pi_t) + (1-\theta_w)\log(w_t^*(s)). \tag{13}$$

Export production function:

$$\bar{y}_t^X(s) = \int y_{kt}^X(s)\ \mathrm{d}k = z_t(s)\int n_{kt}^X(s)\ \mathrm{d}k. \tag{14}$$

Non-tradeable production function:

$$\bar{y}_t^N(s) = \int y_{kt}^N(s)\ \mathrm{d}k = z_t(s)\int n_{kt}^N(s)\ \mathrm{d}k. \tag{15}$$

Tradeable price index:

$$p_t^T = \left(\int_0^1 p_t^X(s')^{1-\kappa}\mathrm{d}s'\right)^{\frac{1}{1-\kappa}}. \tag{16}$$

Non-tradeables Phillips curve:

$$\log(\hat{p}_t^N(s)) = \beta\theta_p\mathbb{E}_t\log(\hat{p}_{t+1}^N(s)) + (1-\beta\theta_p)(\log(w_t(s)) - \log(z_t(s))). \tag{17}$$

Evolution of non-tradeables price:

$$\log(p_t^N(s)) = \theta_p\log(p_{t-1}^N(s)) + (1-\theta_p)\log(\hat{p}_t^N(s)). \tag{18}$$

Tradeables production Phillips curve:

$$\log(\hat{p}_t^X(s)) = \beta\theta_p\mathbb{E}_t\log(\hat{p}_{t+1}^X(s)) + (1-\beta\theta_p)(\log(w_t(s)) - \log(z_t(s))). \tag{19}$$

Evolution of tradeables production price:

$$\log(p_t^X(s)) = \theta_p\log(p_{t-1}^X(s)) + (1-\theta_p)\log(\hat{p}_t^X(s)). \tag{20}$$

Price of final goods:

$$p_t(s) = \left(\omega p_t^N(s)^{1-\sigma} + (1-\omega)\left(p_t^T\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{21}$$

Export demand:

$$y_t^X(s) = (1-\omega)\left(\frac{p_t^X(s)}{p_t^T}\right)^{-\kappa}\left(\int_0^1\left(\frac{p_t^T}{p_t(s')}\right)^{-\sigma} y_t(s')\mathrm{d}s'\right). \tag{22}$$

Non-traded demand:

$$y_t^N(s) = \omega\left(\frac{p_t^N(s)}{p_t(s)}\right)^{-\sigma} y_t(s). \tag{23}$$

Resource constraint:

$$y_t^X(s) = \bar{y}_t^X(s). \tag{24}$$

Labor market clearing:

$$n_t(s) = \int n_{kt}^X(s)\ \mathrm{d}k + \int n_{kt}^T(s)\ \mathrm{d}k. \tag{25}$$

Total output:

$$y_t = \int_0^1 p_t(s) y_t(s) / p_t \, \mathrm{d}s. \tag{26}$$

Aggregate price level:

$$p_t = \int_0^1 p_t(s) \mathrm{d}s. \tag{27}$$

Inflation:

$$\pi_t = p_t / p_{t-1}. \tag{28}$$

Output gap:

$$\log \hat{y}_t = \log y_t - \log y_t^*. \tag{29}$$

Asset market clearing:

$$\int_0^1 a_{t+1}(s) \, \mathrm{d}s = \int_0^1 b_{t+1}(s) \, \mathrm{d}s. \tag{30}$$

Phillips curve:

$$\log(\pi_t/\bar{\pi}) = \bar{\beta} \mathbb{E}_t \log(\pi_{t+1}/\bar{\pi}) + \frac{(1-\theta_p)(1-\theta_p \bar{\beta})}{\theta_p} \left(\log \left(\int w_t(s) \, \mathrm{d}s \right) - \log(z_t) \right). \tag{31}$$

Monetary policy rule:

$$1 + i_t = (1 + i_{t-1})^{\alpha_r} \left[(1 + \bar{\imath}) \left(\pi_t / \bar{\pi} \right)^{\alpha_\pi} \hat{y}_t^{\alpha_y} \right]^{1-\alpha_r} \left(\hat{y}_t / \hat{y}_{t-1} \right)^{\alpha_x}. \tag{32}$$

The corresponding flexible-price equilibrium equations determine the natural rate of output y_t^*. $T_t(s)$ are transfers that arise from the government to offset the steady state price and wage markup distortions.

2 Alternative Sources of Idiosyncratic Risk

Here we discuss alternative approaches to introducing idiosyncratic risk. Though all these approaches would mimic qualitatively the approach we pursue in the paper, they are less tractable analytically. We start by deriving the liquid asset supply curve in a simple version of our baseline model with Pareto-distributed taste shocks, and then discuss the alternative approaches.

2.1 Pareto Taste Shocks

For transparency, we focus on a simple closed-economy, flexible price version of our model with one-period assets. We also abstract from housing and labor supply, and assume a borrowing limit $b_{t+1} \leq \bar{b}$. The problem of the representative household is to maximize its life-time utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \log \left(c_{it} \right) \mathrm{d}i,$$

s.t.

$$x_t = (1 + r_{t-1}) (a_t - b_t) + y + b_{t+1},$$

$$b_{t+1} \leq \bar{b},$$

$$c_{it} \leq x_t,$$

$$a_{t+1} = x_t - \int c_{it}\,\mathrm{d}i.$$

The first-order optimality conditions are

$$\frac{v_{it}}{c_{it}} = \beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1} + \xi_{it},$$

$$\lambda_t = \beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1} + \int \xi_{it}\,\mathrm{d}i,$$

$$\lambda_t = \beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1} + \mu_t.$$

Here λ_t is the multiplier on the budget constraint, ξ_{it} is the multiplier on the liquidity constraint and μ_t is the multiplier on the borrowing constraint. Let

$$\hat{c}_t = \frac{1}{\beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1}},$$

be the consumption of the agents with $v_{it} = 1$. We then have

$$c_{it} = v_{it}\hat{c}_t \text{ if } v_{it}\hat{c}_t \leq x_t,$$

and

$$c_{it} = x_t,$$

otherwise. We next calculate the average multiplier on the liquidity constraints. We have

$$\frac{\int \xi_{it}\,\mathrm{d}i}{\beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1}} = \int v_{it}\frac{\hat{c}_t}{c_{it}}\,\mathrm{d}i - 1.$$

Using the assumption of Pareto-distributed taste shocks, this expression simplifies to

$$\int v_{it}\frac{\hat{c}_t}{c_{it}}\,\mathrm{d}i - 1 = \int_1^{\frac{x_t}{\hat{c}_t}} (1-1)\;\mathrm{d}F\left(v\right) + \alpha\frac{\hat{c}_t}{x_t}\int_{\frac{x_t}{\hat{c}_t}}^{\infty} v^{-\alpha}\,\mathrm{d}v - \left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha} = \frac{1}{\alpha-1}\left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha}.$$

We can therefore write

$$\frac{\lambda_t}{\beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1}} - 1 = \frac{1}{\alpha-1}\left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha}.$$

Let Δ_t be the wedge in the aggregate Euler equation, implicitly defined as

$$\lambda_t = \left(1+\Delta_t\right)\beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1}.$$

Clearly,

$$\Delta_t = \frac{1}{\alpha-1}\left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha}.$$

The household's total consumption expenditure is

$$\frac{c_t}{\hat{c}_t} = \alpha\int_1^{\frac{x_t}{\hat{c}_t}} v^{-\alpha}\,\mathrm{d}v + \frac{x_t}{\hat{c}_t}\left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha} = \frac{\alpha}{1-\alpha}\left[\left(\frac{x_t}{\hat{c}_t}\right)^{1-\alpha} - 1\right] + \left(\frac{x_t}{\hat{c}_t}\right)^{1-\alpha}$$

, or

$$\frac{c_t}{\hat{c}_t} = \frac{\alpha}{\alpha - 1} - \frac{1}{\alpha - 1}\left(\frac{x_t}{\hat{c}_t}\right)^{1-\alpha}.$$

Finally, savings are

$$a_{t+1} = x_t - c_t = \hat{c}_t\left(\frac{x_t}{\hat{c}_t} - \frac{c_t}{\hat{c}_t}\right),$$

and scaling by consumption (or equivalently income), we have

$$\frac{a_{t+1}}{c_t} = \frac{x_t}{c_t} - 1 = \frac{x_t/\hat{c}_t}{c_t/\hat{c}_t} - 1 = \frac{x_t}{\hat{c}_t}\left(\frac{\alpha}{\alpha - 1} - \frac{1}{\alpha - 1}\left(\frac{x_t}{\hat{c}_t}\right)^{1-\alpha}\right)^{-1} - 1.$$

2.2 Gaussian Taste Shocks

All of our analysis goes through with alternative distributions of idiosyncratic shocks, but at the loss of some analytical tractability. To illustrate this, we next assume that idiosyncratic shocks are normally distributed, with $\log v_{it} \sim N(\mu_v, \sigma_v^2)$. The wedge in the Euler equation is now equal to

$$\Delta_t = \frac{\hat{c}_t}{x_t}\int_{\frac{x_t}{\hat{c}_t}}^{\infty} v \, \mathrm{d}\Phi\left(\frac{\log v - \mu_v}{\sigma_v}\right) + \Phi\left(\frac{\log \frac{x_t}{\hat{c}_t} - \mu_v}{\sigma_v}\right) - 1,$$

where $\Phi(\cdot)$ is the cdf of the standard normal. Since

$$\int_{\frac{x_t}{\hat{c}_t}}^{\infty} v \, \mathrm{d}\Phi\left(\frac{\log v - \mu_v}{\sigma_v}\right) = \exp\left(\mu_v + \frac{\sigma_v^2}{2}\right)\Phi\left(\frac{\mu_v + \sigma_v^2 - \ln \frac{x_t}{\hat{c}_t}}{\sigma_v}\right),$$

we have

$$\Delta_t = \left(\frac{x_t}{\hat{c}_t}\right)^{-1}\exp\left(\mu_v + \frac{\sigma_v^2}{2}\right)\Phi\left(\frac{\mu_v + \sigma_v^2 - \ln \frac{x_t}{\hat{c}_t}}{\sigma_v}\right) + \Phi\left(\frac{\log \frac{x_t}{\hat{c}_t} - \mu_v}{\sigma_v}\right) - 1.$$

Given $\frac{x_t}{\hat{c}_t}$ we can find aggregate consumption using

$$\frac{c_t}{\hat{c}_t} = \int_0^{\frac{x_t}{\hat{c}_t}} v \, \mathrm{d}\Phi\left(\frac{\log v - \mu_v}{\sigma_v}\right) + \frac{x_t}{\hat{c}_t}\left(1 - \Phi\left(\frac{\log \frac{x_t}{\hat{c}_t} - \mu_v}{\sigma_v}\right)\right),$$

or

$$\frac{c_t}{\hat{c}_t} = \exp\left(\mu_v + \frac{\sigma_v^2}{2}\right)\Phi\left(\frac{\ln \frac{x_t}{\hat{c}_t} - \mu_v - \sigma_v^2}{\sigma_v}\right) + \frac{x_t}{\hat{c}_t}\left(1 - \Phi\left(\frac{\log \frac{x_t}{\hat{c}_t} - \mu_v}{\sigma_v}\right)\right),$$

and

$$a_{t+1} = x_t - \hat{c}_t,$$
$$a_{t+1} = x_t - c_t = \hat{c}_t\left(\frac{x_t}{\hat{c}_t} - \frac{c_t}{\hat{c}_t}\right),$$

so the asset to income ratio is

$$\frac{a_{t+1}}{c_t} = \left(\frac{c_t}{\hat{c}_t}\right)^{-1}\left(\frac{x_t}{\hat{c}_t} - \frac{c_t}{\hat{c}_t}\right).$$

Though this liquid asset supply curve is more involved, the model with Gaussian taste shocks produces very similar responses to a tightening of credit, provided one recalibrates the volatility of taste shocks, σ_v^2, appropriately.

2.3 Persistent Shocks

One can also allow for serially correlated taste shocks, though once again at the expense of analytical tractability. For example, suppose

$$\log v_{it} = (1-\rho)\,\mu_v + \rho \log v_{it-1} + \left(1-\rho^2\right)^{1/2} \sigma_v \varepsilon_{it},$$

where $\varepsilon_{it} \sim N(0,1)$. Then the conditional mean is

$$\mathbb{E} \log v_{it} | \log v_{it-1} = (1-\rho)\,\mu_v + \rho \log v_{it-1},$$

and the conditional variance is

$$\mathbb{V} \log v_{it} | \log v_{it-1} = \left(1-\rho^2\right) \sigma_v^2,$$

and the formula determining the amount transfered to a consumer who had a taste v_{it-1} in the previous period is

$$\begin{aligned} 1+\Delta_t = \frac{\lambda_t}{\beta\,(1+r_t)\,\mathbb{E}_t \lambda_{t+1}} = \Phi\left(\frac{\log \frac{x_t(v_{it-1})}{\hat{c}_t} - (1-\rho)\,\mu_v - \rho \log v_{it-1}}{(1-\rho^2)^{1/2}\,\sigma_v}\right) + \\ \left(\frac{x_t\,(v_{it-1})}{\hat{c}_t}\right)^{-1} \exp\left((1-\rho)\,\mu_v + \rho \log v_{it-1} + \frac{(1-\rho^2)\,\sigma_v^2}{2}\right) \times \\ \Phi\left(\frac{(1-\rho)\,\mu_v + \rho \log v_{it-1} + (1-\rho^2)\,\sigma_v^2 - \ln \frac{x_t(v_{it-1})}{\hat{c}_t}}{(1-\rho^2)^{1/2}\,\sigma_v}\right). \end{aligned}$$

This is more involved, since it requires solving a non-linear equation for each v_{it-1}, but conceptually the problem is unchanged.

2.4 Income Shocks

We now assume that the idiosyncratic uncertainty takes the form of income, rather than preference shocks. In particular, we assume that income y_{it} is an i.i.d. random variable, realized after the household decides how much funds x_t to transfer to individual household members. The agent's consumption is thus limited by the sum of the transfer it receives and its idiosyncratic income realization. The representative household's problem is therefore

The problem of the household is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \log\left(c_{it}\right)\,\mathrm{d}i,$$

subject to

$$\begin{aligned} x_t &= (1+r_{t-1})\,(a_t - b_t) + b_{t+1}, \\ b_{t+1} &\leq \bar{b}, \\ c_{it} &\leq x_t + v_{it}, \\ a_{t+1} &= x_t + \int v_{it}\,\mathrm{d}i - \int c_{it}\,\mathrm{d}i. \end{aligned}$$

The first-order conditions are

$$\frac{1}{c_{it}} = \beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1} + \xi_{it},$$

$$\lambda_t = \beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1} + \int \xi_{it}\,\mathrm{d}i,$$

$$\lambda_t = \beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1} + \mu_t,$$

so letting

$$\hat{c}_t = \frac{1}{\beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1}},$$

denote the unconstrained level of consumption, we have

$$c_{it} = \min\left(\hat{c}_t, x_t + v_{it}\right).$$

To find the multipliers we have

$$\frac{\lambda_t}{\beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1}} = 1 + \hat{c}_t\int \xi_{it}\,\mathrm{d}i = 1 + \int_0^{\hat{c}_t - x_t}\left(\frac{\hat{c}_t}{x_t+v} - 1\right)\mathrm{d}F\left(v\right),$$

where $F(\cdot)$ is the distribution of income shocks. To find aggregate consumption we use

$$c_t = \int_0^{\hat{c}_t - x_t}\left(x_t + v\right)\,\mathrm{d}F\left(v\right) + \hat{c}_t\left(1 - F\left(\hat{c}_t - x_t\right)\right).$$

Finally, total savings are

$$a_{t+1} = x_t + \int v_{it}\,\mathrm{d}i - c_t = \int_{\hat{c}_t - x_t}^{\infty}\left(x_t + v - \hat{c}_t\right)\,\mathrm{d}F\left(v\right).$$

We prefer our approach based on taste shocks to this alternative approach based on income shocks for two reasons. First, with Pareto-distribution taste shocks the wedge Δ_t can be computed in closed form. Second, with income taste shocks, one can easily show that average savings are necessarily below average income, so the model would require additional sources of heterogeneity to match the average liquid asset holdings observed in the data.

2.5 Expense Shocks

Suppose instead that individual agents are subject to idiosyncratic expense shocks, so they maximize

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\int \log\left(c_{it} - v_{it}\right)\,\mathrm{d}i,$$

where $c_{it} - v_{it}$ is the amount consumed net of the required expense shocks. The constraints are s.t.

$$x_t = \left(1+r_{t-1}\right)\left(a_t - b_t\right) + y + b_{t+1},$$

$$b_{t+1} \leq \bar{b},$$

$$c_{it} \leq x_t,$$

$$a_{t+1} = x_t - \int c_{it}\,\mathrm{d}i.$$

The first-order conditions are

$$\frac{1}{c_{it} - v_{it}} = \beta\left(1 + r_t\right)\mathbb{E}_t\lambda_{t+1} + \xi_{it},$$

and

$$\lambda_t = \beta\left(1 + r_t\right)\mathbb{E}_t\lambda_{t+1} + \int \xi_{it}\,\mathrm{d}i,$$

$$\lambda_t = \beta\left(1 + r_t\right)\mathbb{E}_t\lambda_{t+1} + \mu_t,$$

The constraint is that

$$c_{it} \leq x_t.$$

Let

$$\tilde{c}_{it} = c_{it} - v_{it},$$

which implies that the Euler equation is

$$\frac{1}{\tilde{c}_{it}} = \beta\left(1 + r_t\right)\mathbb{E}_t\lambda_{t+1} + \xi_{it},$$

and the liquidity constraint is

$$\tilde{c}_{it} \leq x_t - v_{it}.$$

We need to bound the support of v here due to the Inada conditions. So assume $F\left(v\right) = \left(\frac{v}{\bar{v}}\right)^{\alpha}$ where $\bar{v}$ is the upper bound and α determines the shape of the distribution.

The wedge Δ_t satisfies

$$\Delta_t = \frac{\lambda_t}{\beta\left(1 + r_t\right)\lambda_{t+1}} - 1 = \frac{\int \xi_{it}\,\mathrm{d}i}{\beta\left(1 + r_t\right)\lambda_{t+1}}.$$

Since

$$\frac{\xi_{it}}{\beta\left(1 + r_t\right)\lambda_{t+1}} = \frac{\hat{c}_t}{c_{it} - v_{it}} - 1,$$

and

$$c_{it} = \min\left(\hat{c}_t + v_{it}, x_t\right),$$

where

$$\hat{c}_t = \frac{1}{\beta\left(1 + r_t\right)\lambda_{t+1}},$$

we have

$$\Delta_t = \int_{x_t - \hat{c}_t}^{\bar{v}} \left(\frac{\hat{c}_t}{x_t - v} - 1\right)\mathrm{d}F\left(v\right).$$

Consumption is

$$c_t = \int_0^{x_t - \hat{c}_t} \left(\hat{c}_t + v\right)\,\mathrm{d}F\left(v\right) + x_t\left(1 - \left(\frac{x_t - \hat{c}_t}{\bar{v}}\right)^{\alpha}\right),$$

or

$$c_t = \hat{c}_t\left(\frac{x_t - \hat{c}_t}{\bar{v}}\right)^{\alpha} + x_t\left(1 - \left(\frac{x_t - \hat{c}_t}{\bar{v}}\right)^{\alpha}\right) + \frac{\alpha}{1 + \alpha}\left(x_t - \hat{c}_t\right)\left(\frac{x_t - \hat{c}_t}{\bar{v}}\right)^{\alpha},$$

or

$$c_t = x_t - \frac{1}{1+\alpha}\left(x_t - \hat{c}_t\right)\left(\frac{x_t - \hat{c}_t}{\bar{v}}\right)^{\alpha}.$$

So savings are

$$a_{t+1} = x_t - c_t = \frac{x_t - \hat{c}_t}{1+\alpha}\left(\frac{x_t - \hat{c}_t}{\bar{v}}\right)^{\alpha}.$$

Though more tractable, we found this approach less numerically stable and therefore less well-suited for estimation.

3 Model with Default

Here we describe in greater detail the model with default. For expositional convenience, we abstract from regional heterogeneity.

Let q_t be the price of a mortgage loan, which pays one unit next period if the borrower does not default, and h_t denote the housing stock. We assume that in addition to the idiosyncratic liquidity shocks, members of the representative household experience idiosyncratic shocks to the quality of housing they own, ω_{it}, which are i.i.d draws from $G(w)$ with $\int_0^{\bar{\omega}} \omega \mathrm{d}G(\omega) = 1$. Each member therefore has housing wealth $\omega_{it} e_t h_t$ and is responsible for an equal share of the family's debt b_t. An individual member has the option to default on its debt and does so if the value of its home is below the value of its mortgage debt:

$$\omega_{it} e_t h_t < b_t.$$

This determines a threshold

$$\hat{\omega}_t = \frac{b_t}{e_t h_t},$$

below which the agent defaults. The budget constraint, integrated over all members, is

$$x_t + e_t h_{t+1} = w_t n_t + \int_{\hat{\omega}_t}^{\bar{\omega}} \left(\omega e_t h_t - b_t\right) \mathrm{d}G(\omega) + q_t b_{t+1} + \left(1 + r_{t-1}\right) a_t.$$

Financial intermediaries are perfectly competitive and owned by the representative household. In period t the intermediary receives liquid assets from households and lends these funds in the mortgage market at price q_t.

We assume a dead-weight loss from default. When the lender seizes the collateral on a property that defaults, with value $\omega e_t h_t$, it only recovers a fraction $\theta \leq 1$ of it.

3.1 Bond Price

The expected value of what the lender will receive next period, in exchange for lending $q_t b_{t+1}$, is

$$\beta \mathbb{E}_t \lambda_{t+1} \left[\left(1 - G\left(\hat{\omega}_{t+1}\right)\right) b_{t+1} + \theta e_{t+1} h_{t+1} \int_0^{\hat{\omega}_{t+1}} \omega \, \mathrm{d}G(\omega)\right],$$

or

$$\beta \mathbb{E}_t \lambda_{t+1} \left[1 - G\left(\hat{\omega}_{t+1}\right) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega \, \mathrm{d}G(\omega)\right] b_{t+1}.$$

next periods in exchange for lending $q_t b_{t+1}$ to the household. The intermediary borrows these resources from households who save in the liquid asset at an interest rate r_t, so its cost of lending is

$$\beta \mathbb{E}_t \lambda_{t+1} (1 + r_t) q_t b_{t+1},$$

where λ_t is the shadow value of wealth of the representative household. We also assume that there is a transaction cost of issuing new loans loans, proportional to the loan amount, so the total cost of the intermediary is

$$\beta \mathbb{E}_t \lambda_{t+1} (1 + r_t) q_t b_{t+1} + \tau_t \lambda_t q_t b_{t+1}.$$

We think of τ_t as capturing, in a parsimonious way, various frictions that lead to fluctuations in the spread at which lenders are willing to lend in the mortgage market, in short, *credit supply* shocks.

The expected profits of the intermediary are therefore

$$\beta \mathbb{E}_t \lambda_{t+1} \left[1 - G(\hat{\omega}_{t+1}) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega \mathrm{d}G(\omega) \right] b_{t+1} - \beta \mathbb{E}_t \lambda_{t+1} (1 + r_t) q_t b_{t+1} - \tau_t \lambda_t q_t b_{t+1}.$$

Competition drives these expected profits to zero, so we have

$$\beta \mathbb{E}_t \lambda_{t+1} \left[1 - G(\hat{\omega}_{t+1}) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega \, \mathrm{d}G(\omega) \right] = \left[\beta \mathbb{E}_t \lambda_{t+1} (1 + r_t) + \tau_t \lambda_t \right] q_t.$$

Recall that the households' FOC for savings in the liquid account is

$$\lambda_t = \beta (1 + r_t) (1 + \Delta_t) \mathbb{E}_t \lambda_{t+1},$$

where Δ_t depends on the multipliers on the liquidity constraint. Using this expression, we can express the bond price as

$$q_t = \frac{1 + \Delta_t}{1 + (1 + \Delta_t) \tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(1 - G(\hat{\omega}_{t+1}) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega \, \mathrm{d}G(\omega) \right).$$

This simplifies to

$$q_t = \frac{1 + \Delta_t}{1 + (1 + \Delta_t) \tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(1 - \left(1 - \frac{\theta}{\bar{\omega}} \right) \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^{\chi} \right),$$

where we made use of

$$G(\omega) = \left(\frac{\omega}{\bar{\omega}} \right)^{\chi},$$

and

$$\int_0^{\hat{\omega}} \omega \, \mathrm{d}G(\omega) = \left(\frac{\hat{\omega}}{\bar{\omega}} \right)^{1+\chi}.$$

3.2 Optimal Choice of Debt

Consider next the household's optimal debt choice. We follow Hatchondo and Martinez and study a Markov Perfect equilibrium. Since there are no refinancing frictions, the price at which the household borrows tomorrow, q_{t+1} does not depend on the amount it borrows today, b_{t+1}. The borrowing FOC is therefore:

$$\lambda_t \left[q_t + \frac{\partial q_t}{\partial b_{t+1}} b_{t+1} \right] = \beta \mathbb{E}_t \lambda_{t+1} \int_{\hat{\omega}_{t+1}}^{\bar{\omega}} \mathrm{d}G\left(\omega\right).$$

Using

$$\int_{\hat{\omega}}^{\bar{\omega}} \mathrm{d}G\left(\omega\right) = 1 - G\left(\hat{\omega}\right) = 1 - \left(\frac{\hat{\omega}}{\bar{\omega}}\right)^{\chi},$$

allows us to write

$$\lambda_t \left[q_t + \frac{\partial q_t}{\partial b_{t+1}} b_{t+1} \right] = \beta \mathbb{E}_t \lambda_{t+1} \left[1 - \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{\chi} \right].$$

Since

$$\frac{\partial q_t}{\partial b_{t+1}} = -\chi \left(1 - \frac{\theta}{\bar{\omega}}\right) \frac{1 + \Delta_t}{1 + \left(1 + \Delta_t\right)\tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{\chi} \frac{1}{b_{t+1}},$$

we can write the debt FOC as

$$q_t - \chi \left(1 - \frac{\theta}{\bar{\omega}}\right) \frac{1 + \Delta_t}{1 + \left(1 + \Delta_t\right)\tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{\chi} = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[1 - \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{\chi} \right],$$

which implies that

$$q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[1 - \left(1 - \chi \left(1 - \frac{\theta}{\bar{\omega}}\right) \frac{1 + \Delta_t}{1 + \left(1 + \Delta_t\right)\tau_t} \right) \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{\chi} \right].$$

Though we no longer assume a limit on how much the household can borrow, the household recognizes that by borrowing more it increases the interest rate, which leads to an interior solution for b_{t+1}. We now attribute the fluctuations in household credit in the data to credit shocks, τ_t.

3.3 Optimal Choice of Housing

The housing FOC is:

$$\lambda_t e_t = \beta \mathbb{E}_t \frac{\eta_h}{h_{t+1}} + \beta \mathbb{E}_t \lambda_{t+1} e_{t+1} \left(1 - \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{1+\chi} \right) + \lambda_t \frac{\partial q_t}{\partial h_{t+1}} b_{t+1}.$$

Since

$$\frac{\partial q_t}{\partial h_{t+1}} = \chi \left(1 - \frac{\theta}{\bar{\omega}}\right) \frac{1 + \Delta_t}{1 + \left(1 + \Delta_t\right)\tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{\chi} \frac{1}{h_{t+1}},$$

we can write

$$\lambda_t e_t = \beta \mathbb{E}_t \frac{\eta_h}{h_{t+1}} + \beta \mathbb{E}_t \lambda_{t+1} e_{t+1} \left(1 - \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{1+\chi} \right) + \chi \left(1 - \frac{\theta}{\bar{\omega}}\right) \frac{1 + \Delta_t}{1 + \left(1 + \Delta_t\right)\tau_t} \beta \mathbb{E}_t \lambda_{t+1} \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}}\right)^{\chi} \frac{b_{t+1}}{h_{t+1}}.$$

The rest of the model is identical to that described in text. In the aggregate the total amount of liquid assets, a_{t+1}, is equal to the overall amount of mortgage debt, $q_t b_{t+1}$.

Table 1: Marginal Propensity to Consume

	$\alpha = 1.5$	$\alpha = 2.5$	$\alpha = 3.5$	$\alpha = 4.5$	$\alpha = 5.5$
Fraction constrained, %	3.60	0.97	0.26	0.07	0.02
Rate of time preference, $1/\beta - 1$, annual	0.309	0.046	0.024	0.021	0.020
Marginal propensity to consume	0.127	0.061	0.032	0.017	0.010

4 Marginal Propensities to Consume

We show here that our model implies relatively low marginal propensities to consume out of a transitory income change, in line with the predictions of the frictionless model. Table 1 traces out the effect of varying α for several measures of the severity of liquidity constraints in our baseline model. As we vary α, we recalibrate the discount factor β to ensure that the steady-state equilibrium interest rate stays constant at 2% (annualized).

As the table reports, reducing α all the way to 1.5 increases the fraction of household members whose liquidity constraint binds to 3.6%. More consequential is the impact of these constraints on the rate of time preference, $1/\beta - 1$, required to match a 2% interest rate. This rate increases to 31%, a sizable amount, reflecting the households' strong precautionary savings motive. Even with such an extreme parameterization, the marginal propensity to consume out of a transitory income shock is only equal to 12.7%. As we increase α to more empirically plausible values, the fraction of constrained household members fall, as does the rate of time preference and the MPC. For example, when $\alpha = 3.5$, the mean estimate in our baseline model, the fraction of constrained household members falls to 0.26%, the discount rate falls to 2.4%, only a bit higher than the interest rate of 2%, and the MPC falls to 3.2%. As we further increase α, the fraction of constrained household members falls to nearly zero, as does the gap between the interest rate and the rate of time preference and the MPC.

We thus conclude that our results do not rely on implausibly large marginal propensities to consume. Indeed, our model's predictions along this dimension are similar to those of the frictionless consumption-savings model.

5 Likelihood Function

We use Bayesian likelihood methods to estimate the parameters of the island economy and the shocks. We use a panel dataset across states together with aggregate data and the ZLB. We formulate the state-space of the model so as to separate our estimation into a regional component and aggregate component and make it computationally feasible.

We discuss first the likelihood function of the state/regional component and then the likelihood function of the aggregate component. In the paper, we show how we arrive at the state-space representations that we use below to form the likelihood function.

5.1 Likelihood of the Relative State Component

We use Bayesian methods. We first log-linearize the model. The log-linearized model for the relative regional-level variables has the state space representation:

$$\hat{\mathbf{x}}_t(s) = \mathbf{Q}\hat{\mathbf{x}}_{t-1}(s) + \mathbf{G}\epsilon_t(s) \tag{33}$$

$$\hat{\mathbf{z}}_t(s) = \mathbf{H}\hat{\mathbf{x}}_t(s). \tag{34}$$

The state vector is $\hat{\mathbf{x}}_t(s)$. The error is distributed $\epsilon_t(s) \sim N(0, \Omega)$ where Ω is the covariance matrix of $\epsilon_t(s)$. We assume no observation error of the data $\hat{\mathbf{z}}_t(s)$.

Denote by ϑ the vector of parameters to be estimated. Denote by $\mathcal{Z}(s) = \{\hat{\mathbf{z}}_\tau(s)\}_{\tau=1}^{T}$ the sequence of $N_z \times 1$ vectors of observable variables. By Bayes law, letting $\mathcal{P}(\vartheta)$ denote the prior of ϑ, the posterior $\mathcal{P}(\vartheta \mid \mathcal{Z}(s))$ satisfies:

$$\mathcal{P}(\vartheta \mid \mathcal{Z}(s)) \propto L(s) \times \mathcal{P}(\vartheta).$$

The likelihood function $L(s)$ is computed using the sequence of structural matrices and the Kalman filter, described below, and is, for any individual state:

$$\log \mathcal{L}(s) = -\left(\frac{\bar{N}\bar{T}}{2}\right)\log 2\pi - \frac{1}{2}\sum_{t=1}^{\bar{T}} \log\det \mathbf{S}_t - \frac{1}{2}\sum_{t=1}^{\bar{T}} \tilde{\mathbf{x}}_t'(s)\mathbf{S}_t^{-1}\tilde{\mathbf{x}}_t(s).$$

where $\tilde{\mathbf{x}}_t(s)$ is the vector of forecast errors and $\mathbf{S}_t$ is its associated covariance matrix.

5.2 Kalman Filter

The Kalman filter recursion is given by the following equations. For a given region, the state of the system is $(\bar{\mathbf{x}}_t(s), \mathbf{P}_{t-1})$. In the 'predict' step of the filter, the structural matrices $\mathbf{Q}$ and $\mathbf{G}$ are used to compute a forecast of the state $\bar{\mathbf{x}}_{t|t-1}(s)$ and the forecast covariance matrix $\mathbf{P}_{t|t-1}$ as:

$$\begin{aligned} \bar{\mathbf{x}}_{t|t-1}(s) &= \mathbf{Q}\bar{\mathbf{x}}_t(s) \\ \mathbf{P}_{t|t-1} &= \mathbf{Q}\mathbf{P}_{t-1}\mathbf{Q}^\top + \mathbf{G}\Omega\mathbf{G}^\top. \end{aligned} \tag{35}$$

We update these forecasts with imperfect observations of the state vector. This update step involves computing forecast errors $\tilde{\mathbf{x}}_t(s)$ and its associated covariance matrix $\mathbf{S}_t$ as:

$$\begin{aligned} \tilde{\mathbf{x}}_t(s) &= \hat{\mathbf{z}}_t(s) - \mathbf{H}\bar{\mathbf{x}}_{t|t-1}(s) \\ \mathbf{S}_t &= \mathbf{H}\mathbf{P}_{t|t-1}\mathbf{H}^\top. \end{aligned}$$

The Kalman gain matrix is given by:

$$\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{H}^\top\mathbf{S}_t^{-1}.$$

With $\tilde{\mathbf{x}}_t(s)$, $\mathbf{S}_t$ and $\mathbf{K}_t$ in hand, the optimal filtered update of the state $\bar{\mathbf{x}}_t(s)$ is

$$\bar{\mathbf{x}}_t(s) = \bar{\mathbf{x}}_{t|t-1}(s) + \mathbf{K}_t\tilde{\mathbf{x}}_t(s),$$

and for its associated covariance matrix:

$$\mathbf{P}_t = (I - \mathbf{K}_t\mathbf{H})\,\mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with $\bar{\mathbf{x}}_0(s)$ and $\mathbf{P}_0$ determined from their unconditional moments, and is computed until the final time period T of data. We can show that the stationary $\mathbf{P}_0$ has the expression:

$$\mathrm{vec}(\mathbf{P}_0) = (\mathbf{I} - \mathbf{Q}\otimes\mathbf{Q})^{-1}\,\mathrm{vec}(\mathbf{G}\Omega\mathbf{G}^\top) \tag{36}$$

5.3 Kalman Smoother

With the estimates of the parameters on a sample up to time period T, the Kalman smoother gives an estimate of $\mathbf{x}_{t|T}(s)$, or an estimate of the state vector at each point in time given all available information. With $\bar{\mathbf{x}}_{t|t-1}(s)$, $\mathbf{P}_{t|t-1}$, $\mathbf{K}_t$ and $\mathbf{S}_t$ in hand from the Kalman filter, the vector $\mathbf{x}_{t|T}(s)$ is computed by:

$$\mathbf{x}_{t|T}(s) = \bar{\mathbf{x}}_{t|t-1}(s) + \mathbf{P}_{t|t-1}\mathbf{r}_{t|T}(s),$$

where the vector $\mathbf{r}_{T+1|T}(s) = 0$ and is updated with the recursion:

$$\mathbf{r}_{t|T}(s) = \mathbf{H}^\top \mathbf{S}_t^{-1}\left(\hat{\mathbf{z}}_t(s) - \mathbf{H}\bar{\mathbf{x}}_{t|t-1}(s)\right) + (I - \mathbf{K}_t\mathbf{H})^\top \mathbf{P}_{t|t-1}^\top \mathbf{r}_{t+1|T}(s).$$

Finally, to get an estimate of the shocks to each state variable under this model's shock structure, denoted by $\hat{\epsilon}_t(s)$, we can compute:

$$\hat{\epsilon}_t(s) = \mathbf{G}\mathbf{r}_{t|T}(s).$$

5.4 Block structure

The regional component of the model has a block structure. For example, consider two states so that the log-linearized state-space representation of the state variables relative to the aggregate is:

$$\begin{bmatrix} x_t(1) \\ x_t(2) \end{bmatrix} = \begin{bmatrix} \mathbf{Q} & 0 \\ 0 & \mathbf{Q} \end{bmatrix} \begin{bmatrix} x_{t-1}(1) \\ x_{t-1}(2) \end{bmatrix} + \begin{bmatrix} \mathbf{G} & 0 \\ 0 & \mathbf{G} \end{bmatrix} \begin{bmatrix} \epsilon_t(1) \\ \epsilon_t(2) \end{bmatrix}$$

Under this block structure, the forecast covariance matrix $\mathbf{P}_{t|t-1}$ also has a block structure, which can be seen from the expressions (35) and (36).

The log-likelihood becomes a weighted sum of state-by-state log-likelihood functions. To show this: because $\mathbf{P}_{t|t-1}$ has a block structure, so does $\mathbf{S}_t$. And because $\mathbf{S}_t$ has a block structure:

$$\log\det \mathbf{S}_t = \log \prod_j \det \mathbf{S}_t^j = \sum_j \log\det \mathbf{S}_t^j.$$

Also, because $\mathbf{S}_t$ has a block structure, so does its inverse, so that the last term in the log-likehood can also be separated by state. The log-likelihood of the state-level components is then:

$$\log \mathcal{L} = \sum_s \log \mathcal{L}(s).$$

5.5 Weighting

We weight the contributions of the state likelihoods to account for differences in the size of states. Weights can, in principle, depend on the sample and the model's parameters. Agostinelli and Greco (2012) discuss the asymptotic properties of the weighting function which are needed for the weighted likelihood to share the same asymptotic properties as the genuine likelihood function. Using population weights for state subsamples which are constant over time is a simple weighting function which satisfies these properties.

6 Likelihood of the Aggregate Component

6.1 Solution with Zero Lower Bound

The equilibrium conditions that characterize the evolution of aggregate variables $\mathbf{x}_t$ are a function of time-varying structural matrices $\mathbf{A}_t$, $\mathbf{B}_t$, $\mathbf{C}_t$, $\mathbf{D}_t$ and $\mathbf{F}_t$:

$$\mathbf{A}_t\mathbf{x}_t = \mathbf{C}_t + \mathbf{B}_t\mathbf{x}_{t-1} + \mathbf{D}_t\mathbb{E}_t x_{t+1} + \mathbf{F}_t\epsilon_t.$$

For example, if the ZLB binds, the equation describing the Taylor rule becomes $i_t = 0$, changing the structural matrices $\mathbf{A}_t$, and so on. In the paper, we describe two monetary policy regimes, $r = 0$ when the ZLB does not bind, and $r = 1$ when the ZLB binds. With time-varying structural matrices, the solution is a time-varying VAR:

$$\mathbf{x}_t = \mathbf{J}_t + \mathbf{Q}_t\mathbf{x}_{t-1} + \mathbf{G}_t\epsilon_t, \tag{37}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$ are conformable matrices which are functions of the evolution of beliefs about the time-varying structural matrices $\mathbf{A}_t$, $\mathbf{B}_t$, $\mathbf{C}_t$, $\mathbf{D}_t$ and $\mathbf{F}_t$ (Kulish and Pagan, 2017). These matrices satisfy the recursion:

$$\begin{aligned}
\mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1}\mathbf{B}_t \\
\mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1}\left(\mathbf{C}_t + \mathbf{D}_t\mathbf{J}_{t+1}\right) \\
\mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1}\mathbf{E}_t,
\end{aligned}$$

where the final structures $\mathbf{Q}_T$ and $\mathbf{J}_T$ are known and computed from the time invariant structure above under the terminal period's structural parameters, ie the no-ZLB case.

Given a sequence of ZLB durations, the state-space of the model is:

$$\begin{aligned}
\mathbf{x}_t &= \mathbf{J}_t + \mathbf{Q}_t\mathbf{x}_{t-1} + \mathbf{G}_t\epsilon_t \\
\mathbf{z}_t &= \mathbf{H}_t\mathbf{x}_t.
\end{aligned}$$

The observation equation and matrix $\mathbf{H}_t$ is time-varying because the nominal interest rate becomes unobserved when it is at the ZLB.

Denote by ϑ the vector of parameters to be estimated and by $\mathbf{T}$ the vector of ZLB durations that are observed each period. Denote by $\mathcal{Z} = \{z_\tau\}_{\tau=1}^{T}$ the sequence of vectors of observable variables. With Gaussian errors, the likelihood function $\mathcal{L}^a(\mathcal{Z}, \mathbf{T} \mid \vartheta)$ for the aggregate component is computed using the sequence of structural matrices associated with the sequence of ZLB durations, and the Kalman filter:

$$\log \mathcal{L}^a(\mathcal{Z}, \mathbf{T} \mid \vartheta) = -\left(\frac{N_z T}{2}\right)\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log\det \mathbf{H}_t\mathbf{S}_t\mathbf{H}_t^\top - \frac{1}{2}\sum_{t=1}^{T}\tilde{\mathbf{x}}_t^\top\left(\mathbf{H}_t\mathbf{S}_t\mathbf{H}_t^\top\right)^{-1}\tilde{\mathbf{x}}_t,$$

where $\tilde{\mathbf{x}}_t$ is the vector of forecast errors and $\mathbf{S}_t$ is its associated covariance matrix.

6.2 Kalman filter

The state of the system is $(\bar{\mathbf{x}}_t, \mathbf{P}_{t-1})$. In the predict step, the structural matrices $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$ are used to compute a forecast of the state $\hat{x}_{t|t-1}$ and the forecast covariance matrix $\mathbf{P}_{t|t-1}$ as:

$$\begin{aligned}
\bar{\mathbf{x}}_{t|t-1} &= \mathbf{J}_t + \mathbf{Q}_t\bar{\mathbf{x}}_t \\
\mathbf{P}_{t|t-1} &= \mathbf{Q}_t\mathbf{P}_{t-1}\mathbf{Q}_{t|t-1}^\top + \mathbf{G}_t\Omega\mathbf{G}_t^\top.
\end{aligned}$$

This formulation differs from the time-invariant Kalman filter used at the state-level because in the forecast stage the matrices $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$ can vary over time. We update these forecasts with imperfect observations of the state vector. This update step involves computing forecast errors $\tilde{y}_t$ and its associated covariance matrix $\mathbf{S}_t$ as:

$$\begin{aligned}\tilde{\mathbf{x}}_t &= \mathbf{z}_t - \mathbf{H}_t \bar{\mathbf{x}}_{t|t-1} \\ \mathbf{S}_t &= \mathbf{H}_t \mathbf{P}_{t|t-1} \mathbf{H}_t^\top.\end{aligned}$$

The Kalman gain matrix is given by:

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^\top \mathbf{S}_t^{-1}.$$

With $\tilde{\mathbf{x}}_t$, $\mathbf{S}_t$ and $\mathbf{K}_t$ in hand, the optimal filtered update of the state $\mathbf{x}_t$ is

$$\bar{\mathbf{x}}_t = \bar{\mathbf{x}}_{t|t-1} + \mathbf{K}_t \tilde{\mathbf{x}}_t,$$

and for its associated covariance matrix:

$$\mathbf{P}_t = (I - \mathbf{K}_t \mathbf{H}_t)\, \mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with $\bar{\mathbf{x}}_0$ and $\mathbf{P}_0$ determined from their unconditional moments, and is computed until the final time period T of data.

6.3 Kalman smoother

With the estimates of the parameters and durations in hand at time period T, the Kalman smoother gives an estimate of $\mathbf{x}_{t|T}$. With $\bar{\mathbf{x}}_{t|t-1}$, $\mathbf{P}_{t|t-1}$, $\mathbf{K}_t$ and $\mathbf{S}_t$ in hand from the Kalman filter, the vector $\mathbf{x}_{t|T}$ is computed by:

$$\mathbf{x}_{t|T} = \bar{\mathbf{x}}_{t|t-1} + \mathbf{P}_{t|t-1} \mathbf{r}_{t|T},$$

where the vector $\mathbf{r}_{T+1|T} = 0$ and is updated with the recursion:

$$\mathbf{r}_{t|T} = \mathbf{H}_t^\top \mathbf{S}_t^{-1} \left(\mathbf{z}_t - \mathbf{H}_t \bar{\mathbf{x}}_{t|t-1}\right) + \left(I - \mathbf{K}_t \mathbf{H}_t\right)^\top \mathbf{P}_{t|t-1}^\top \mathbf{r}_{t+1|T}.$$

Finally, to get an estimate of the shocks to each state variable under this model's shock structure, denoted by $\hat{\epsilon}_t$, we compute:

$$\hat{\epsilon}_t = \mathbf{G}_t \mathbf{r}_{t|T}.$$

7 Posterior Sampler

This section describes the sampler used to obtain the posterior distribution. We compute the likelihood function at the state-level, the likelihood at the aggregate level, and the prior. The posterior of our full model $\mathcal{P}(\vartheta \mid \mathbf{T}, \mathcal{Z})$ satisfies:

$$\mathcal{P}(\vartheta \mid \mathbf{T}, \mathcal{Z}) \propto L(\mathcal{Z}, \mathbf{T} \mid \vartheta) \times \mathcal{P}(\vartheta).$$

We discuss the specification of our priors below.

We use a Markov Chain Monte Carlo procedure to sample from the posterior. It has a single block, corresponding to the parameters ϑ. The sampler at step j is initialized with the last accepted draw of the structural parameters ϑ_{j-1}.

The block is a standard Metropolis-Hastings random walk. First start by selecting which parameters to propose new values. For those parameters, draw a new proposal ϑ_j from a thick-tailed proposal density centered at ϑ_{j-1} to ensure sufficient coverage of the parameter space and an acceptance rate of roughly 20% to 25%. The proposal ϑ_j is accepted with probability $\frac{\mathcal{P}(\vartheta_j|\mathbf{T},\mathcal{Z})}{\mathcal{P}(\vartheta_{j-1}|\mathbf{T},\mathcal{Z})}$. If ϑ_j is accepted, then set $\vartheta_{j-1} = \vartheta_j$.

8 Data

8.1 State-level

We use state-level data on employment, consumption spending, compensation, government spending, debt-to-income, and house prices. The observed state data is annual. The model is quarterly. So we used a mixed-frequency estimation procedure. The data is only used to compute forecast errors in the first quarter of the year.

To construct the data, we first take each state's series relative to its 1999 value, compute the devation of each state's observation from the state mean, regress that series on time dummies, weighted by the state's relative population, and work with the residuals. We then take out a linear trend from the resulting series. As discussed in the text, this formulation allows us to separate the state-based and aggregate- components of the state-space model. The resulting series used in our baseline specification for each state from 1999 to 2015 are plotted in Figures 18 to 22.

Here, we provide more details on each series:

- Consumption: We use state-level data on Total Personal Consumption Expenditures by State from the BEA, net of housing. The data is available for download at the BEA website.
- Employment: We use state-level data on Total Employment net of employment in the construction sector from the BEA annual table SA4. In our empirical analysis we scale this measure of employment by each state's population.
- Population: We use state-level data on Population from the BEA annual table SA1-3.
- Labor Compensation: We use state-level data on Compensation of Employees by Industry from the BEA annual table SA6N. net of construction compensation
- Wages: We divide total labor compensation by the number of employed individuals using the two series described above.
- Income: We use state-level data on Personal Income from the BEA annual table SA4.
- Household Debt: We use data from the FRBNY Consumer Credit Panel Q4 State statistics by year. Our measures of debt include auto loans, credit card debt, mortgage debt and student loans. This database also provides information on the number of individuals with credit scores in each state, which we use to express the debt data in per-capita terms.

We then construct a debt-to-income series by dividing this measure of per-capita debt by per-capita income using the data described above on income and population from the BEA.

- House Prices: We used data on the Not Seasonally Adjusted House Price Index available on the FHFA website.
- Government Spending: We use data from the BEA Table SAGDP2N gross domestic product (GDP) by state: Government and government enterprises (Millions of current dollars).

8.2 Aggregate level

At the aggregate level, we use data on inflation, employment, output, household debt, house prices, wages, government spending, the Fed Funds rate, and ZLB durations from NY Federal Reserve Survey Data. The codes for the raw data series are as follows:

- Gross Domestic Product: Implicit Price Deflator (GDPDEF).
- Personal Consumption Expenditures (BEA Table 2.4.5U). Current, $. We subtract housing from consumption.
- Cumulated nonfarm business section compensation (PRS85006062) minus employment growth (PRS85006012) and deflated by the GDP deflator.
- Total employment net of construction, over the civilian noninstitutional population.
- Household Debt from FRED (code CMDEBT) deflated by PCE deflator, and expressed relative to income (from the BEA Table 2.1). U.S. household debt to income ratio exhibits a trend, starting from about 0.5 in 1975 to about 1 in the last decade. Since we do not allow for trends in our model, we de-trend the data by subtracting a linear trend. We smooth this series to eliminate high frequency noise, by projecting it on a cubic spline of order 15 — the smoothed series is reported with dotted lines in the figure.
- House Prices from Case-Logic.
- Government Spending: Real government spending (GCEC1).
- Fed Funds rate: The interest rate is the Federal Funds Rate, taken from the Federal Reserve Economic Database.
- ZLB Durations: We follow the approach of Kulish, Morley and Ronbinson (2017) and use the ZLB durations extracted from the New York Federal Reserve Survey of Primary Dealers, conducted eight times a year, from 2011Q1 onwards.[1] We take the mode of the distribution implied by these surveys. Before 2011, we use responses from the Blue Chip Financial Forecasts survey.

We plot the aggregate data used in our baseline specification in Figure 23.

[1]See the website here. For example in 2011, the survey conducted on January 18, one of the questions asked was: "Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase." (Question 2b). Responses were given in terms of a probability distribution across future quarters.

9 Priors and Posteriors

The priors and posterior estimates in the baseline estimation are given in Table 2.

9.1 Priors

For the persistence and standard deviation of the AR(1) shocks, we use the same priors as Smets and Wouters (2007) use in their aggregate estimation. The persistence parameters are centered around 1/2. We use wide priors on the standard deviations of the shocks.

For the wage and price Calvo stickiness parameters, we use a more diffuse prior than Smets and Wouters (2007) but centered around the same mode of $\lambda_p = \lambda_w = 1/2$. This is because we use a more recent sample and wider priors are consistent with a flattening of the aggregate Phillips Curve.

For the degree of idiosyncratic uncertainty α, we use a wide prior, centered around a value of 2.5. As the first panel of Figure 9 shows, this prior is wide enough to allow the data to find strong or very weak effects of credit shocks, as we discuss in the paper. In estimations with uniform priors over α we find similar results.

9.2 Posteriors

Kernel density estimates for the posterior distributions are plotted in red in Figure 9. They display single peaks, and are largely different from the priors.

The convergence of the posterior distributions for each parameter is analyzed in Figure 10. Here we plot the Gelman-Rubin convergence diagnostic, along the length of the chain on the x-axes, up to the maximum length 100,000. The diagnostic compares the estimated between-chain and within-chain variances. As originally suggested by Brooks and Gelman (1998), values of the diagnostic below 1.2 are suggestive of convergence. We find values well below 1.2 for our baseline posterior estimates with a chain of 100,000.

We discard half of the draws in each chain as a burn-in.

10 Theoretical Variance Decompositions

The unconditional forecast error variance decomposition is shown in Table 10. The theoretical forecast error variance decompositions at the 2Q, 4Q, and 12Q horizons are shown in Tables 11 to 13. Credit shocks account for a nontrivial fraction of the differential changes in employment and consumption at the state-level at all frequencies. The aggregate component of credit shocks has a small role in accounting for aggregate consumption or employment.

Unconditionally, monetary policy shocks account for 11 percent of the variation in aggregate consumption and 13 percent of the variation in employment. To provide a point of comparison, we compare our variance decompositions with those obtained using the Smets and Wouters (2007) model, presented in Table 14. When a similar degree of price and wage stickiness is imposed in the Smets and Wouters model and all the remaining parameters reestimated, we find that about 14 percent of the variation in output is explained by policy shocks. We find wage markup shocks account for the bulk of the variation in wages, similar to our leisure preference shocks, and that inflation is almost entirely accounted for by price markup shocks, as in our model at the aggregate level.

11 Fiscal Policy

We add government spending in a simple way to the model by imposing lump-sum taxes that finance an exogeous level of government spending that we assume is zero in steady-state. Government spending follows an AR(1) process

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t}.$$

The resource constraint in our model is now simply

$$c_t + g_t = y_t.$$

We use spending by the government and government enterprises in the cross-section from BEA table SAGDP2N and treat the state-level data in the same way as the other state-level observables. At the aggregate-level, we use real government spending from the FRED (GCEC1), demeaned over 1984Q1 to 2015Q1, our sample period. Our estimated parameters are given in Table 6.

The variance decompositions in this model are shown in Tables 15 and 16. At the 2 quarter horizon, government spending accounts for about 9 percent of the variation in cross-sectional relative spending and 6 percent of the variation in cross-sectional relative employment. At the aggregate level, government spending shocks account for about 12 percent of the variation in consumption and employment. Unconditionally, government spending shocks account for about one-fifth to one-quarter of the variation in consumption and employment at the aggregate level, and a smaller fraction of relative state-level variation in consumption spending and employment. Compared to our baseline variance decompositions, the contribution of fiscal policy shocks largely comes at the expense of a smaller contribution from discount factor shocks, which suggests that the discount factor shocks in our baseline specification capture much of the cross-sectional and aggregate variation caused by perturbations in government spending.

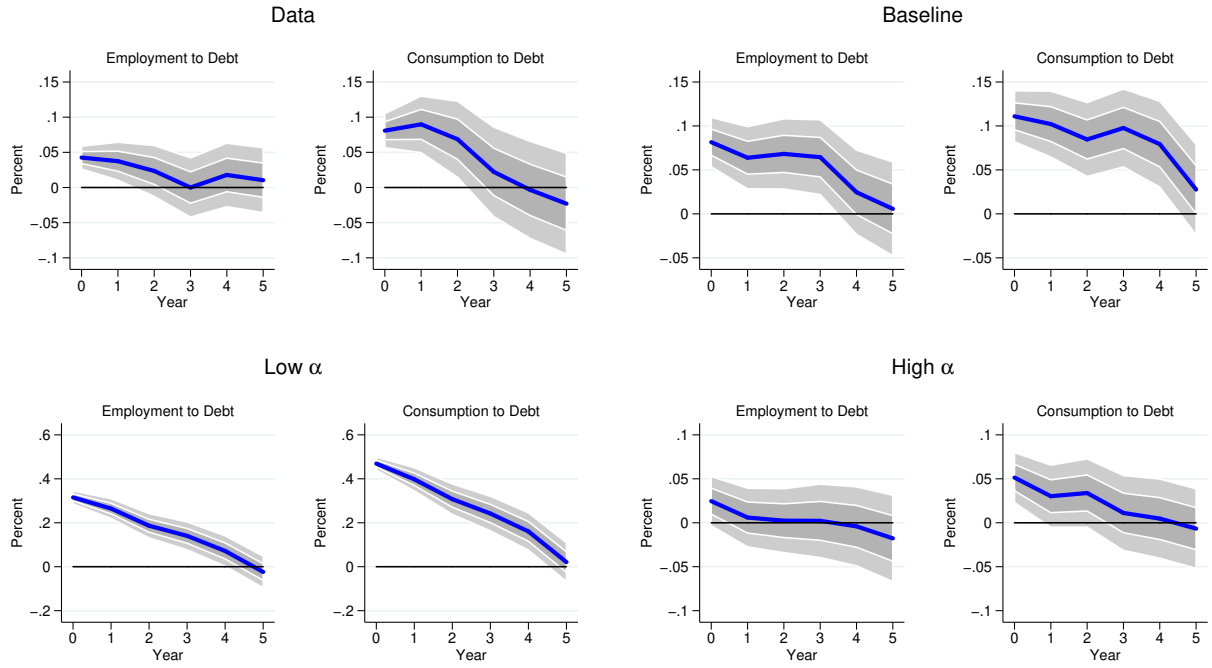
12 Estimated Taylor Rule

In a final robustness exercise, we estimated the Taylor Rule parameters, using the same priors as in Smets and Wouters (2007). We find a higher estimate of the response to the output gap α_y and to the growth rate in the output gap α_x but otherwise similar estimates to that of Justiniano et al. (2011). This is likely due to the fact our estimation is conducted over the period of the ZLB, which through the lens of the model was a period with a significant negative output gap. Because our estimates of the Taylor Rule are similar to the ones that we calibrated in our baseline estimations, the contribution of credit shocks at the regional and aggregate level are also very similar to contribution found in our baseline.

13 Identification of α: Local Projections

In the paper, we discussed what identifies the parameter α by examining what the model predicts for the comovement between employment, consumption, and household debt for different values of α. Here, we examine those predictions for longer leads and lags. Figure 1 plots the impulse response of employment and consumption to a change in debt computed using the local projections method of Jordà (2005). The first two panels show the impulse responses computed using the data. Following a change in household debt, employment increase on average by 0.05

Figure 1: Local Projections of Effect of Debt Shock, State-Level



percent and consumption increases by almost 0.1 percent, and mean revert after approximately 3 to 4 years. We find that model simulated series show very similar patterns. However, a panel simulated using $\alpha = 2$ greatly overstates the response of employment and consumption to changes in credit, while a panel simulated using $\alpha = 10$ understates the responses.

14 Identification of α: GMM

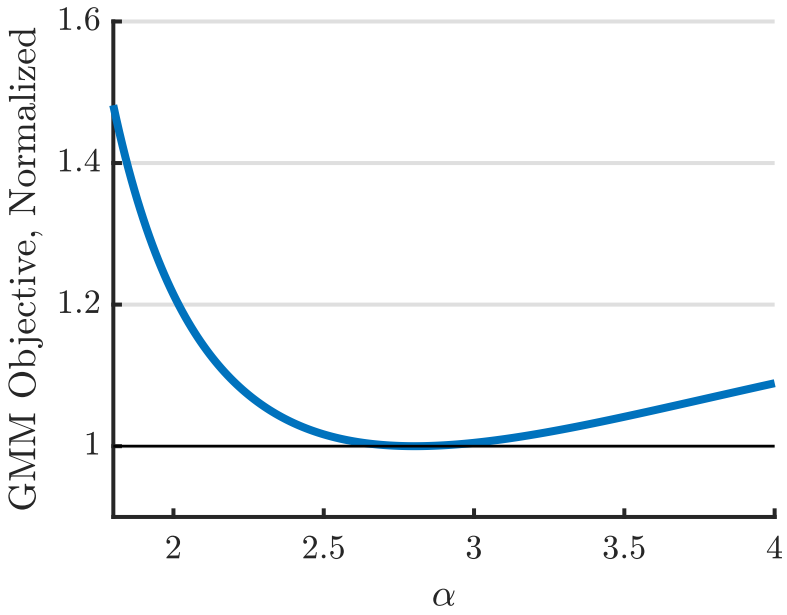
We also consider an alternative, perhaps more transparent limited-information approach to estimating the model's parameters. Specifically, we choose the model's parameters by minimizing the distance between moments computed using state-level data and those implied by the model. The moments we target are the the standard deviation of the variables, the contemporaneous second moments and the persistence in the data. Hence, denote

$$M_t \equiv \begin{bmatrix} \text{vech}(\mathbf{z}_t \mathbf{z}_t') \\ \text{diag}(\mathbf{z}_t \mathbf{z}_t') \\ \text{diag}(\mathbf{z}_t \mathbf{z}_{t-1}') \end{bmatrix}, \tag{38}$$

where $\mathbf{z}_t$ denotes the five state-level time series we described above at an annual frequency, the vech($\bullet$) operator selects the lower triangular elements of a matrix and orders them in a vector, and the diag($\bullet$) operator selects the diagonal elements of a matrix. Let Θ denote the vector of structural parameters that we wish to estimate, then the GMM estimator is given by:

$$\hat{\Theta}_{GMM} = \arg\min \left(\frac{1}{T} \sum_{t=1}^{T} M_t - \mathbb{E}[\mathbf{M}(\Theta)] \right)' \mathbf{W} \left(\frac{1}{T} \sum_{t=1}^{T} M_t - \mathbb{E}[\mathbf{M}(\Theta)] \right), \tag{39}$$

Figure 2: GMM Objective Across Parameters



where $\mathbb{E}([\mathbf{M}(\Theta)]$ denotes the model-implied moments that are counterparts to M_t when taking a first-order approximation to the model conditions and evaluate them at Θ. $\mathbf{W}$ is a positive definite weighting matrix, which is positive definite. We use a conventional two-step approach. First, we use an identity matrix for $\mathbf{W}$ to obtain an initial estimate of the parameters denoted by Θ_0. Then, we use the inverse of the variance-covariance matrix of $\left(\frac{1}{T}\sum_{t=1}^{T} M_t - \mathbb{E}[\mathbf{M}(\Theta_0)]\right)$ as the weighting matrix, which is obtained with a Newey-West estimator with 1 lag (since we are using annual data).

This approach, which we apply to state-level data only, yields an estimate of α of 2.9, very close to the maximum likelihood estimate that uses state-level data only (see the Robustness Table 3). Figure 2 shows how the GMM objective function varies with α and that this parameter is indeed well identified by state-level data.

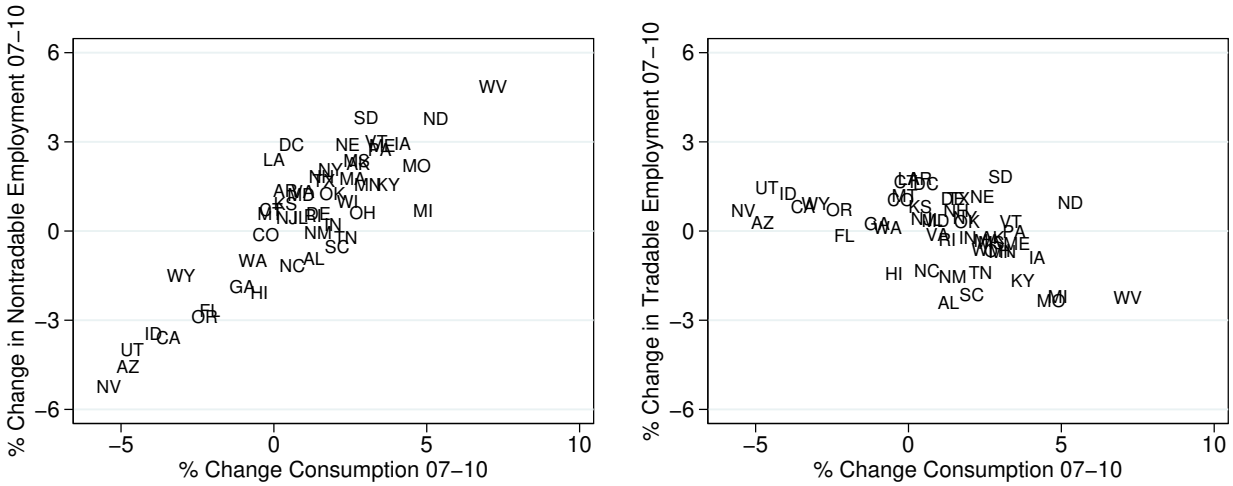
15 Other Model Implications

We next study our model's ability to reproduce several additional variables that we have not directly used in estimation. We also look at the predictions for default rates from the model with default, which was not used in estimation.

15.1 Tradeables and Non-tradeables

Consider first Figure 3 which shows how in our model state-level tradable and nontradable employment comove with consumption during the Great Recession. As Mian and Sufi report, most of the decline in employment at the state level was due to a decline in nontradable employment. Our model reproduces this fact well: the elasticity of nontradable employment to consumption is equal to 0.75 in the model (0.55 in the data, as reported by Kehoe et al. (2019)). Similarly, tradable employment comoves little with state-level consumption, both in the model

Figure 3: Dynamics of Sectoral Employment During the Recession



(the elasticity of tradable employment to consumption is -0.21) and in the data (an elasticity of -0.03).

15.2 Mortgage Rate

Consider next the model's implications for the mortgage interest rate. Since in our model mortgages are long-term perpetuities with decaying coupon payments, the return on such securities is not directly comparable to the interest rate on 30-year mortgages in the data. Nevertheless, we can derive the implied rate at which the flows underlying these securities are discounted as the rate i_t^{m} that rationalizes the price of the security q_t. This rate is defined by

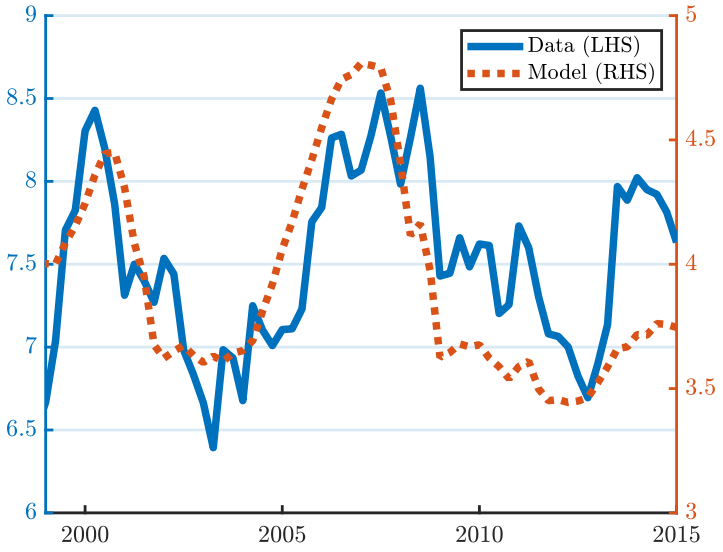
$$q_t = \frac{1}{1+i_t^{\mathrm{m}}} \sum_{k=0}^{\infty} \left(\frac{\gamma}{1+i_t^{\mathrm{m}}} \right)^k,$$

which gives

$$1 + i_t^{\mathrm{m}} = \frac{1}{q_t} + \gamma.$$

Figure 4 compares this implied long-term rate in our model with the average interest rate on 30-year mortgages in the data. Since the latter series has a trend, we detrend both the model and the data (and add the average rate over 2001 to 2015 in both the model and data). The model does a reasonable job at reproducing medium-term movements in the mortgage rate in the data, though it misses the high-frequency variation. Since the model abstracts from several sources of risk embedded in mortgages rates, such as default and prepayment risk, we do not view the model's failure to match these high frequency fluctuations as critical. Indeed, since our Kalman filter isolates the credit shocks from the dynamics of mortgage debt in the data, changes in mortgage spreads in the data are captured in a reduced-form way as shifts in the borrowing constraint. In our Robustness section below we extend our model to explicitly model mortgage default and show that our conclusions are robust to adding a time-varying default spread between mortgage rates and the interest rate on liquid assets.

Figure 4: Dynamics of the Mortgage Rate



15.3 Default Rate in Model with Default

Figure 5 plots the default rate in the model with the option to default in the left panel, obtained using the Kalman smoother and the estimated parameters. In the right panel, the model's default rate is plotted against a series from FRED on the delinquency rate on consumer loans from all commercial banks (FRED Code DRCLACBS). The two series show similar patterns.

In addition, the Mortgage Bankers Association's National Delinquency Survey shows that mortgage delinquencies rose to about 4% between 2009 and 2012, slightly higher than the default rate implied by our model.[2]

16 Full Decompositions and Shocks

16.1 Decomposition of State Employment/Consumption for all States

We plot in Figures 25 to 42 the decomposition of relative state-level employment and consumption into the state-level structural shocks in our baseline specification.

16.2 State-Level and Aggregate Shocks

Figures 43 to 48 plot the full time series of state-level structural shocks implied by our baseline estimation. And Figure 49 plots the time series of each of the structural shocks extracted from the model for the aggregate variables.

[2]See the chart at https://www.mortgagemedia.com/featured/mortgage-delinquency-and-foreclosure-rates-mba-chart-of-the-week.

Table 2: Estimated Structural Parameters, Baseline Specification

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Mean	10%	90%
α	N	2.5	1.5	3.8	3.23	3.44	3.05	3.93
λ_p	B	0.5	0.2	0.8	0.97	0.96	0.94	0.98
λ_w	B	0.5	0.2	0.8	0.86	0.86	0.82	0.89
B. Regional Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.77	0.66	0.39	0.87
ρ_m	B	0.5	0.2	0.8	0.70	0.61	0.41	0.79
ρ_h	B	0.5	0.2	0.8	0.89	0.87	0.80	0.93
ρ_n	B	0.5	0.2	0.8	0.65	0.59	0.38	0.76
ρ_b	B	0.5	0.2	0.8	0.87	0.80	0.65	0.92
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.43	0.55	0.34	0.79
σ_m	IG	0.6	0.3	1.9	0.85	0.83	0.54	1.18
σ_h	IG	0.6	0.3	1.9	0.56	0.71	0.35	1.20
σ_n	IG	0.6	0.3	1.9	1.20	2.17	0.79	4.38
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	0.67	1.56	0.49	3.08
C. Aggregate Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.97	0.96	0.94	0.98
ρ_m	B	0.5	0.2	0.8	0.49	0.50	0.38	0.62
ρ_h	B	0.5	0.2	0.8	0.96	0.96	0.94	0.97
ρ_n	B	0.5	0.2	0.8	0.07	0.10	0.04	0.17
ρ_b	B	0.5	0.2	0.8	0.93	0.93	0.91	0.94
ρ_p	B	0.5	0.2	0.8	0.68	0.65	0.49	0.77
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.65	0.65	0.58	0.73
σ_m	IG	0.6	0.3	1.9	0.90	0.94	0.83	1.05
σ_h	IG	0.6	0.3	1.9	0.18	0.19	0.13	0.26
$\frac{1}{100} \times \sigma_n$	IG	0.6	0.3	1.9	0.14	0.16	0.10	0.23
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	1.20	1.22	1.03	1.42
$1000 \times \sigma_p$	IG	0.6	0.3	1.9	0.68	0.76	0.50	1.08
$100 \times \sigma_r$	IG	0.6	0.3	1.9	0.78	0.80	0.70	0.90

Table 3: Estimated Structural Parameters: Robustness

	A. Baseline Estimates			B. No Population Weighting		
Parameter	Mean	10%	90%	Mean	10%	90%
α	3.44	3.05	3.93	3.45	3.01	3.90
λ_p	0.96	0.94	0.98	0.97	0.94	0.98
λ_w	0.86	0.82	0.89	0.86	0.83	0.89

	C. State Data Only			D. $\alpha = 5$		
Parameter	Mean	10%	90%	Mean	10%	90%
α	3.62	3.23	4.08	-	-	-
λ_p	0.93	0.89	0.96	0.97	0.94	0.98
λ_w	0.57	0.47	0.66	0.86	0.83	0.89

	E. $\alpha = 2$			F. $\gamma = 0.965$		
Parameter	Mean	10%	90%	Mean	10%	90%
α	-	-	-	2.99	2.67	3.41
λ_p	0.96	0.93	0.98	0.96	0.94	0.98
λ_w	0.87	0.84	0.91	0.85	0.82	0.88

	G. $\gamma = 0$			H. $\psi = 5$		
Parameter	Mean	10%	90%	Mean	10%	90%
α	2.65	2.41	2.87	3.34	2.98	3.78
λ_p	0.98	0.96	0.99	0.96	0.94	0.98
λ_w	0.87	0.84	0.89	0.92	0.91	0.94

	I. Construction Sector			J. Heterogeneous Housing Elasticities		
Parameter	Mean	10%	90%	Mean	10%	90%
α	3.62	2.96	4.41	3.57	3.10	4.12
λ_p	0.96	0.94	0.98	0.92	0.88	0.96
λ_w	0.86	0.83	0.88	0.56	0.44	0.67

	K. Government Spending			L. Option to Default		
Parameter	Mean	10%	90%	Mean	10%	90%
α	3.18	2.88	3.40	2.87	2.65	3.01
λ_p	0.96	0.95	0.98	0.96	0.93	0.98
λ_w	0.84	0.82	0.87	0.86	0.82	0.89

	M. Estimated Taylor Rule			N. Remove 5 Largest States		
Parameter	Mean	10%	90%	Mean	10%	90%
α	3.43	3.17	3.85	3.28	2.94	3.62
λ_p	0.97	0.94	0.98	0.96	0.94	0.98
λ_w	0.85	0.82	0.88	0.88	0.85	0.90

Table 4: Estimated Structural Parameters, $\gamma = 0.965$

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Median	10%	90%
α	N	2.6	1.5	3.8	2.85	2.99	2.67	3.41
λ_p	B	0.5	0.2	0.8	0.97	0.96	0.94	0.98
λ_w	B	0.5	0.2	0.8	0.86	0.85	0.82	0.88
B. Regional Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.80	0.66	0.37	0.87
ρ_m	B	0.5	0.2	0.8	0.49	0.50	0.27	0.71
ρ_h	B	0.5	0.2	0.8	0.89	0.87	0.80	0.92
ρ_n	B	0.5	0.2	0.8	0.66	0.60	0.42	0.76
ρ_b	B	0.5	0.2	0.8	0.88	0.84	0.74	0.93
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.48	0.54	0.35	0.77
σ_m	IG	0.6	0.3	1.9	0.38	0.46	0.29	0.71
σ_h	IG	0.6	0.3	1.9	0.44	0.63	0.31	1.04
σ_n	IG	0.6	0.3	1.9	1.20	1.81	0.82	3.25
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	0.75	1.21	0.51	2.37
C. Aggregate Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.96	0.96	0.94	0.98
ρ_m	B	0.5	0.2	0.8	0.48	0.46	0.33	0.58
ρ_h	B	0.5	0.2	0.8	0.96	0.95	0.94	0.97
ρ_n	B	0.5	0.2	0.8	0.07	0.10	0.04	0.16
ρ_b	B	0.5	0.2	0.8	0.92	0.92	0.91	0.94
ρ_p	B	0.5	0.2	0.8	0.68	0.66	0.56	0.78
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.64	0.65	0.58	0.73
σ_m	IG	0.6	0.3	1.9	0.39	0.40	0.35	0.45
σ_h	IG	0.6	0.3	1.9	0.16	0.17	0.12	0.23
$\frac{1}{100} \times \sigma_n$	IG	0.6	0.3	1.9	0.13	0.15	0.10	0.22
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	1.28	1.29	1.10	1.50
$1000 \times \sigma_p$	IG	0.6	0.3	1.9	0.80	0.73	0.51	0.96
$100 \times \sigma_r$	IG	0.6	0.3	1.9	0.75	0.79	0.70	0.90

Table 5: Estimated Structural Parameters, $\gamma = 0$

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Median	10%	90%
α	N	2.6	1.5	3.8	2.66	2.65	2.41	2.87
λ_p	B	0.5	0.2	0.8	0.98	0.98	0.96	0.99
λ_w	B	0.5	0.2	0.8	0.87	0.87	0.84	0.89
				B. Regional Shock Processes				
ρ_z	B	0.5	0.2	0.8	0.63	0.59	0.32	0.84
ρ_m	B	0.5	0.2	0.8	0.20	0.19	0.09	0.29
ρ_h	B	0.5	0.2	0.8	0.53	0.54	0.29	0.77
ρ_n	B	0.5	0.2	0.8	0.57	0.56	0.40	0.69
ρ_b	B	0.5	0.2	0.8	0.85	0.85	0.78	0.92
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.68	0.62	0.37	0.92
σ_m	IG	0.6	0.3	1.9	0.14	0.16	0.11	0.22
σ_h	IG	0.6	0.3	1.9	0.29	0.36	0.20	0.55
σ_n	IG	0.6	0.3	1.9	2.62	2.21	1.53	2.91
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	1.24	1.01	0.56	1.43
$\tilde{\rho}_m$	B	0.8	0.6	0.9	0.70	0.71	0.60	0.81
$\tilde{\rho}_h$	B	0.8	0.6	0.9	0.84	0.80	0.70	0.89
				C. Aggregate Shock Processes				
ρ_z	B	0.5	0.2	0.8	0.98	0.98	0.96	0.99
ρ_m	B	0.5	0.2	0.8	0.56	0.53	0.34	0.69
ρ_h	B	0.5	0.2	0.8	0.92	0.60	0.20	0.94
ρ_n	B	0.5	0.2	0.8	0.06	0.11	0.04	0.19
ρ_b	B	0.5	0.2	0.8	0.94	0.94	0.93	0.95
ρ_p	B	0.5	0.2	0.8	0.68	0.66	0.59	0.72
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.61	0.59	0.54	0.62
σ_m	IG	0.6	0.3	1.9	0.03	0.03	0.02	0.03
σ_h	IG	0.6	0.3	1.9	0.09	0.10	0.07	0.14
$\frac{1}{100} \times \sigma_n$	IG	0.6	0.3	1.9	0.15	0.17	0.11	0.24
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	0.96	0.99	0.91	1.06
$1000 \times \sigma_p$	IG	0.6	0.3	1.9	0.64	0.70	0.61	0.81
$100 \times \sigma_r$	IG	0.6	0.3	1.9	0.80	0.81	0.70	0.91
$\tilde{\rho}_m$	B	0.8	0.6	0.9	0.82	0.81	0.72	0.90
$\tilde{\rho}_h$	B	0.8	0.6	0.9	0.95	0.80	0.55	0.96

The state and aggregate shocks to credit and preference for housing are themselves AR(1) processes with coefficients $\tilde{\rho}_m$ and $\tilde{\rho}_h$.

Table 6: Estimated Parameters, With Government Spending

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Mean	10%	90%
A. Structural Parameters								
α	N	2.5	1.5	3.8	3.20	3.18	2.88	3.40
λ_p	B	0.5	0.2	0.8	0.96	0.96	0.95	0.98
λ_w	B	0.5	0.2	0.8	0.86	0.84	0.82	0.87
B. Regional Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.77	0.72	0.55	0.86
ρ_m	B	0.5	0.2	0.8	0.71	0.69	0.57	0.81
ρ_h	B	0.5	0.2	0.8	0.86	0.87	0.83	0.92
ρ_n	B	0.5	0.2	0.8	0.72	0.65	0.54	0.76
ρ_b	B	0.5	0.2	0.8	0.85	0.84	0.80	0.88
ρ_g	B	0.5	0.2	0.8	0.65	0.55	0.30	0.73
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.53	0.52	0.34	0.69
σ_m	IG	0.6	0.3	1.9	0.58	0.62	0.46	0.84
σ_h	IG	0.6	0.3	1.9	0.50	0.62	0.36	0.90
σ_n	IG	0.6	0.3	1.9	1.17	1.14	1.06	1.22
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	1.16	1.13	0.92	1.39
$100 \times \sigma_g$	IG	0.6	0.3	1.9	0.40	0.40	0.31	0.49
C. Aggregate Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.97	0.97	0.95	0.98
ρ_m	B	0.5	0.2	0.8	0.53	0.52	0.41	0.61
ρ_h	B	0.5	0.2	0.8	0.95	0.95	0.94	0.96
ρ_n	B	0.5	0.2	0.8	0.08	0.10	0.04	0.17
ρ_b	B	0.5	0.2	0.8	0.93	0.93	0.91	0.94
ρ_p	B	0.5	0.2	0.8	0.72	0.72	0.67	0.78
ρ_g	B	0.5	0.2	0.8	0.99	0.99	0.97	1.00
$100 \times \sigma_z$	IG	0.6	0.3	1.9	1.05	1.06	0.94	1.17
σ_m	IG	0.6	0.3	1.9	1.20	1.23	1.09	1.42
σ_h	IG	0.6	0.3	1.9	0.21	0.22	0.17	0.28
$\frac{1}{100} \times \sigma_n$	IG	0.6	0.3	1.9	0.10	0.13	0.09	0.19
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	1.25	1.31	1.15	1.55
$1000 \times \sigma_p$	IG	0.6	0.3	1.9	0.60	0.62	0.51	0.75
$100 \times \sigma_r$	IG	0.6	0.3	1.9	0.93	0.91	0.82	0.99
$100 \times \sigma_g$	IG	0.6	0.3	1.9	0.82	0.87	0.75	1.00

Table 7: Estimated Parameters, Construction Model

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Mean	10%	90%
A. Structural Parameters								
α	N	2.5	1.5	3.8	3.47	3.62	2.96	4.41
λ_p	B	0.5	0.2	0.8	0.97	0.96	0.94	0.98
λ_w	B	0.5	0.2	0.8	0.87	0.86	0.83	0.88
ξ	N	1.0	0.4	1.6	1.75	1.74	1.37	2.12
B. Regional Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.83	0.67	0.41	0.87
ρ_m	B	0.5	0.2	0.8	0.69	0.64	0.45	0.81
ρ_h	B	0.5	0.2	0.8	0.90	0.84	0.75	0.91
ρ_n	B	0.5	0.2	0.8	0.68	0.63	0.47	0.77
ρ_b	B	0.5	0.2	0.8	0.89	0.87	0.81	0.92
ρ_{zh}	B	0.5	0.2	0.8	0.87	0.80	0.65	0.92
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.41	0.53	0.34	0.73
σ_m	IG	0.6	0.3	1.9	0.64	0.72	0.48	1.00
σ_h	IG	0.6	0.3	1.9	0.58	1.05	0.52	1.61
σ_n	IG	0.6	0.3	1.9	1.17	1.56	0.79	2.51
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	0.53	0.76	0.43	1.20
$100 \times \sigma_{zh}$	IG	0.6	0.3	1.9	2.14	2.66	1.76	3.76
C. Aggregate Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.96	0.96	0.95	0.98
ρ_m	B	0.5	0.2	0.8	0.55	0.46	0.30	0.61
ρ_h	B	0.5	0.2	0.8	0.96	0.95	0.94	0.97
ρ_n	B	0.5	0.2	0.8	0.07	0.10	0.04	0.17
ρ_b	B	0.5	0.2	0.8	0.93	0.93	0.91	0.94
ρ_p	B	0.5	0.2	0.8	0.73	0.70	0.61	0.79
ρ_{zh}	B	0.5	0.2	0.8	0.97	0.96	0.94	0.98
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.61	0.66	0.59	0.74
σ_m	IG	0.6	0.3	1.9	0.86	0.86	0.77	0.95
σ_h	IG	0.6	0.3	1.9	0.19	0.22	0.15	0.30
$\frac{1}{100} \times \sigma_n$	IG	0.6	0.3	1.9	0.13	0.16	0.10	0.22
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	1.23	1.21	1.04	1.38
$1000 \times \sigma_p$	IG	0.6	0.3	1.9	0.63	0.63	0.46	0.81
$100 \times \sigma_r$	IG	0.6	0.3	1.9	0.78	0.79	0.70	0.90
$100 \times \sigma_{zh}$	IG	0.6	0.3	1.9	2.11	2.18	1.84	2.57

Table 8: Estimated Parameters, Estimated Taylor Rule

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Mean	10%	90%
A. Structural Parameters								
α	N	2.5	1.5	3.8	3.36	3.43	3.17	3.85
λ_p	B	0.5	0.2	0.8	0.97	0.97	0.94	0.98
λ_w	B	0.5	0.2	0.8	0.85	0.85	0.82	0.88
α_r	B	0.8	0.6	0.9	0.83	0.83	0.80	0.86
α_p	N	1.5	1.1	2.0	1.74	1.67	1.37	1.97
α_x	N	0.1	0.0	0.2	0.34	0.34	0.29	0.39
α_y	N	0.1	0.0	0.2	0.13	0.13	0.09	0.16
B. Regional Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.82	0.65	0.34	0.88
ρ_m	B	0.5	0.2	0.8	0.71	0.56	0.29	0.80
ρ_h	B	0.5	0.2	0.8	0.91	0.89	0.84	0.93
ρ_n	B	0.5	0.2	0.8	0.67	0.62	0.48	0.76
ρ_b	B	0.5	0.2	0.8	0.91	0.88	0.83	0.93
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.38	0.53	0.33	0.76
σ_m	IG	0.6	0.3	1.9	0.65	0.95	0.50	1.60
σ_h	IG	0.6	0.3	1.9	0.41	0.55	0.32	0.83
σ_n	IG	0.6	0.3	1.9	0.92	1.60	0.81	2.50
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	0.52	0.67	0.43	1.04
C. Aggregate Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.96	0.96	0.94	0.98
ρ_m	B	0.5	0.2	0.8	0.57	0.54	0.41	0.67
ρ_h	B	0.5	0.2	0.8	0.95	0.96	0.94	0.97
ρ_n	B	0.5	0.2	0.8	0.07	0.09	0.04	0.16
ρ_b	B	0.5	0.2	0.8	0.92	0.92	0.91	0.93
ρ_p	B	0.5	0.2	0.8	0.67	0.64	0.52	0.74
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.65	0.65	0.58	0.71
σ_m	IG	0.6	0.3	1.9	0.94	0.96	0.85	1.08
σ_h	IG	0.6	0.3	1.9	0.20	0.20	0.13	0.28
$\frac{1}{100} \times \sigma_n$	IG	0.6	0.3	1.9	0.12	0.15	0.09	0.21
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	1.22	1.35	1.17	1.55
$1000 \times \sigma_p$	IG	0.6	0.3	1.9	0.64	0.78	0.58	1.03
$100 \times \sigma_r$	IG	0.6	0.3	1.9	0.75	0.78	0.67	0.91

Table 9: Estimated Parameters, Option to Default

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Mean	10%	90%
A. Structural Parameters								
α	N	2.5	1.5	3.8	2.97	2.87	2.65	3.01
λ_p	B	0.5	0.2	0.8	0.97	0.96	0.93	0.98
λ_w	B	0.5	0.2	0.8	0.88	0.86	0.82	0.89
α_r	B	0.8	0.6	0.9	0.81	0.81	0.78	0.84
α_p	N	1.5	1.1	2.0	2.15	2.12	1.93	2.30
α_x	N	0.1	0.0	0.2	0.34	0.34	0.29	0.40
α_y	N	0.1	0.0	0.2	0.10	0.10	0.08	0.13
B. Regional Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.89	0.67	0.42	0.90
ρ_t	B	0.5	0.2	0.8	0.66	0.63	0.31	0.88
ρ_h	B	0.5	0.2	0.8	0.67	0.55	0.25	0.82
ρ_n	B	0.5	0.2	0.8	0.60	0.44	0.14	0.70
ρ_b	B	0.5	0.2	0.8	0.87	0.81	0.69	0.90
$100 \times \sigma_z$	U	0.5	2.5	4.5	0.60	0.57	0.33	0.83
σ_τ	U	0.5	2.5	4.5	0.09	0.23	0.07	0.45
σ_h	U	0.5	2.5	4.5	0.16	0.66	0.11	1.55
σ_n	U	0.5	2.5	4.5	4.86	3.23	1.86	4.88
$1000 \times \sigma_b$	U	0.5	2.5	4.5	0.70	1.49	0.63	2.39
ρ_τ^ϵ	B	0.8	0.6	0.9	0.86	0.80	0.68	0.90
ρ_h^ϵ	B	0.8	0.6	0.9	0.82	0.79	0.65	0.90
C. Aggregate Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.97	0.96	0.94	0.98
ρ_τ	B	0.5	0.2	0.8	0.93	0.73	0.49	0.94
ρ_h	B	0.5	0.2	0.8	0.95	0.81	0.44	0.97
ρ_n	B	0.5	0.2	0.8	0.08	0.11	0.04	0.19
ρ_b	B	0.5	0.2	0.8	0.92	0.91	0.89	0.93
ρ_p	B	0.5	0.2	0.8	0.88	0.86	0.78	0.92
$100 \times \sigma_z$	U	0.5	2.5	4.5	0.63	0.65	0.58	0.73
σ_τ	U	0.5	2.5	4.5	0.18	0.14	0.07	0.21
σ_h	U	0.5	2.5	4.5	0.02	0.05	0.02	0.10
$\frac{1}{100} \times \sigma_n$	U	0.5	2.5	4.5	0.10	0.16	0.09	0.24
$1000 \times \sigma_b$	U	0.5	2.5	4.5	1.25	1.37	1.10	1.71
$1000 \times \sigma_p$	U	0.5	2.5	4.5	0.33	0.41	0.27	0.60
$100 \times \sigma_r$	U	0.5	2.5	4.5	0.78	0.80	0.68	0.93
ρ_τ^ϵ	B	0.8	0.6	0.9	0.91	0.76	0.49	0.93
ρ_h^ϵ	B	0.8	0.6	0.9	0.95	0.87	0.70	0.96

Table 10: Unconditional Variance Decomposition

Variable \ Shock	Collateral	Housing	Productivity	Leisure	Discount	Policy	Markup
A. State-level							
Spending	30.3	0.4	0.0	34.6	34.7	-	-
Employment	7.1	0.1	9.6	71.1	12.2	-	-
Wages	1.9	0.0	0.0	97.9	0.1	-	-
Debt-to-income	98.8	1.1	0.0	0.1	0.0	-	-
House prices	2.3	95.0	0.0	2.5	0.2	-	-
B. Aggregate-level							
Consumption	1.5	0.1	22.7	8.8	47.6	10.7	8.7
Employment	1.8	0.1	4.0	10.9	59.2	13.2	10.8
Wages	0.3	0.0	3.7	69.6	3.0	0.3	23.1
Debt-to-income	78.8	5.9	0.5	2.4	12.0	0.1	0.4
House prices	0.0	87.3	3.3	1.2	5.3	1.6	1.3
Fed Funds rate	3.4	0.3	11.2	2.6	76.5	3.6	2.2
Inflation	0.8	0.1	2.9	12.2	2.6	0.2	81.3

Table 11: Variance Decomposition, 2 Quarter Horizon

Variable \ Shock	Collateral	Housing	Productivity	Leisure	Discount	Policy	Markup
A. State-level							
Spending	13.0	0.1	0.0	3.8	83.0	-	-
Employment	6.7	0.1	45.5	3.7	44.0	-	-
Wages	0.0	0.0	0.0	99.9	0.0	-	-
Debt-to-income	99.8	0.2	0.0	0.0	0.0	-	-
House prices	0.9	98.9	0.0	0.2	0.0	-	-
B. Aggregate-level							
Consumption	0.3	0.0	30.8	0.1	42.3	20.5	6.0
Employment	0.4	0.0	3.6	0.2	58.9	28.5	8.4
Wages	0.0	0.0	0.0	92.9	0.0	0.0	7.0
Debt-to-income	40.9	0.0	0.0	1.5	56.6	0.4	0.6
House prices	0.0	80.5	3.4	0.0	13.1	2.3	0.7
Fed Funds rate	0.6	0.0	7.1	0.1	87.8	3.1	1.2
Inflation	0.0	0.0	1.0	3.6	0.3	0.0	95.1

Table 12: Variance Decomposition, 4 Quarter Horizon

Variable \ Shock	Collateral	Housing	Productivity	Leisure	Discount	Policy	Markup
A. State-level							
Spending	17.3	0.2	0.0	5.3	77.2	-	-
Employment	9.5	0.1	38.7	7.5	44.2	-	-
Wages	0.1	0.0	0.0	99.9	0.0	-	-
Debt-to-income	99.8	0.2	0.0	0.0	0.0	-	-
House prices	1.0	98.8	0.0	0.2	0.0	-	-
B. Aggregate-level							
Consumption	0.3	0.0	28.7	0.4	42.4	18.2	10.0
Employment	0.5	0.0	3.2	0.6	57.5	24.7	13.5
Wages	0.0	0.0	0.1	88.0	0.1	0.0	11.7
Debt-to-income	55.2	0.0	0.0	1.3	42.7	0.3	0.5
House prices	0.0	81.5	3.4	0.0	11.6	2.2	1.2
Fed Funds rate	0.8	0.0	7.6	0.2	86.4	3.2	1.8
Inflation	0.1	0.0	1.5	5.6	0.5	0.0	92.4

Table 13: Unconditional Variance Decomposition, 12 Quarter Horizon

Variable \ Shock	Collateral	Housing	Productivity	Leisure	Discount	Policy	Markup
A. State-level							
Spending	29.0	0.3	0.0	11.0	59.6	-	-
Employment	13.6	0.1	25.0	29.6	31.6	-	-
Wages	0.2	0.0	0.0	99.8	0.0	-	-
Debt-to-income	99.5	0.5	0.0	0.0	0.0	-	-
House prices	1.4	98.1	0.0	0.5	0.1	-	-
B. Aggregate-level							
Consumption	0.5	0.0	25.4	2.4	45.5	14.1	12.1
Employment	0.7	0.0	2.8	3.1	59.3	18.4	15.7
Wages	0.0	0.0	0.7	78.8	0.5	0.1	19.9
Debt-to-income	69.8	0.3	0.1	1.3	27.8	0.2	0.5
House prices	0.0	84.5	3.5	0.3	8.0	2.0	1.7
Fed Funds rate	1.3	0.0	9.4	0.8	82.7	3.5	2.3
Inflation	0.2	0.0	2.6	10.3	1.2	0.1	85.6

Table 14: Variance Decompositions in Smets and Wouters (2007) Model

Variable \ Shock	Tech	Pref	Gov	Inve	Policy	Price Infl	Wage Infl
A. SW Model, 1966 to 2004							
y	27.1	1.7	4.3	7.3	2.0	6.1	51.5
c	9.1	2.4	8.6	2.9	2.0	4.0	71.0
i	17.6	0.2	4.7	45.0	1.1	6.1	25.3
Hours	1.8	2.6	10.2	7.6	2.8	5.8	69.2
Infl	3.4	0.7	0.9	3.4	4.3	27.6	59.7
Wages	4.1	1.3	0.1	2.2	1.8	30.4	60.2
Fed Funds	9.5	8.0	3.5	18.2	14.4	7.8	38.7
B. SW Model, 1966 to 2004 and Less Flexible Prices and Wages							
y	57.1	4.0	5.7	9.4	13.9	9.7	0.2
c	51.7	7.0	11.5	4.1	15.4	3.0	7.4
i	22.6	0.4	10.1	42.9	9.2	12.8	2.0
Hours	4.7	9.1	17.0	17.0	27.2	9.0	16.0
Infl	0.2	0.0	0.0	0.0	0.0	97.0	2.8
Wages	0.3	0.0	0.0	0.0	0.1	32.8	66.8
Fed Funds	5.7	14.3	4.1	12.8	37.0	20.5	5.4

Figure 5: Default Rate in Model and Data

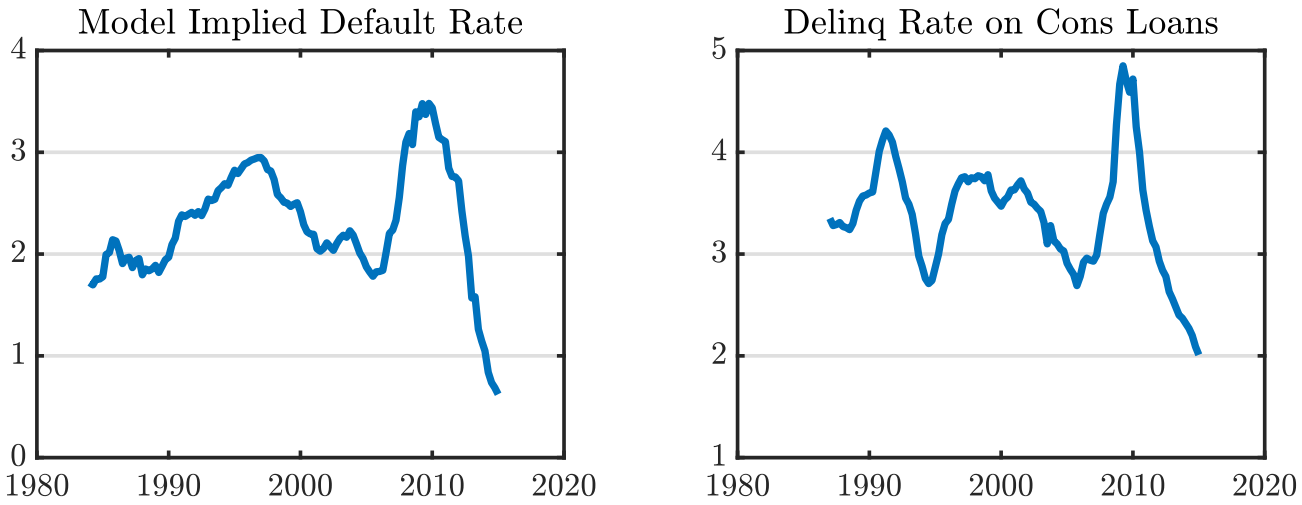


Table 15: Variance Decomposition, Model with Fiscal Policy

Variable \ Shock	Collateral	Housing	Prod	Leisure	Discount	Gov	Policy	Markup
A. State-level								
Spending	41.0	0.5	0.1	22.3	31.4	4.7	-	-
Employment	12.9	0.2	13.6	57.3	14.3	1.7	-	-
Wages	4.1	0.1	0.0	95.3	0.3	0.3	-	-
Debt-to-income	98.7	1.2	0.0	0.1	0.0	0.0	-	-
House prices	3.4	94.4	0.0	1.7	0.3	0.2	-	-
B. Aggregate-level								
Consumption	1.9	0.0	36.3	2.3	19.1	30.8	5.0	4.6
Employment	4.0	0.1	4.2	4.7	39.8	27.3	10.4	9.5
Wages	0.9	0.0	21.2	42.6	3.0	0.2	0.4	31.8
Debt-to-income	79.8	2.3	0.9	0.9	7.9	7.9	0.1	0.3
House prices	0.1	66.6	12.8	0.7	5.5	10.8	1.9	1.6
Fed Funds rate	9.0	0.3	19.7	1.1	62.1	1.8	3.5	2.3
Inflation	2.4	0.1	8.4	6.8	2.3	0.5	0.2	79.3

Table 16: Variance Decomposition, 2 Quarter Horizon, Model with Fiscal Policy

Variable \ Shock	Collateral	Housing	Prod	Leisure	Discount	Gov	Policy	Markup
A. State-level								
Spending	18.7	0.2	0.0	2.6	67.2	11.3	-	-
Employment	9.7	0.1	45.8	2.5	35.8	6.1	-	-
Wages	0.1	0.0	0.0	99.8	0.0	0.0	-	-
Debt-to-income	99.7	0.3	0.0	0.0	0.0	0.0	-	-
House prices	1.4	98.4	0.0	0.1	0.0	0.0	-	-
B. Aggregate-level								
Consumption	0.5	0.0	46.6	0.1	23.1	13.6	13.1	3.0
Employment	1.0	0.0	4.3	0.1	48.8	11.7	27.6	6.4
Wages	0.0	0.0	0.3	89.1	0.1	0.0	0.0	10.5
Debt-to-income	45.0	0.0	0.0	0.7	48.2	5.2	0.4	0.6
House prices	0.1	72.4	8.8	0.0	12.9	2.6	2.6	0.6
Fed Funds rate	1.8	0.0	11.0	0.1	82.1	0.5	3.4	1.2
Inflation	0.1	0.0	2.9	2.3	0.3	0.0	0.0	94.3

Figure 6: Effect of Credit Shocks on State Employment: Model with Construction

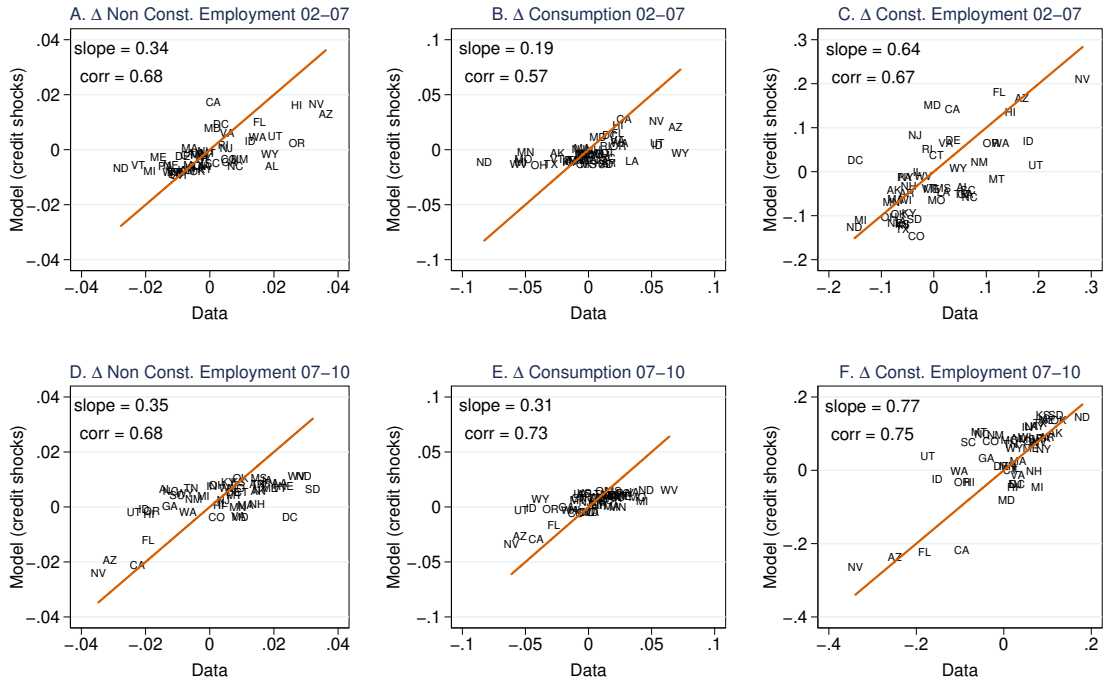


Figure 7: Effect of Credit Shocks on State Employment in the Boom: Robustness

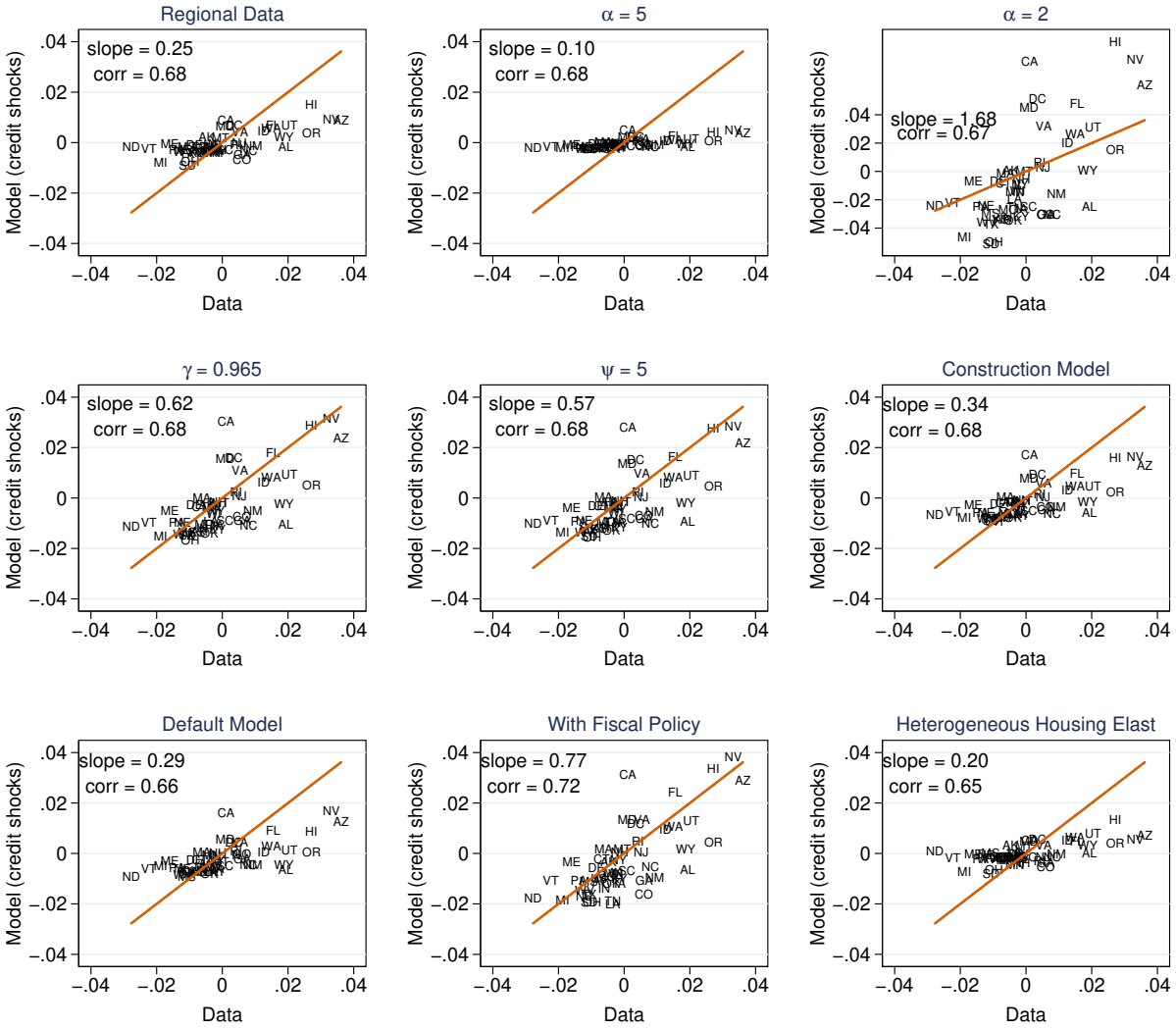


Figure 8: Effect of Credit Shocks on State Employment in the Bust: Robustness

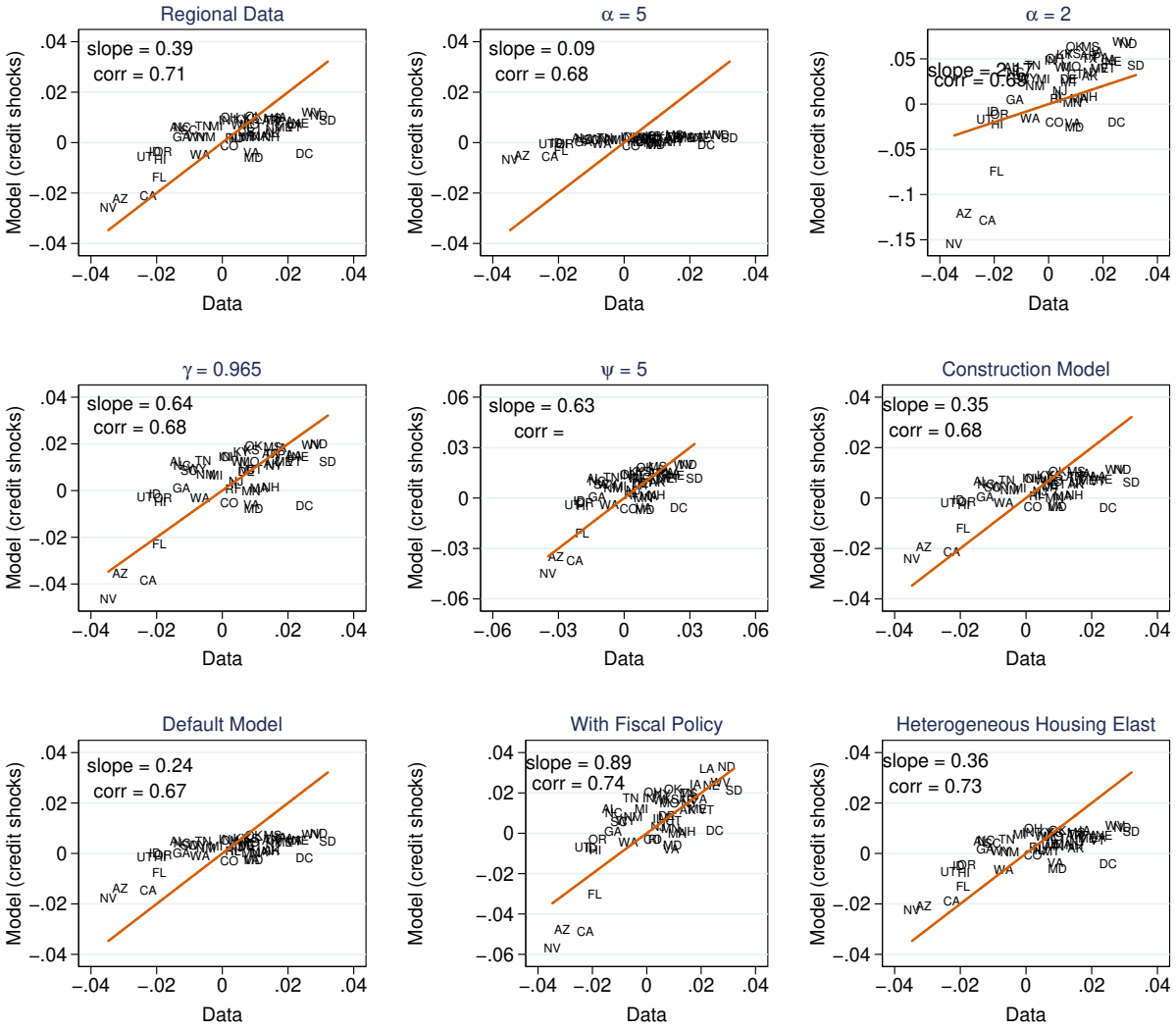


Figure 9: Prior (blue) and Posterior (red) Distributions in Baseline Estimation

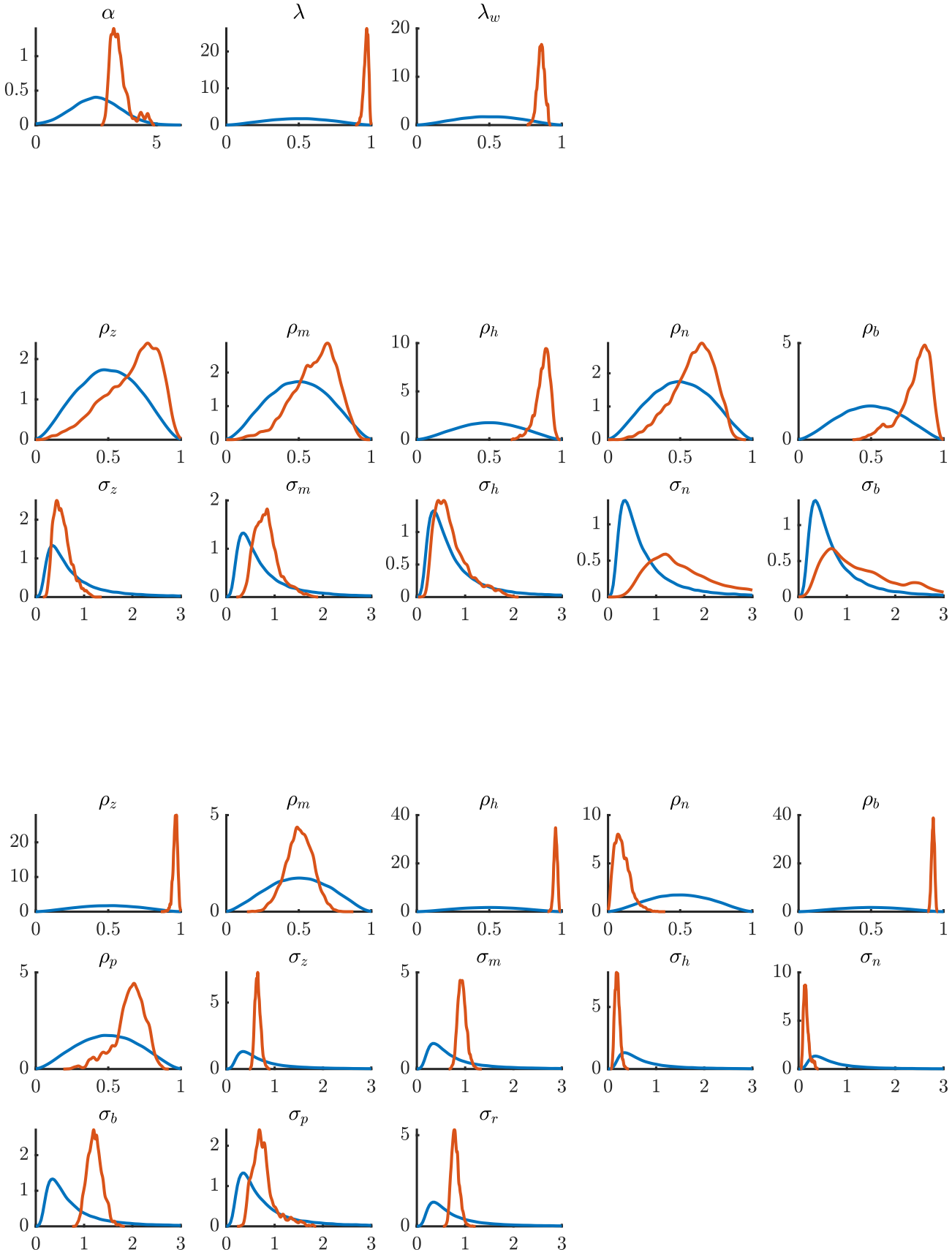


Figure 10: Convergence of Parameter Posterior Distributions Across MCMC Chains

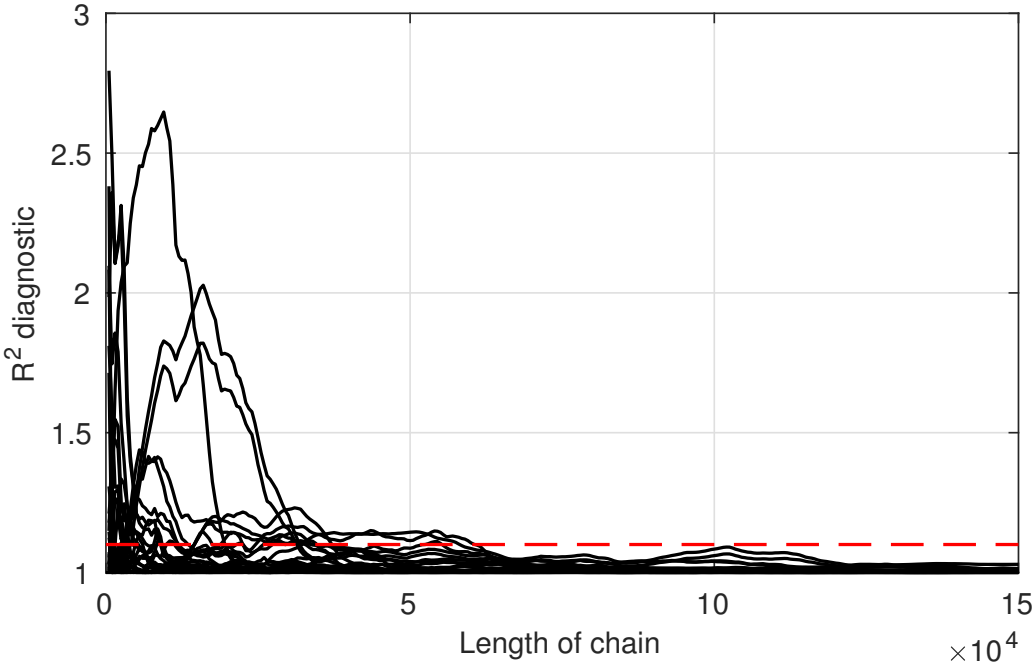


Figure 11: Effects of Credit Shocks at the Regional Level: Estimated Taylor Rule

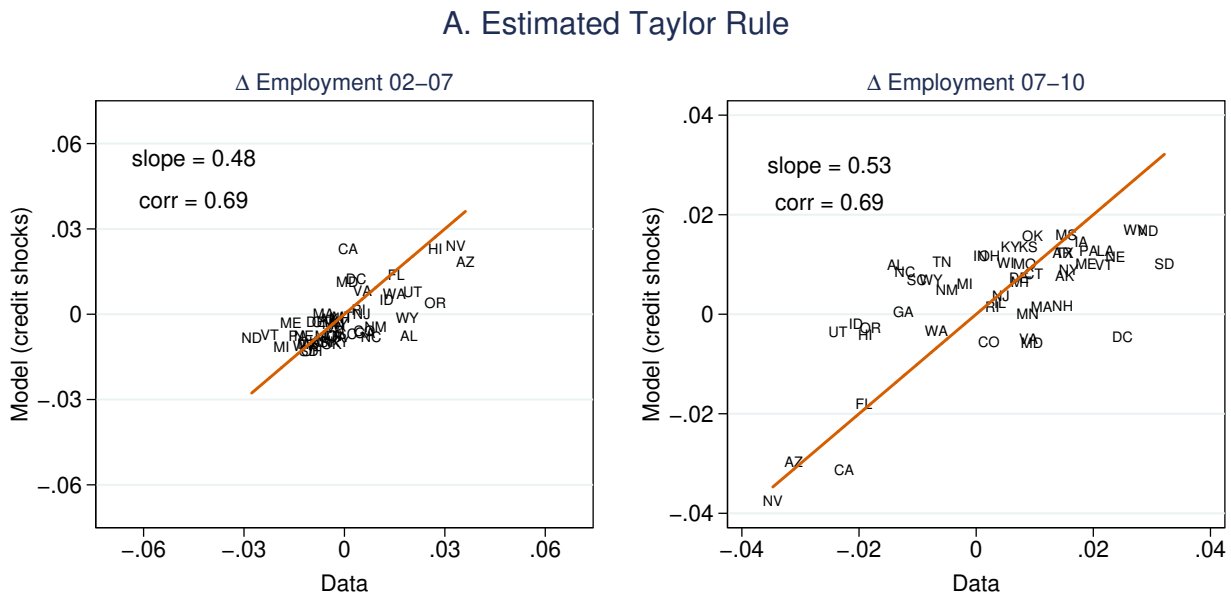


Figure 12: Credit Shocks at Aggregate: Robustness, No Population Weights

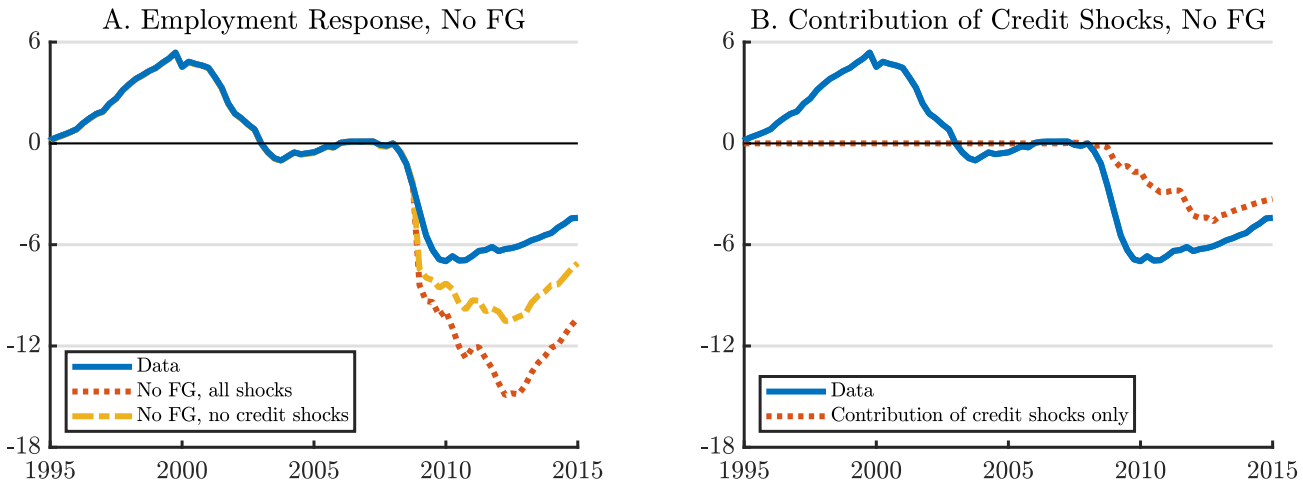


Figure 13: Credit Shocks at Aggregate: Robustness, State Data Only

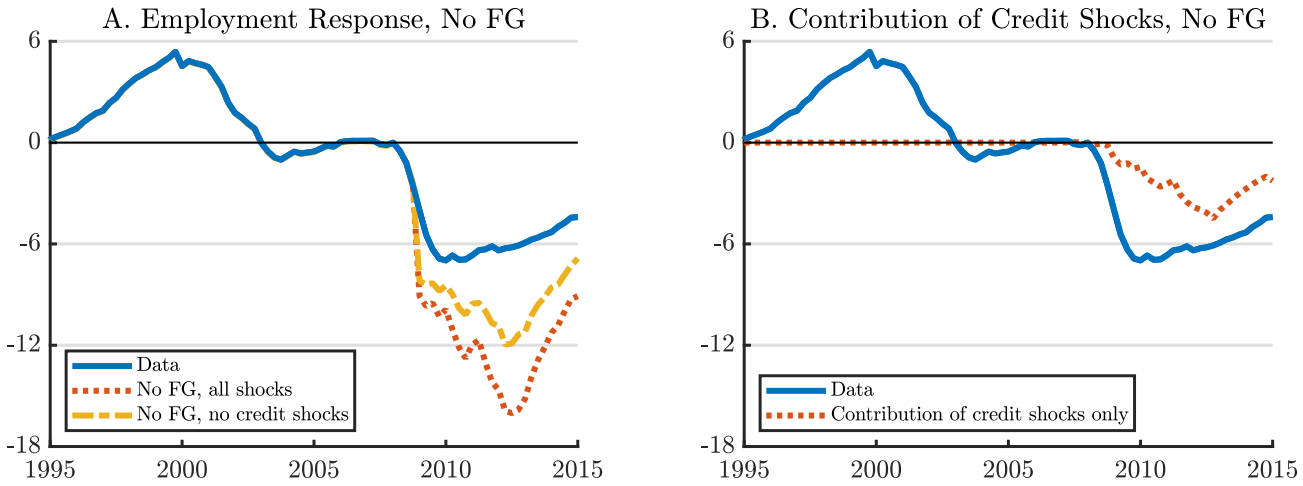


Figure 14: Credit Shocks at Aggregate: Robustness, Model with Construction Sector

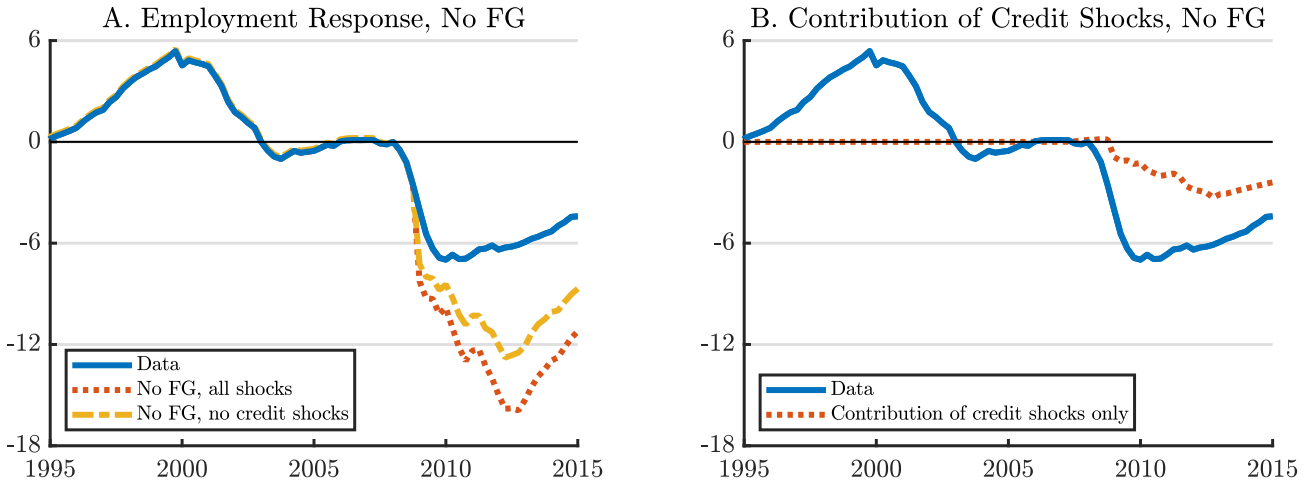


Figure 15: Credit Shocks at Aggregate: Robustness, Government Spending

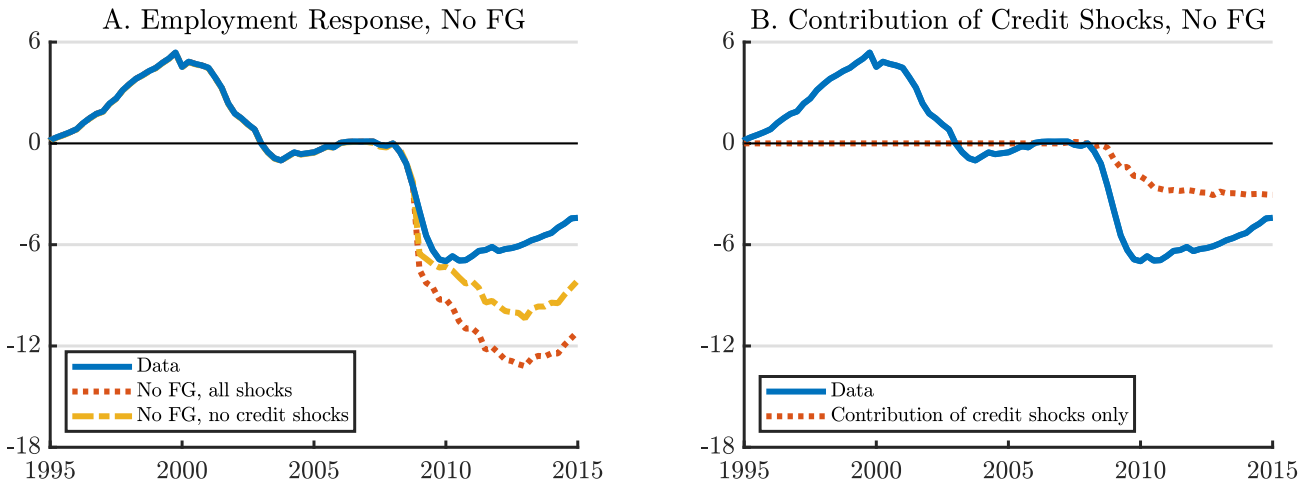


Figure 16: Credit Shocks at Aggregate: Robustness, Option to Default

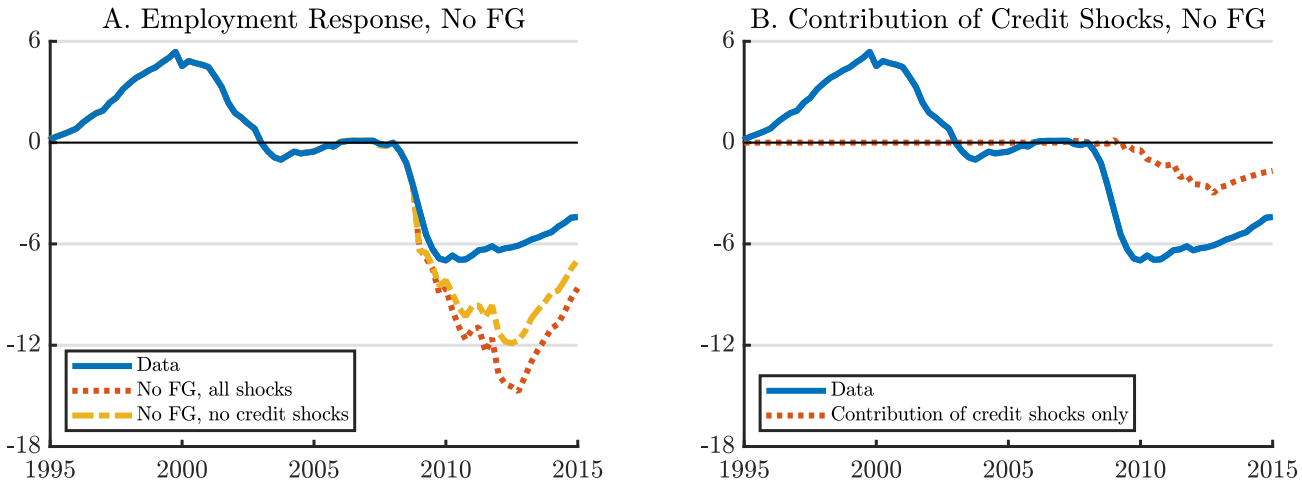


Figure 17: Credit Shocks at Aggregate: Robustness, Estimated Taylor Rule

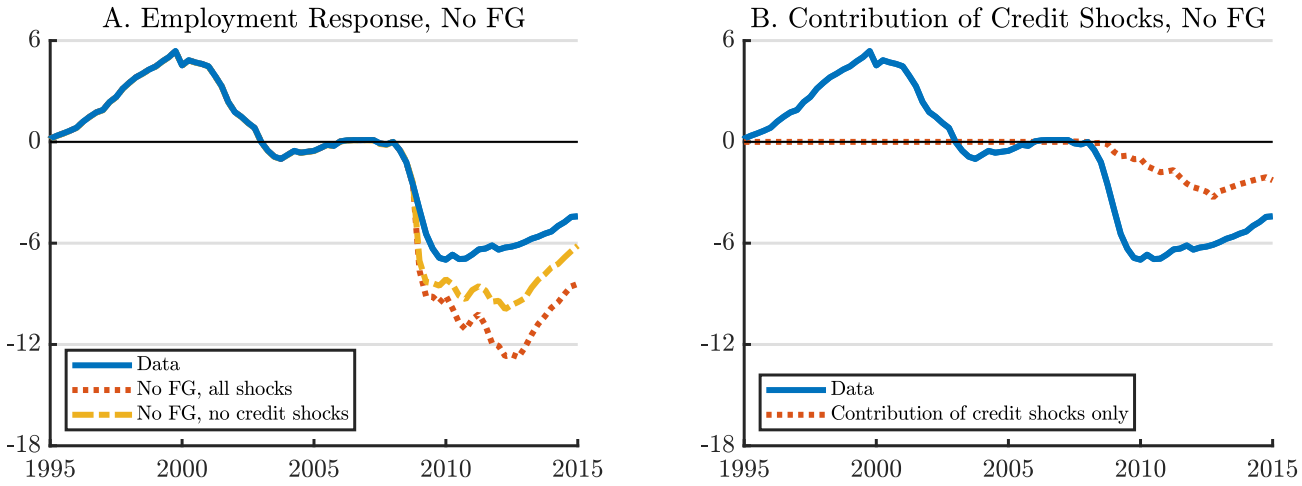


Figure 18: State Data: Relative Employment

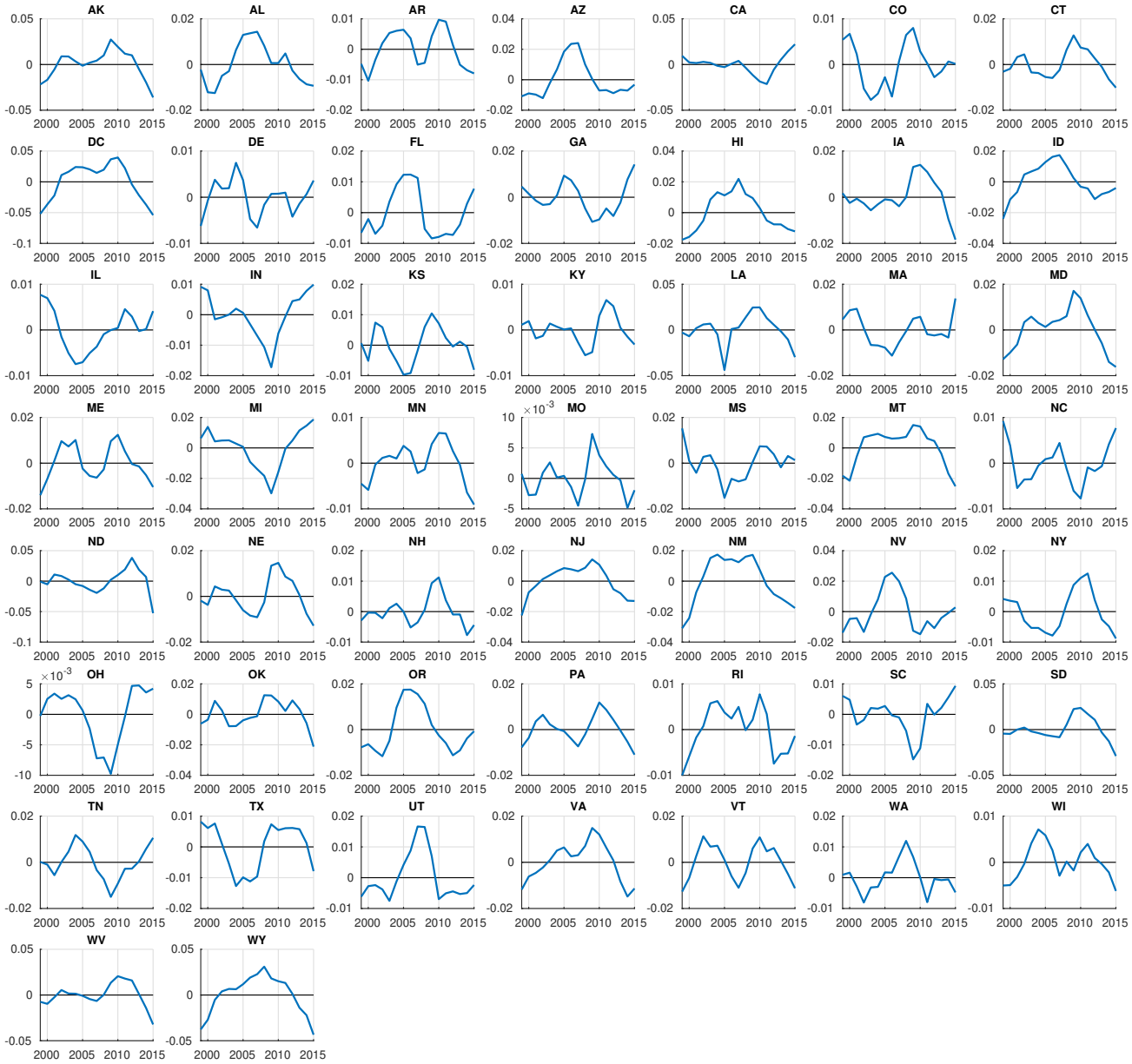


Figure 19: State Data: Relative Household Spending

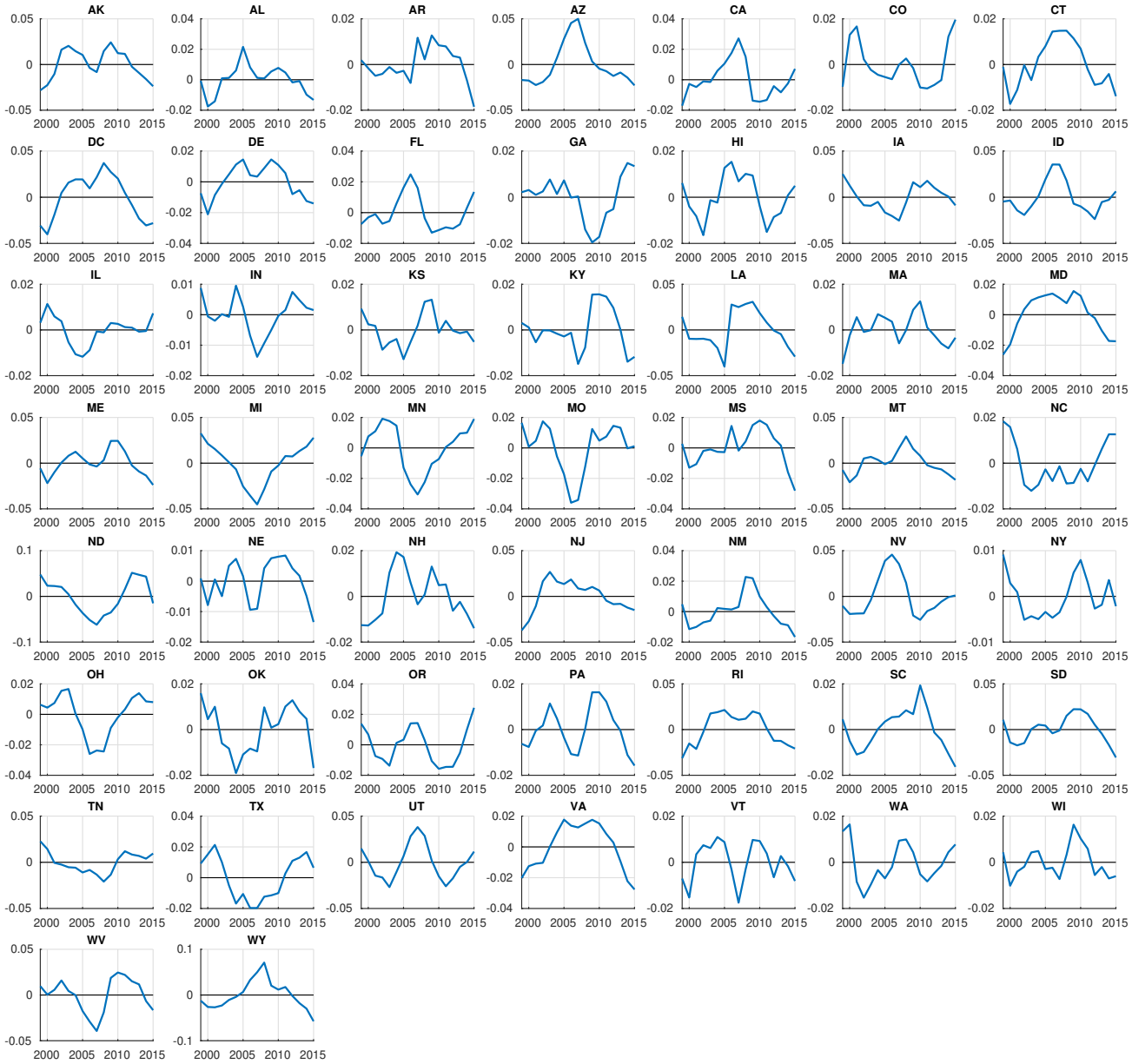


Figure 20: State Data: Relative Wages

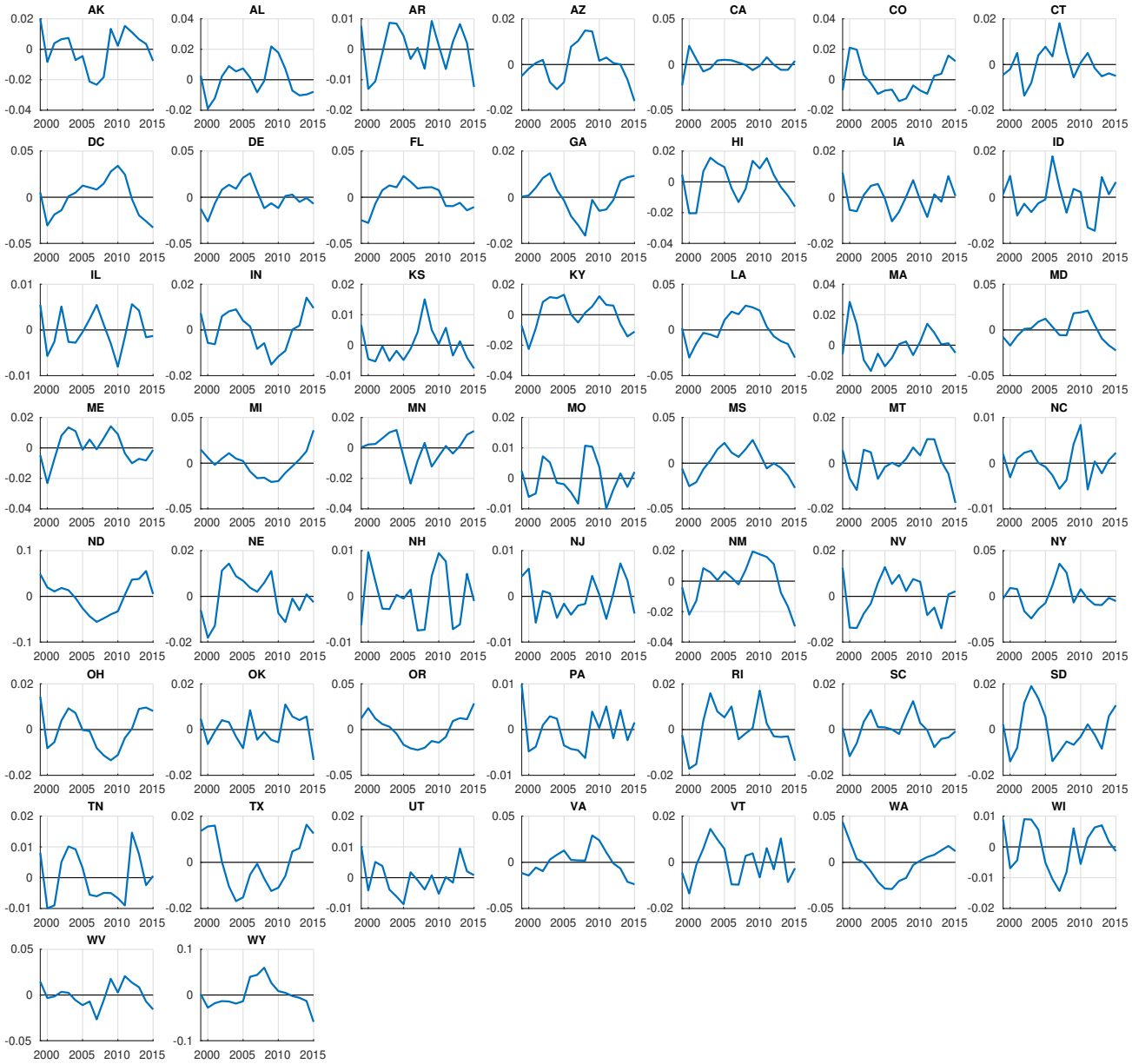


Figure 21: State Data: Relative Household Debt

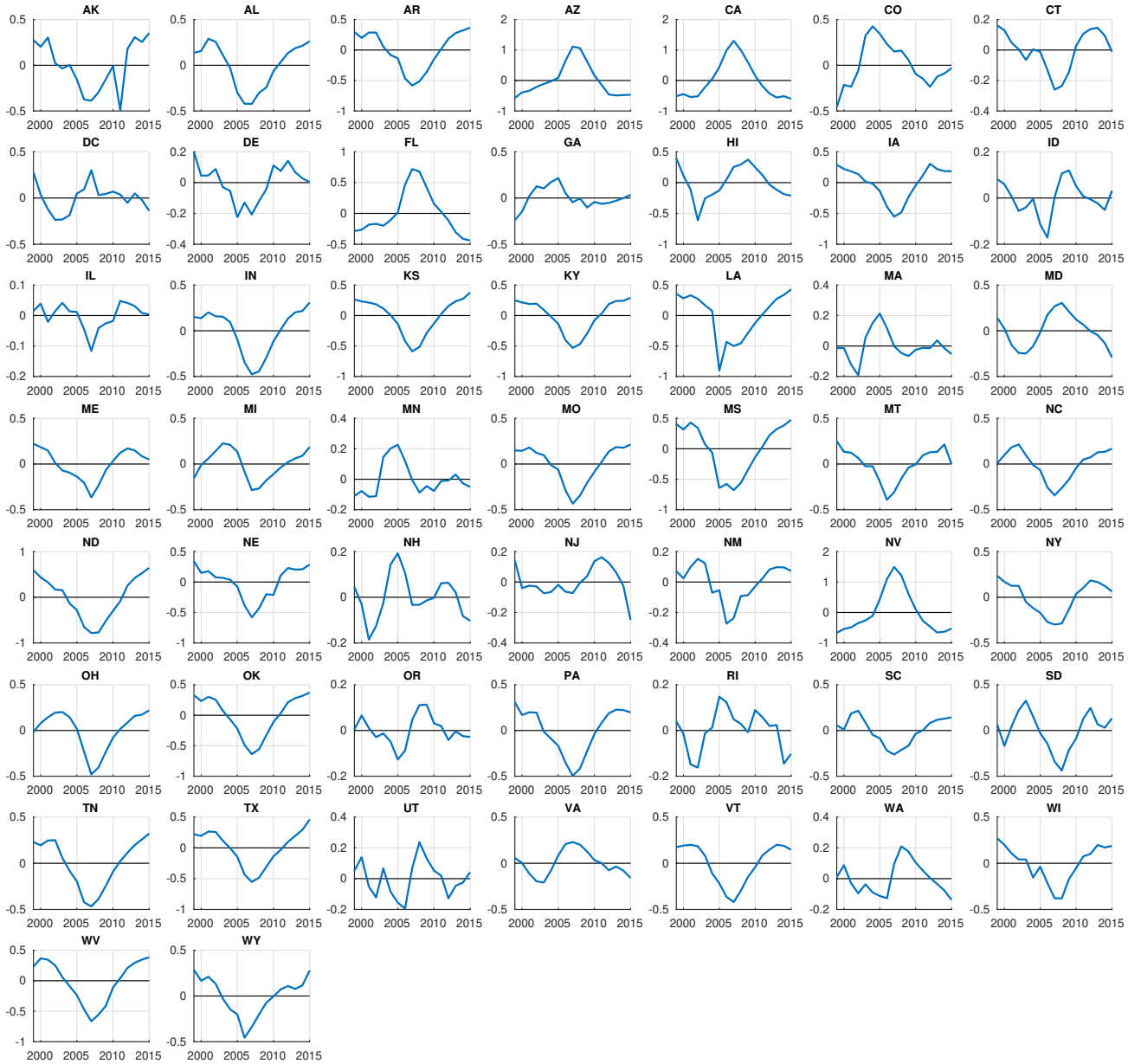


Figure 22: State Data: Relative House Prices

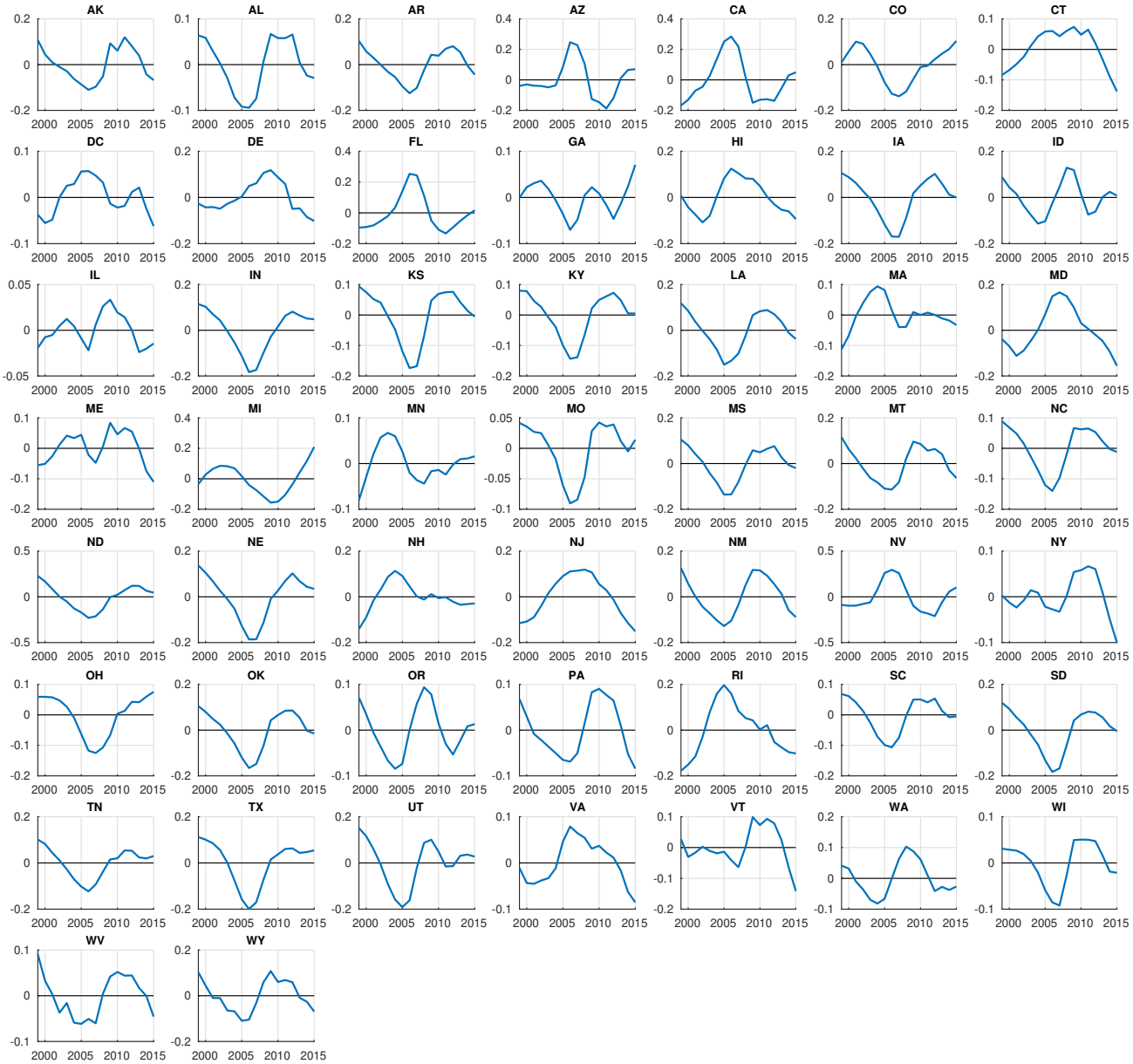


Figure 23: Aggregate Data

Figure 24: ZLB Durations

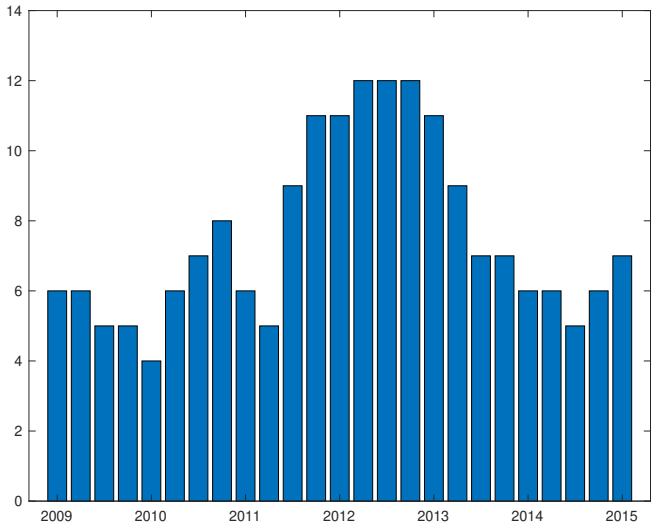


Figure 25: Shock Decomposition, Relative Employment

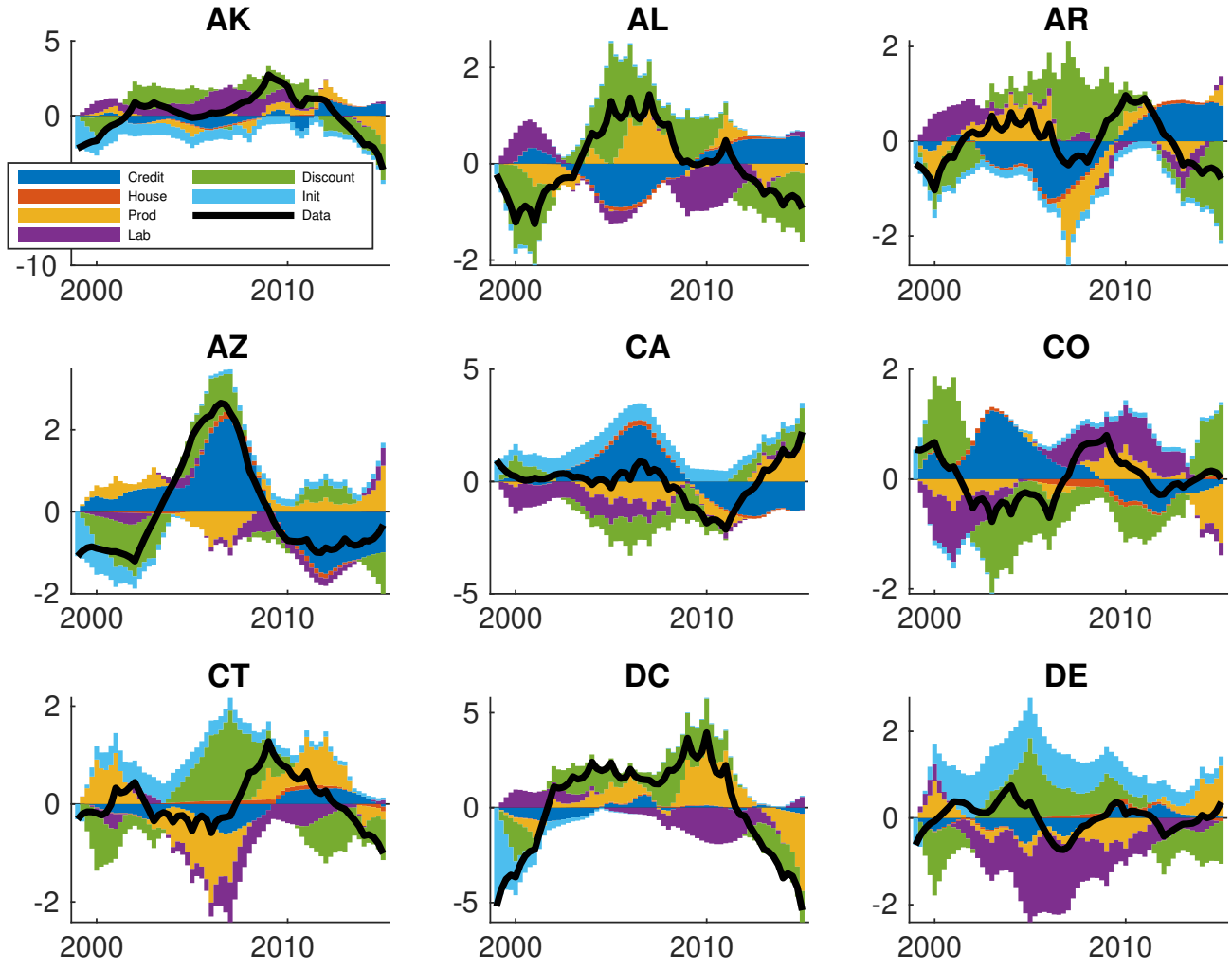


Figure 26: Shock Decomposition, Relative Employment (cont)

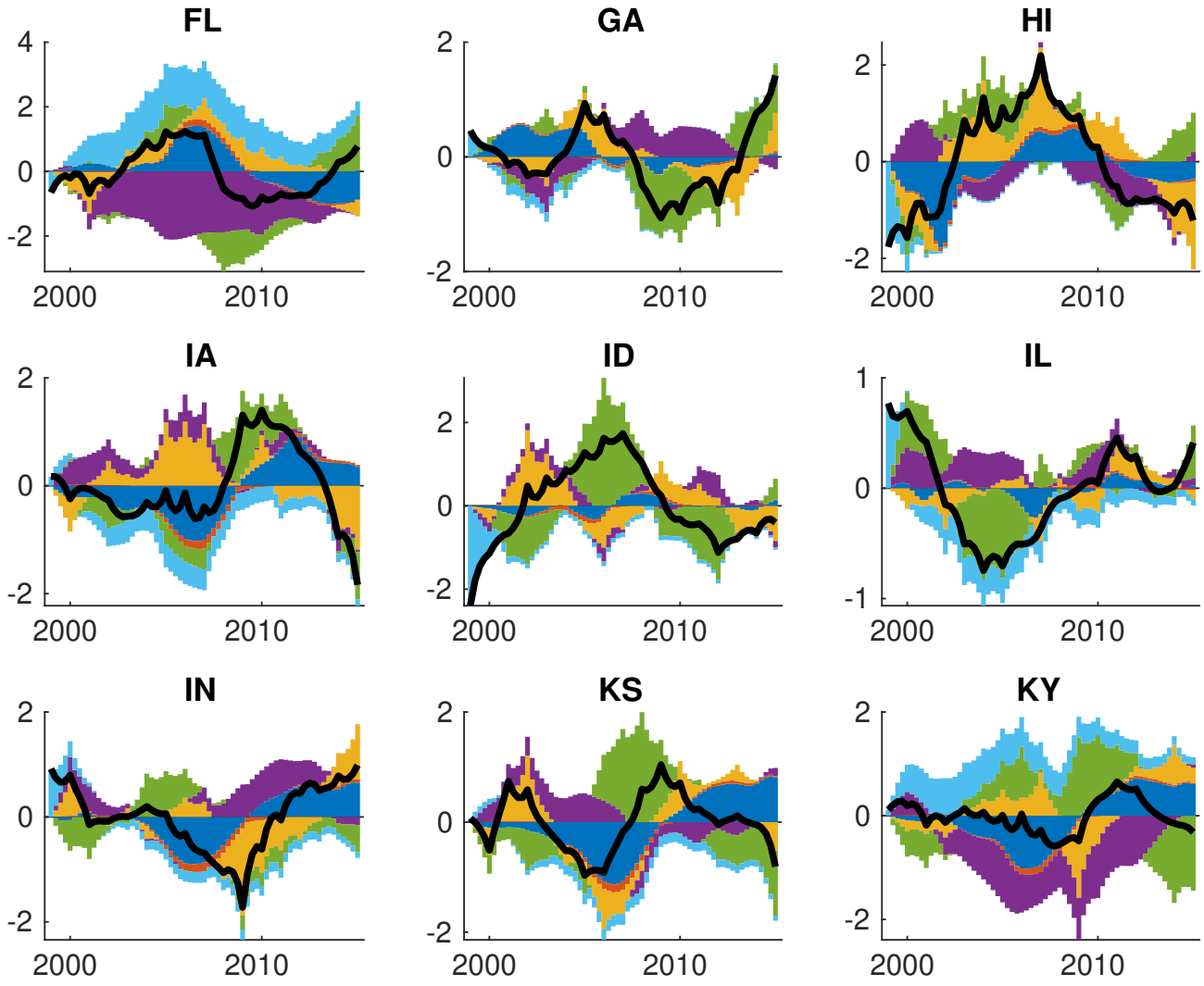


Figure 27: Shock Decomposition, Relative Employment (cont)

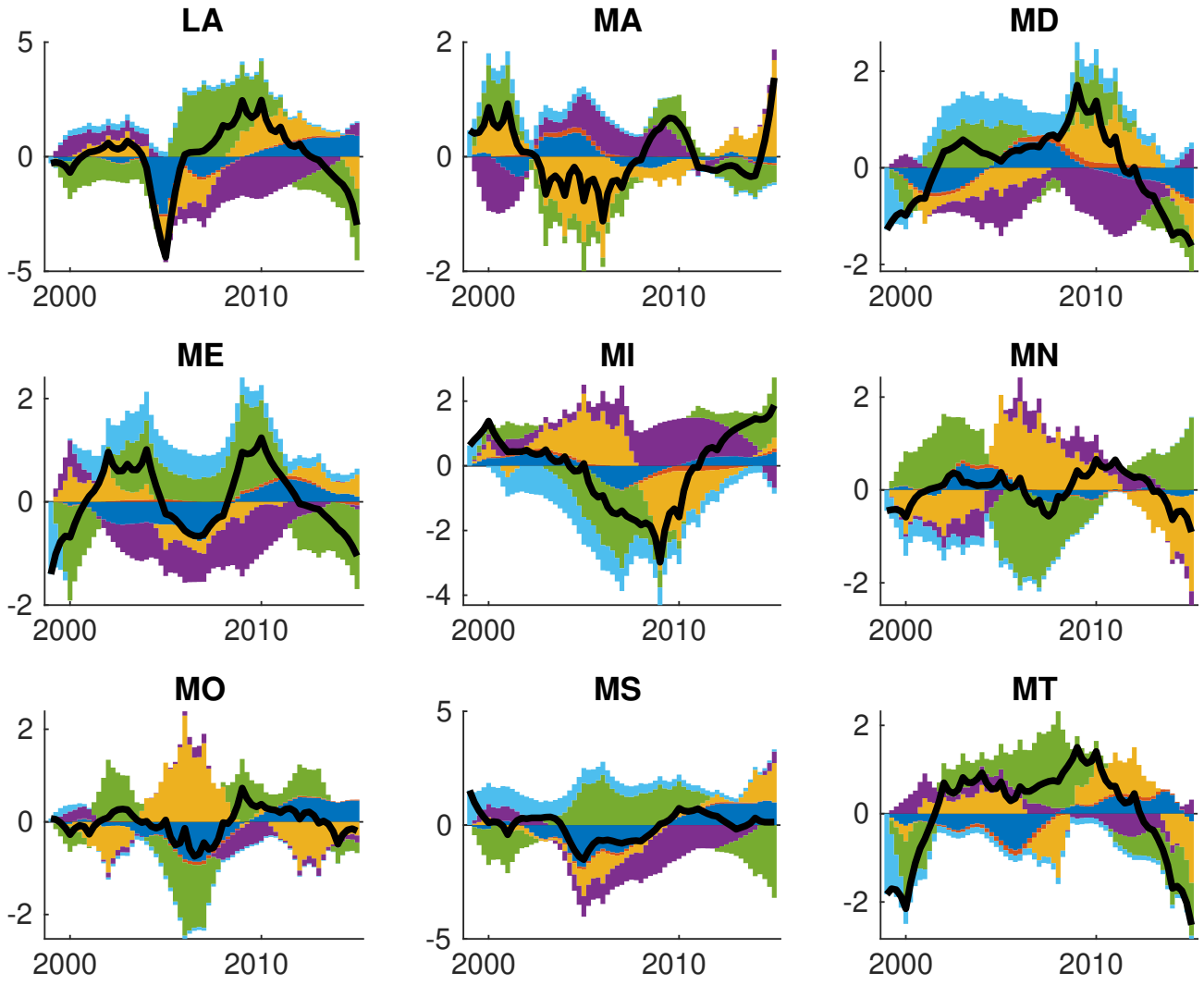


Figure 28: Shock Decomposition, Relative Employment (cont)

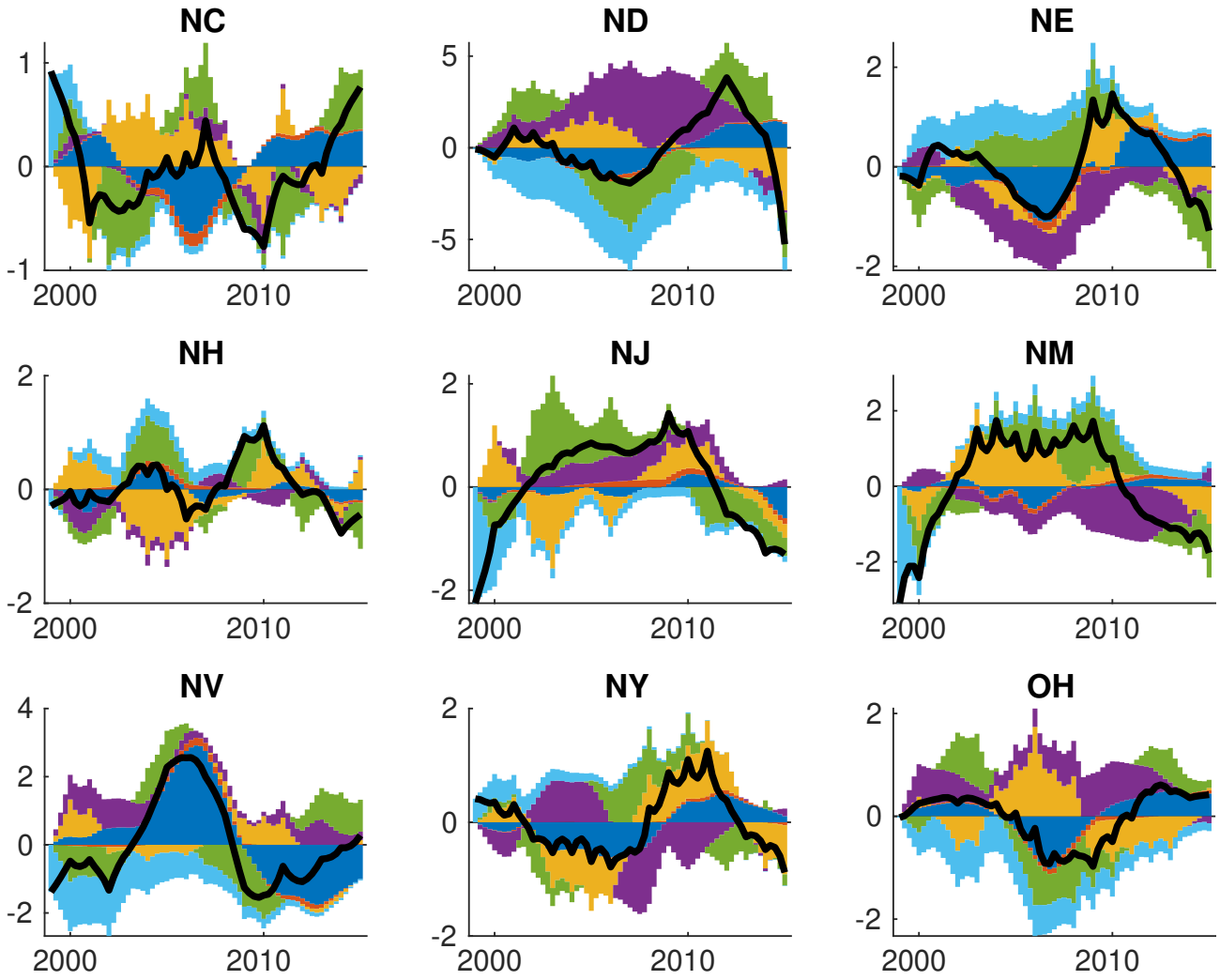


Figure 29: Shock Decomposition, Relative Employment (cont)

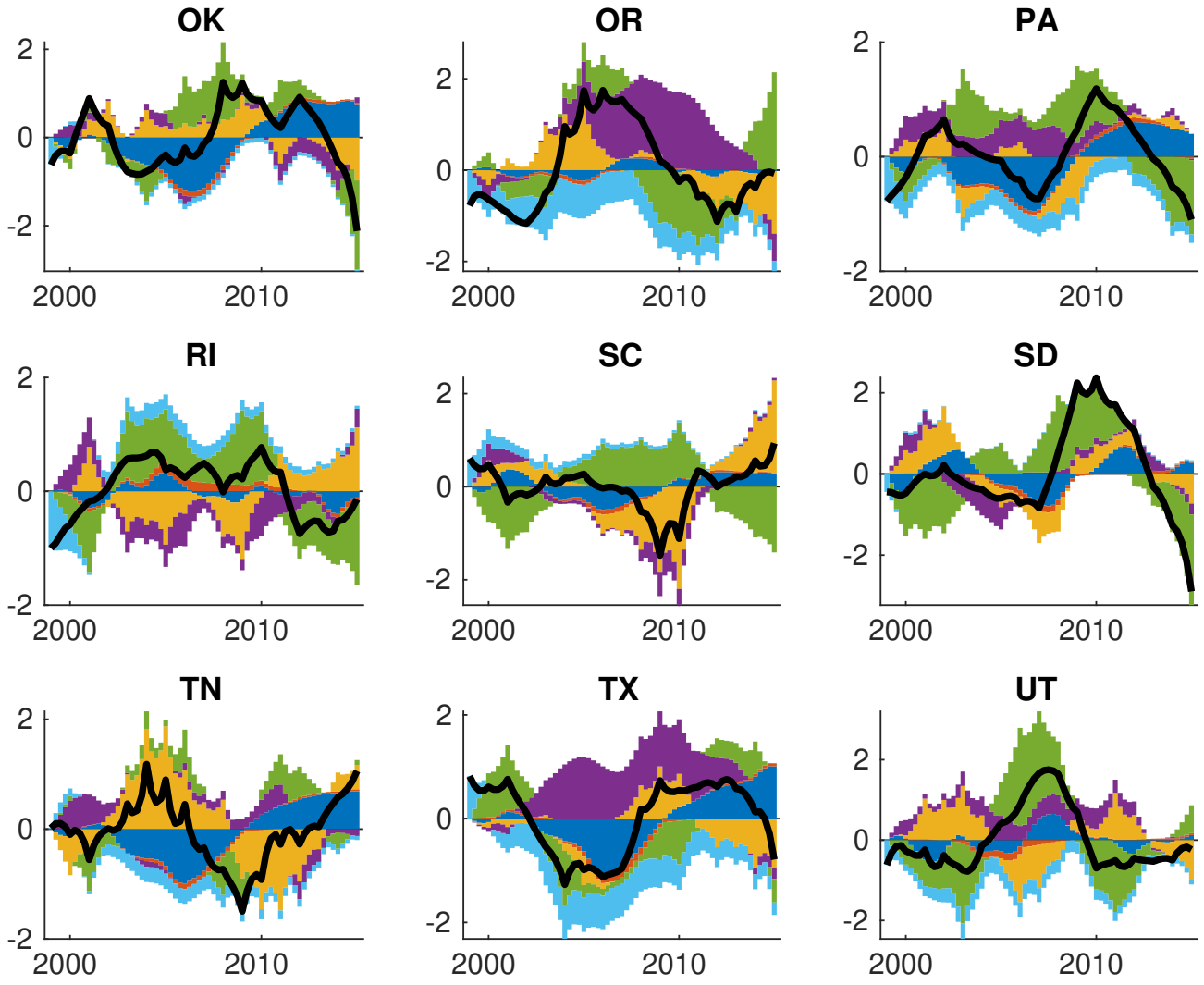


Figure 30: Shock Decomposition, Relative Employment (cont)

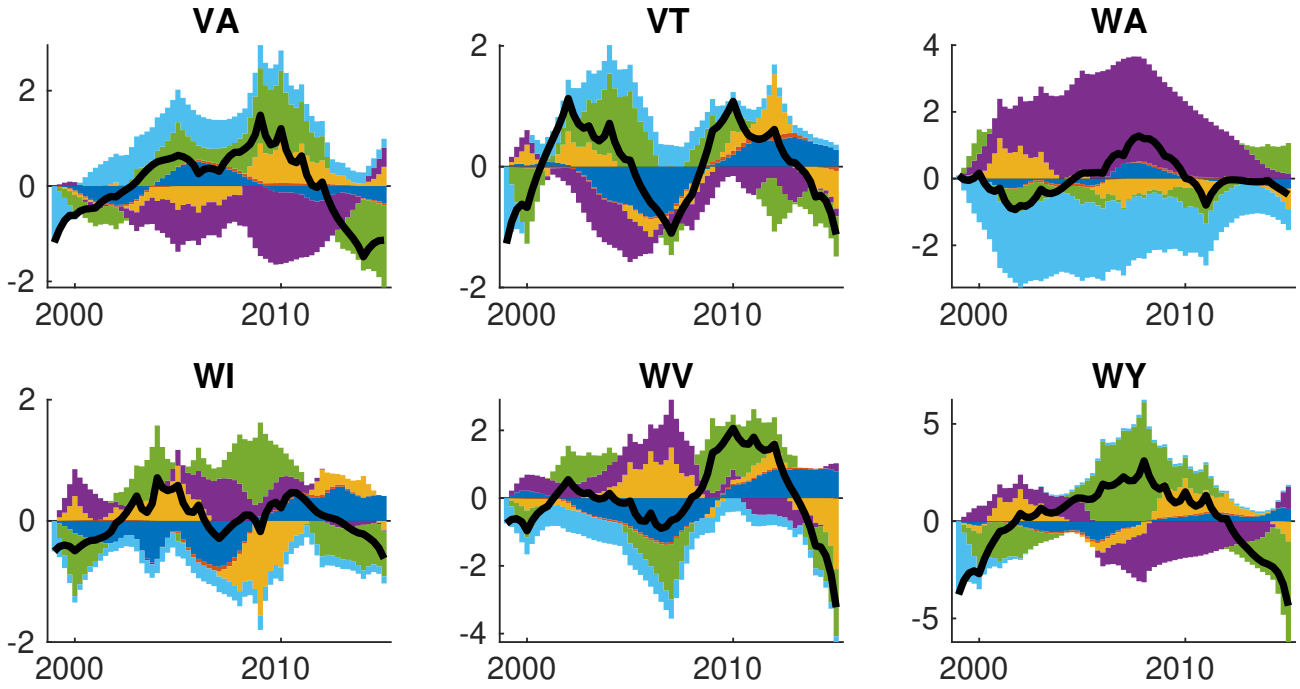


Figure 31: Shock Decomposition, Relative Consumption Spending

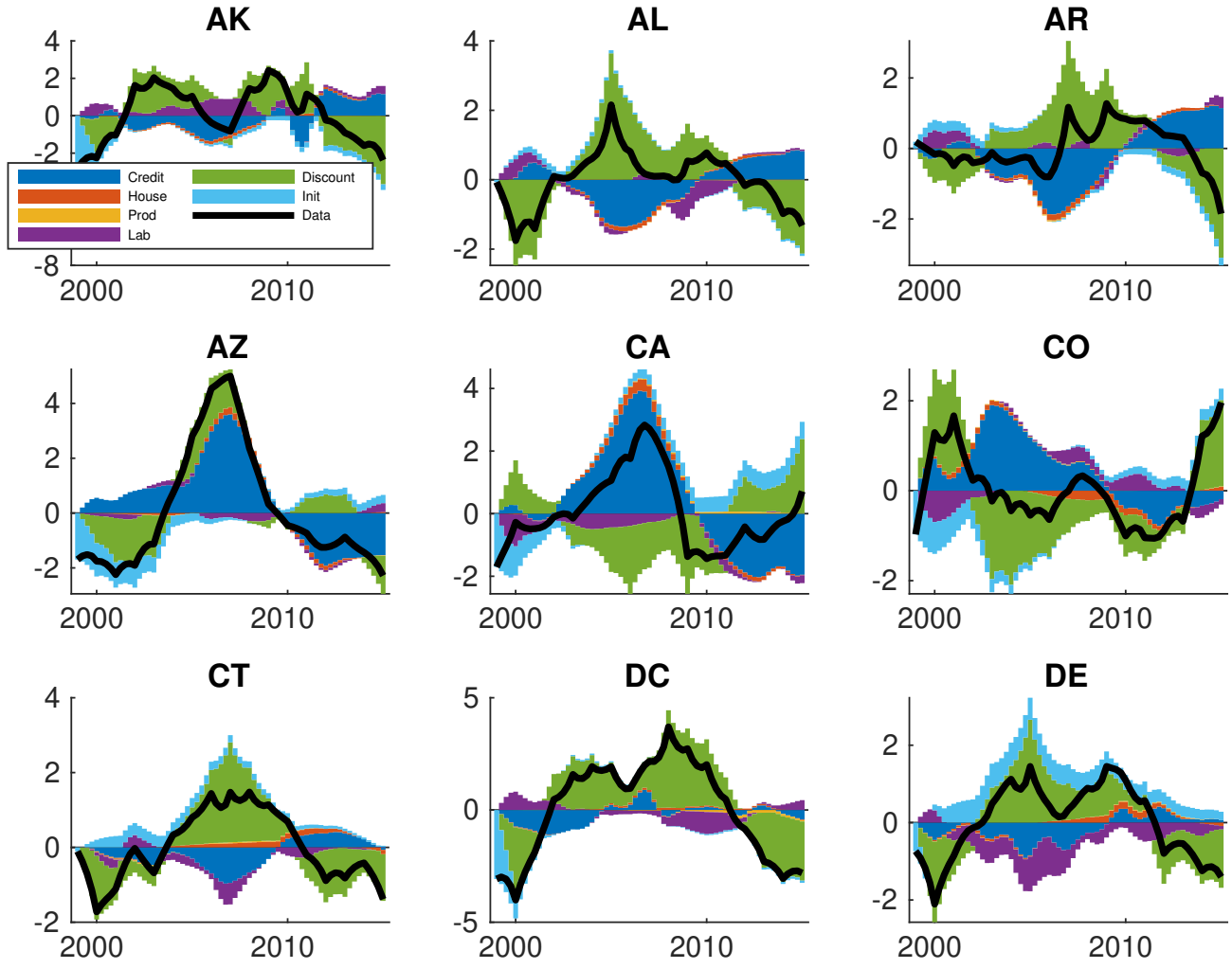


Figure 32: Shock Decomposition, Relative Consumption Spending (cont)

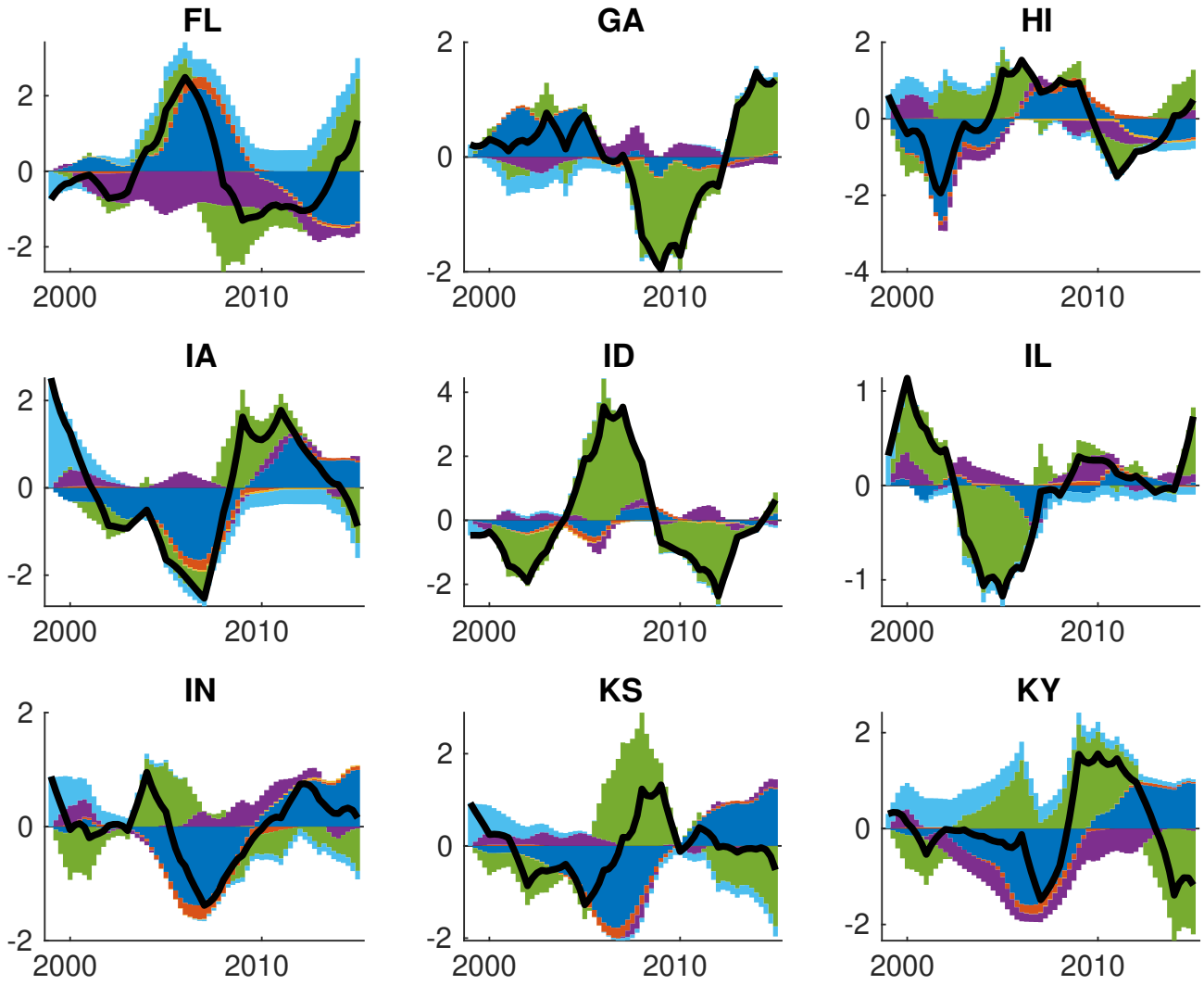


Figure 33: Shock Decomposition, Relative Consumption Spending (cont)

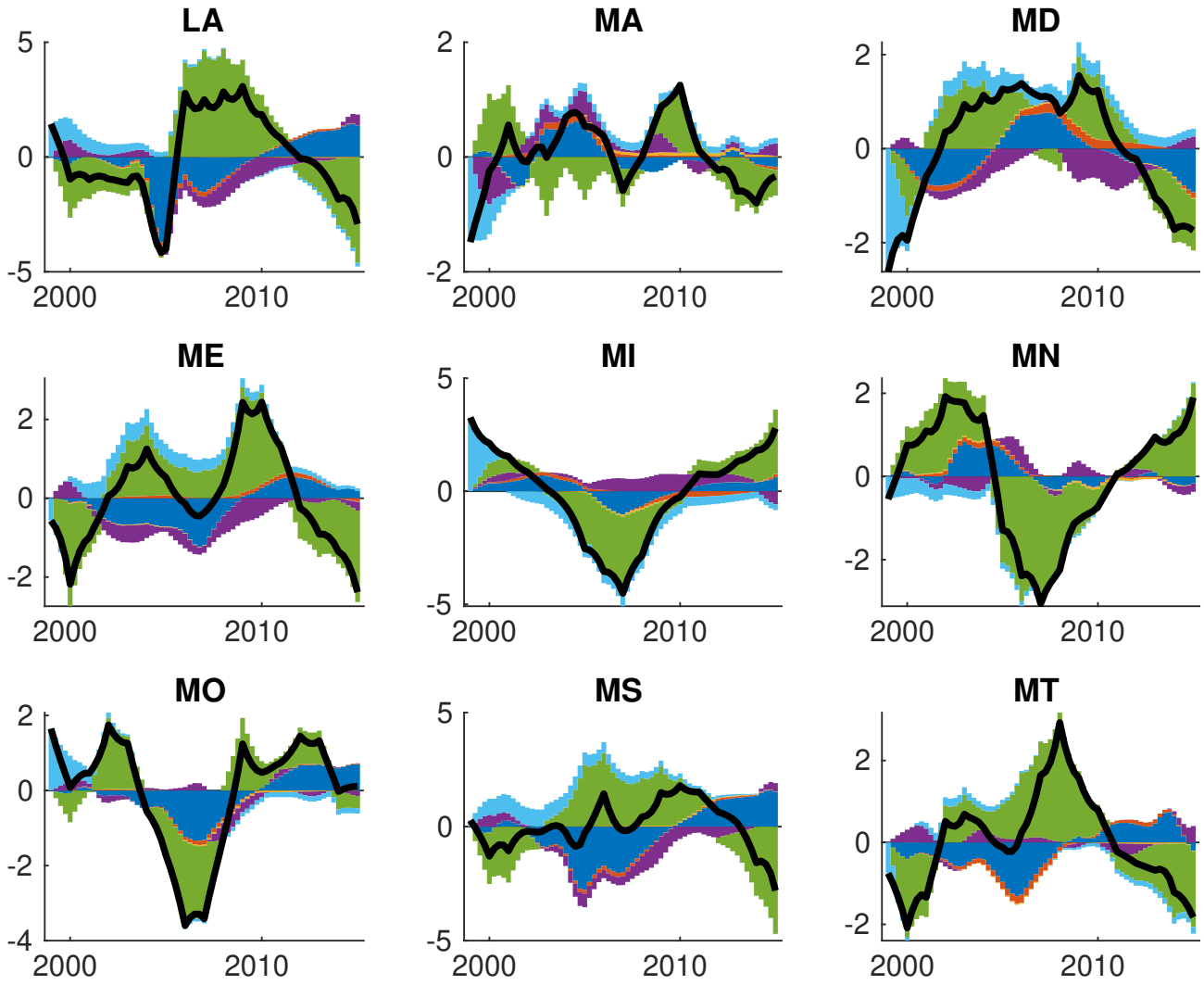


Figure 34: Shock Decomposition, Relative Consumption Spending (cont)

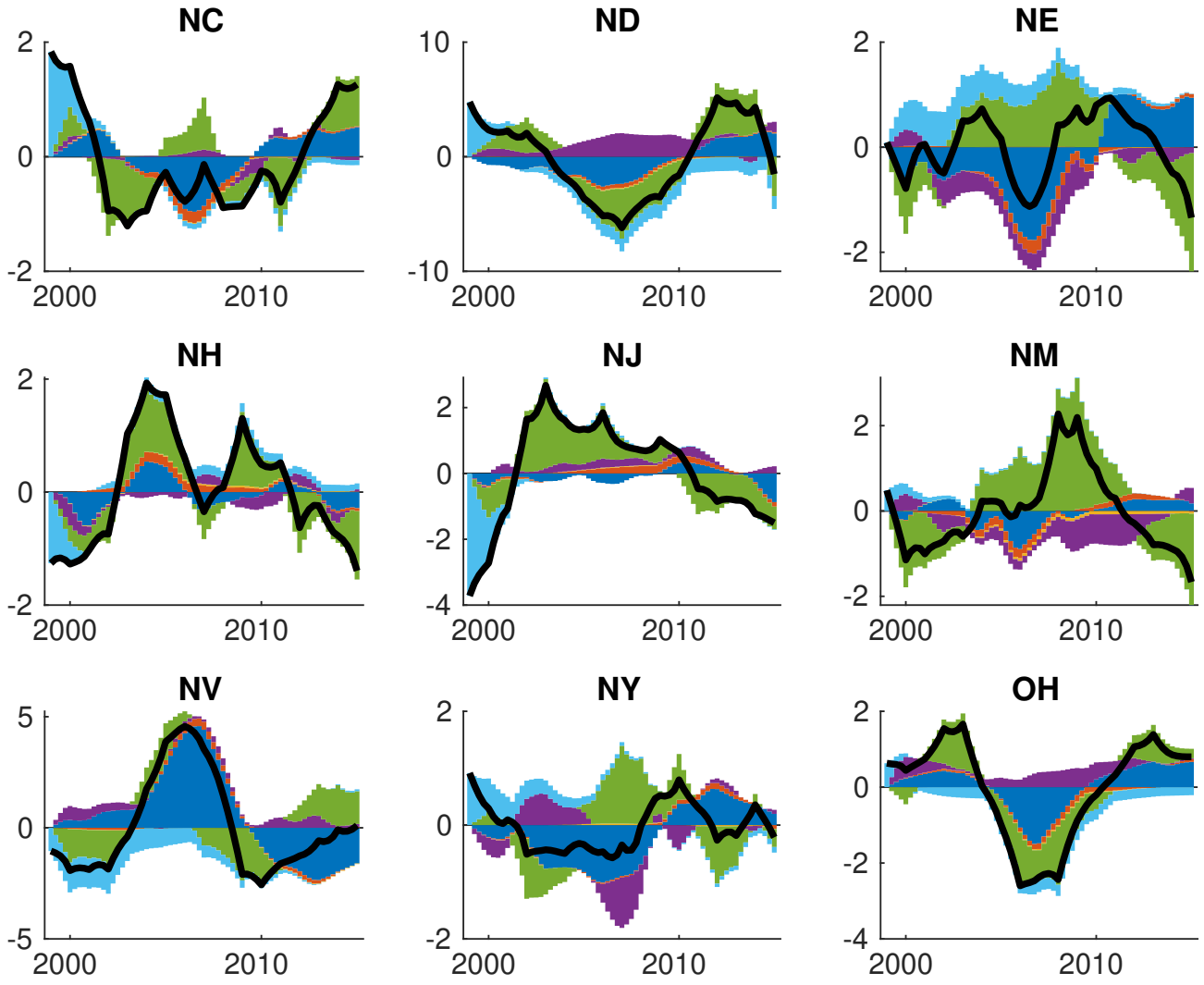


Figure 35: Shock Decomposition, Relative Consumption Spending (cont)

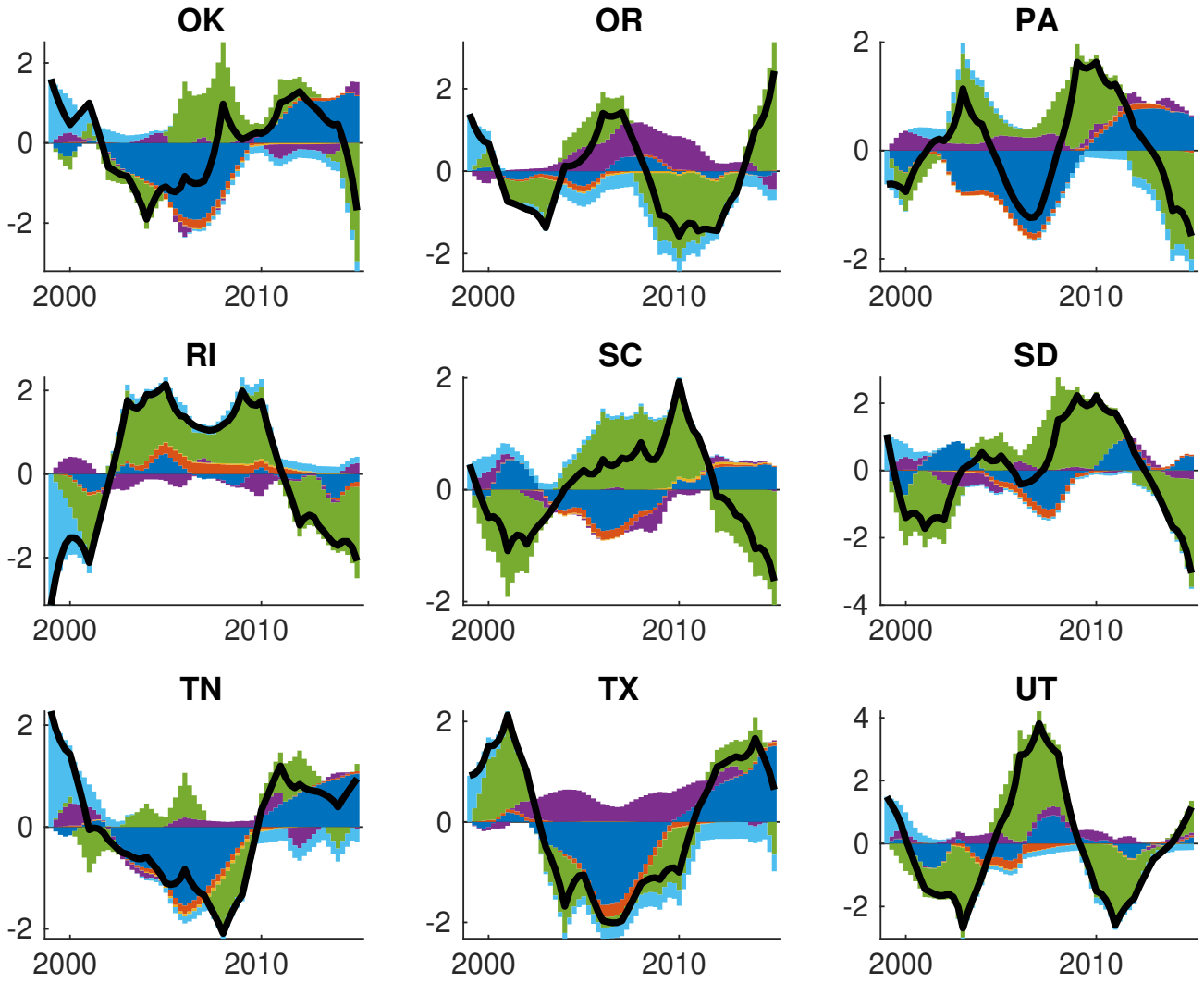


Figure 36: Shock Decomposition, Relative Consumption Spending (cont)

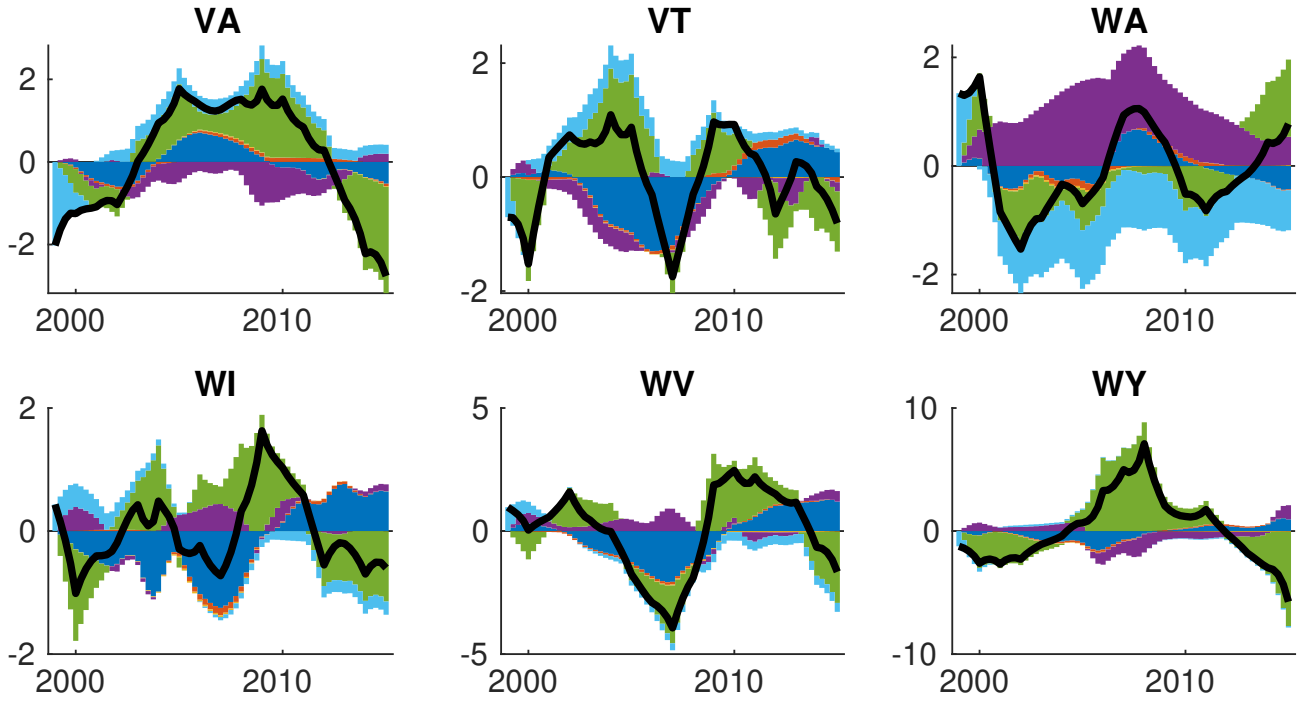


Figure 37: Shock Decomposition, Relative Household Debt

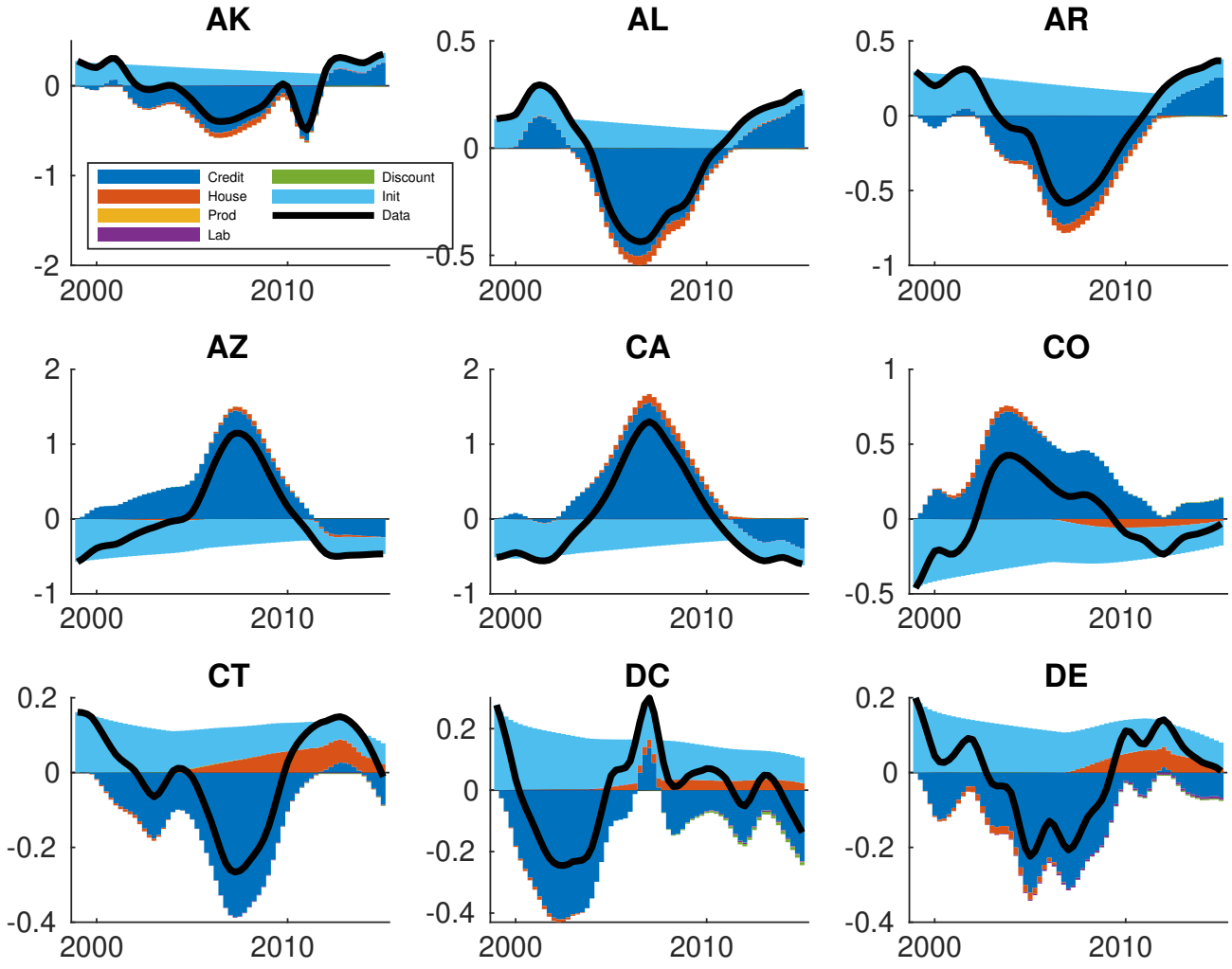


Figure 38: Shock Decomposition, Relative Household Debt (cont)

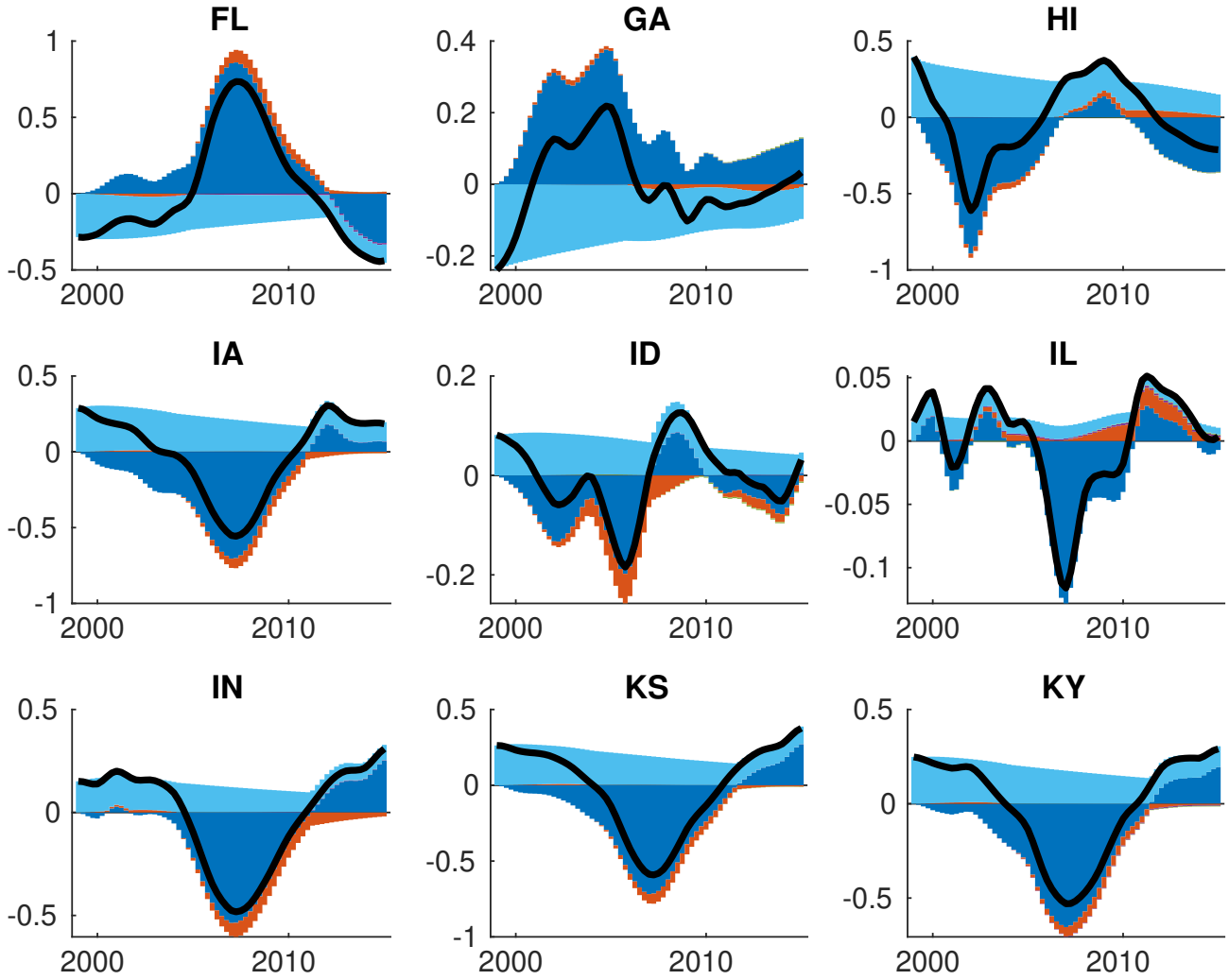


Figure 39: Shock Decomposition, Relative Household Debt (cont)

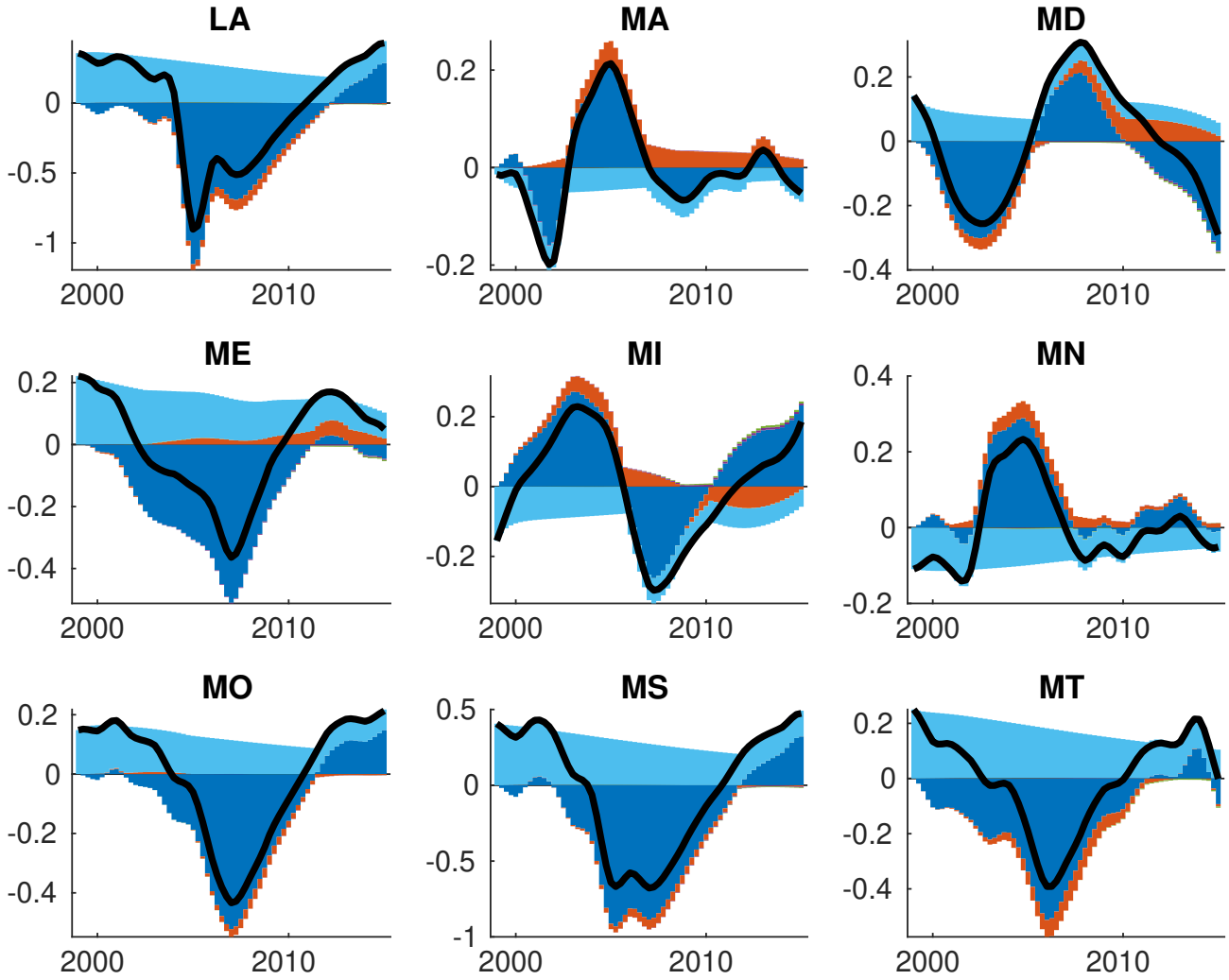


Figure 40: Shock Decomposition, Relative Household Debt (cont)

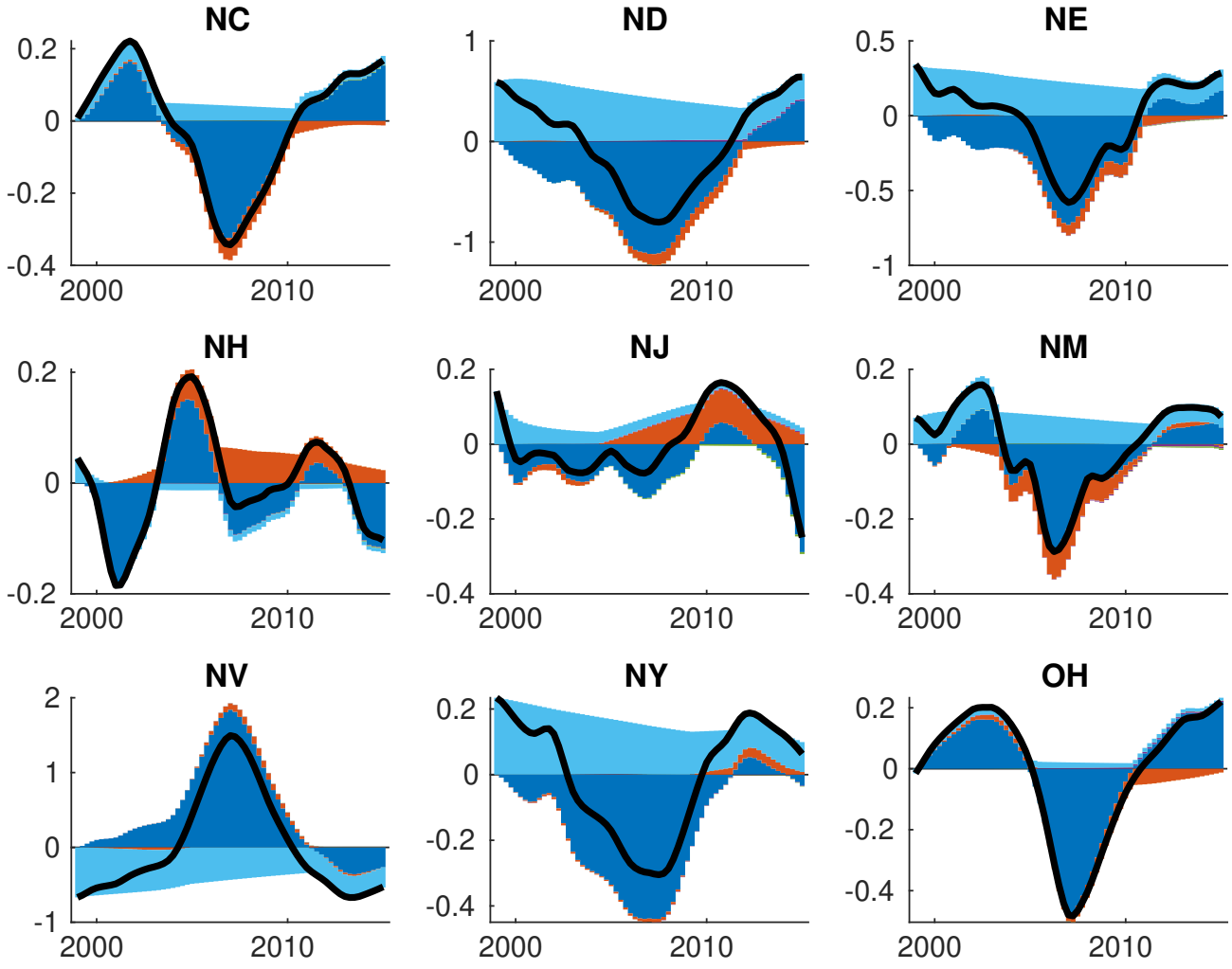


Figure 41: Shock Decomposition, Relative Household Debt (cont)

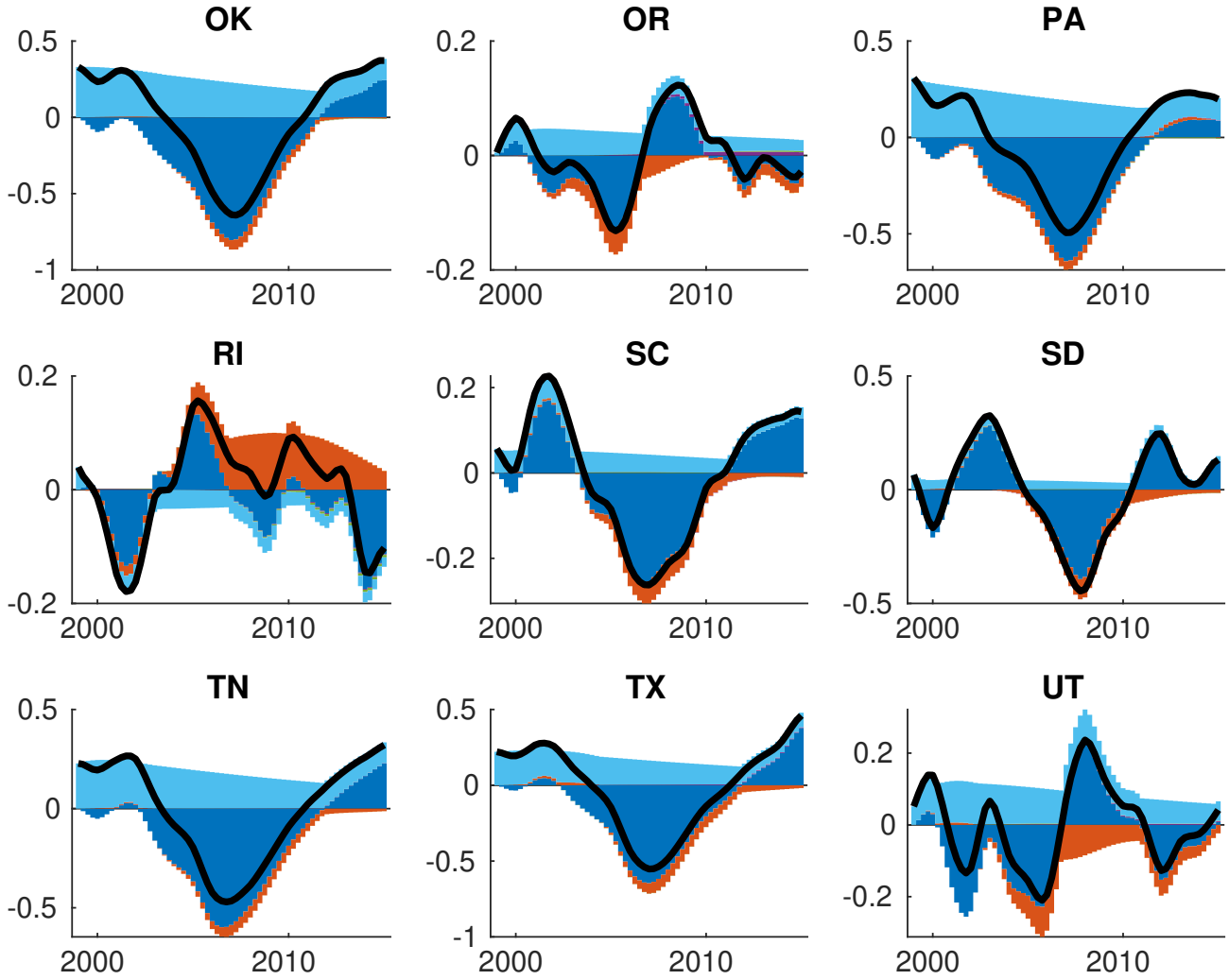


Figure 42: Shock Decomposition, Relative Household Debt (cont)

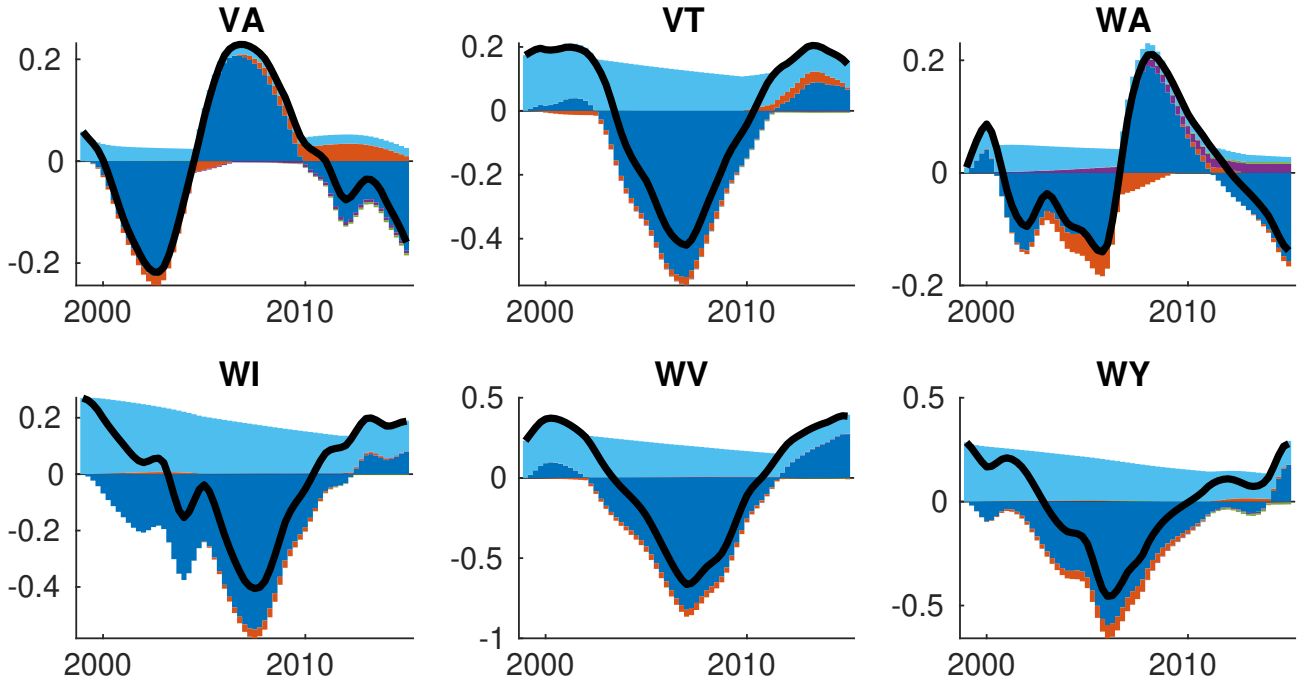


Figure 43: State-level Structural Shocks

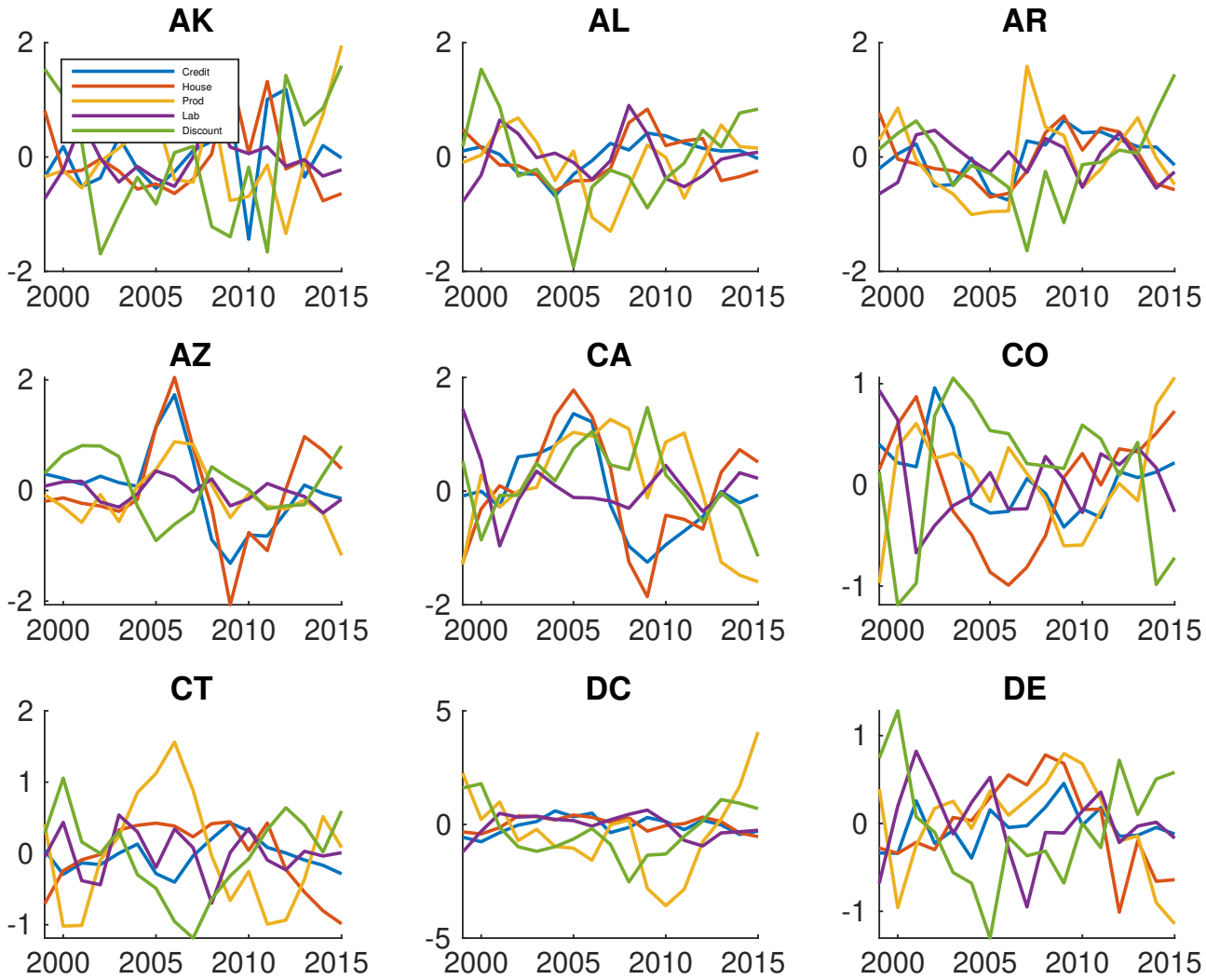


Figure 44: State-level Structural Shocks (cont)

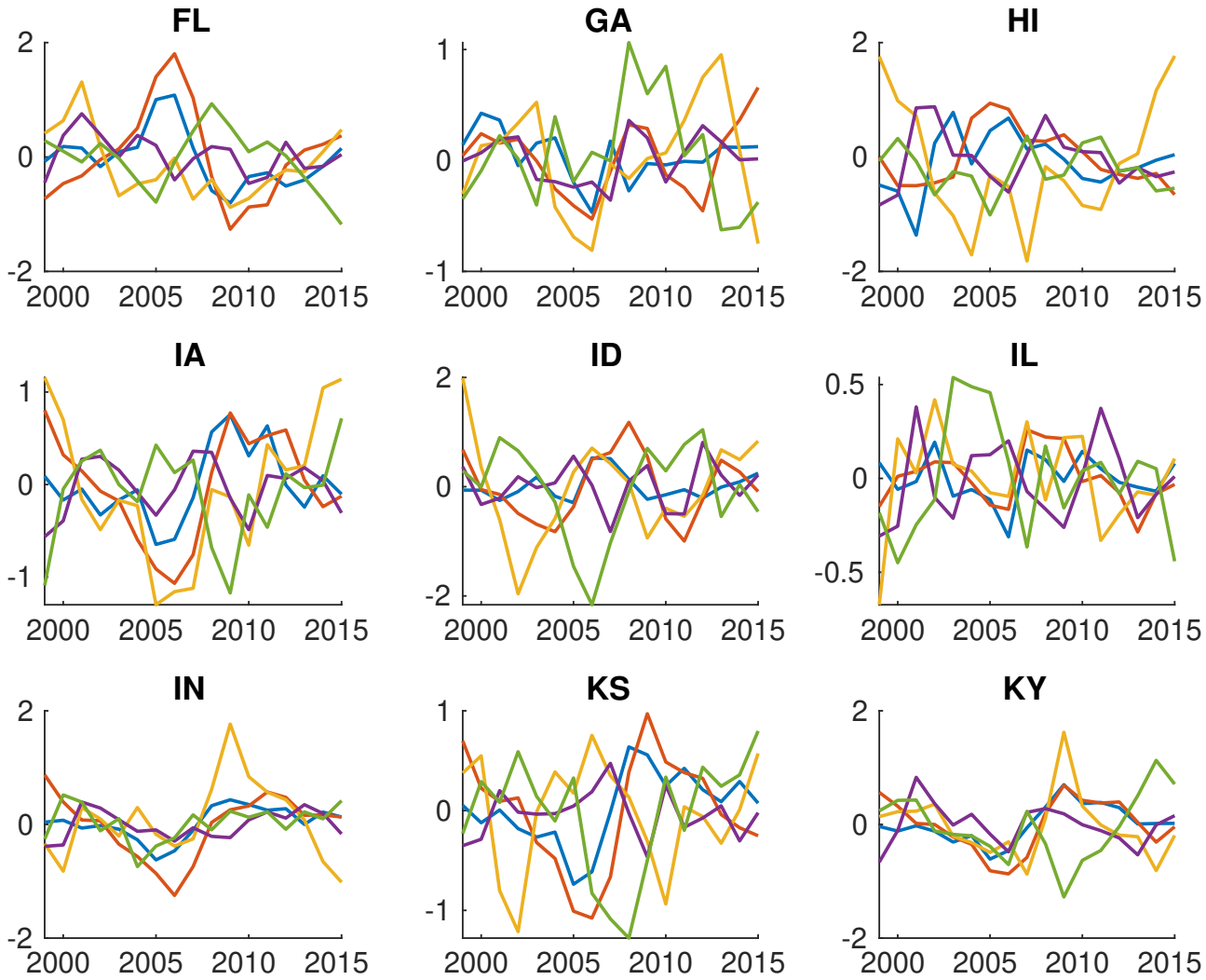


Figure 45: State-level Structural Shocks (cont)

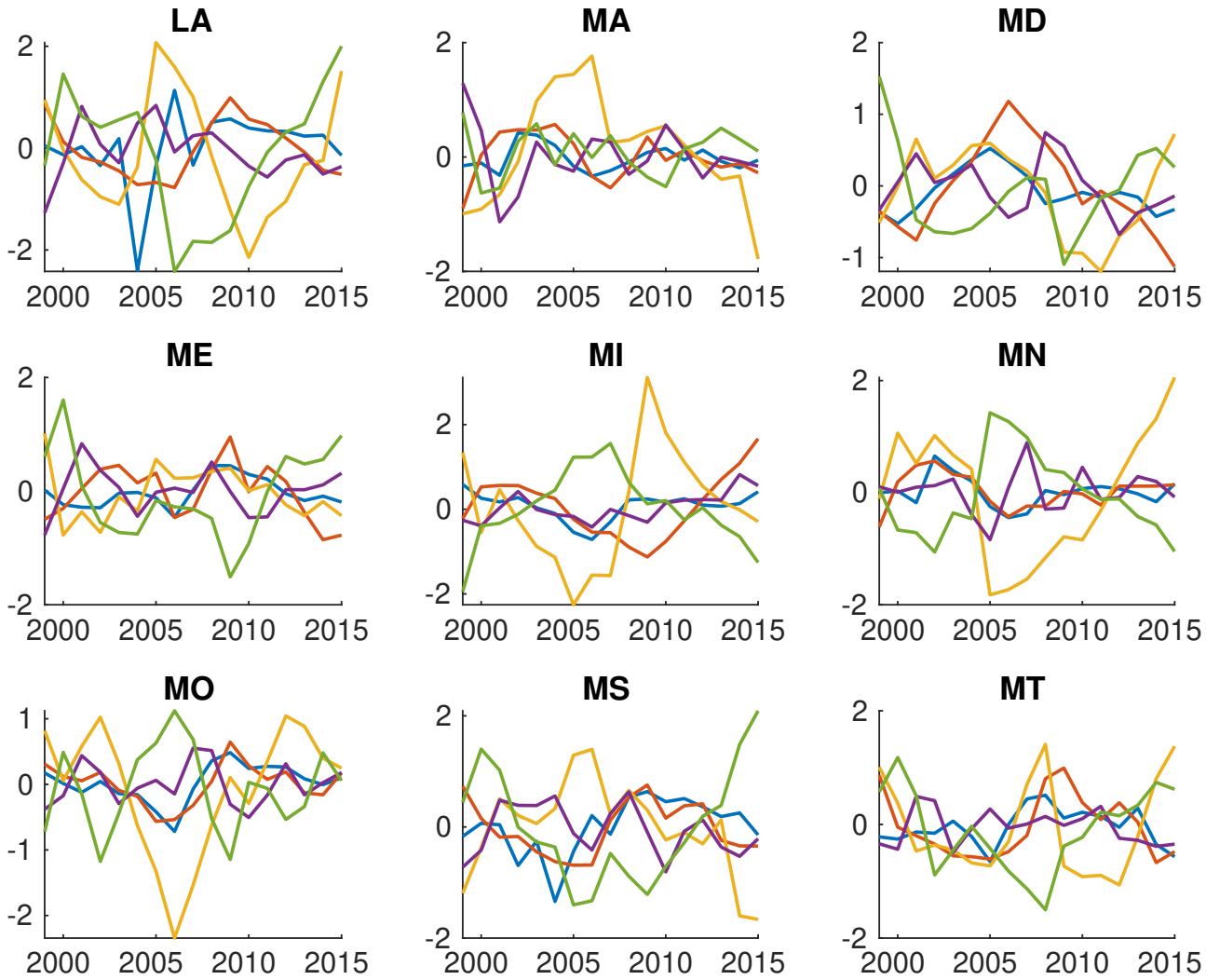


Figure 46: State-level Structural Shocks (cont)

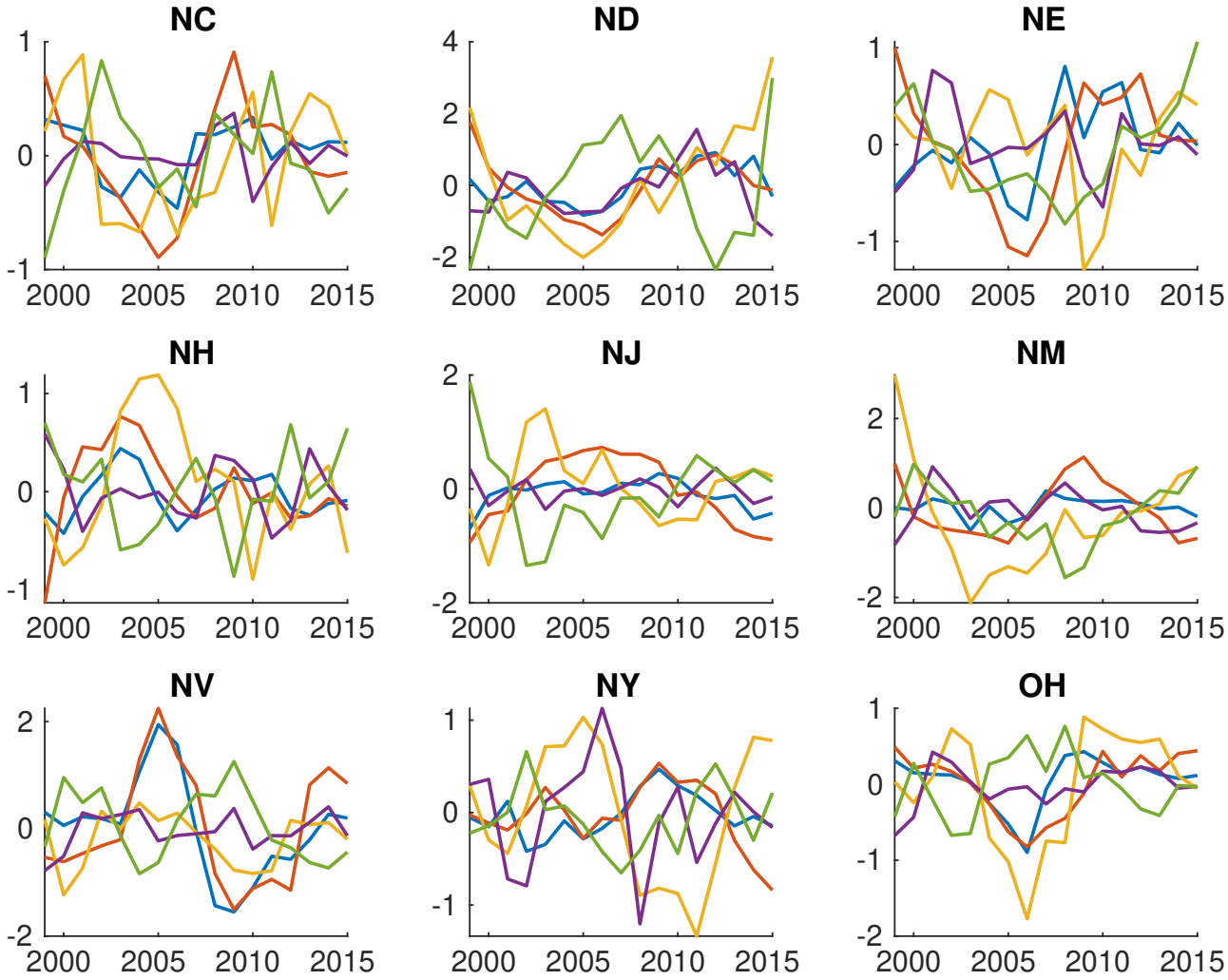


Figure 47: State-level Structural Shocks (cont)

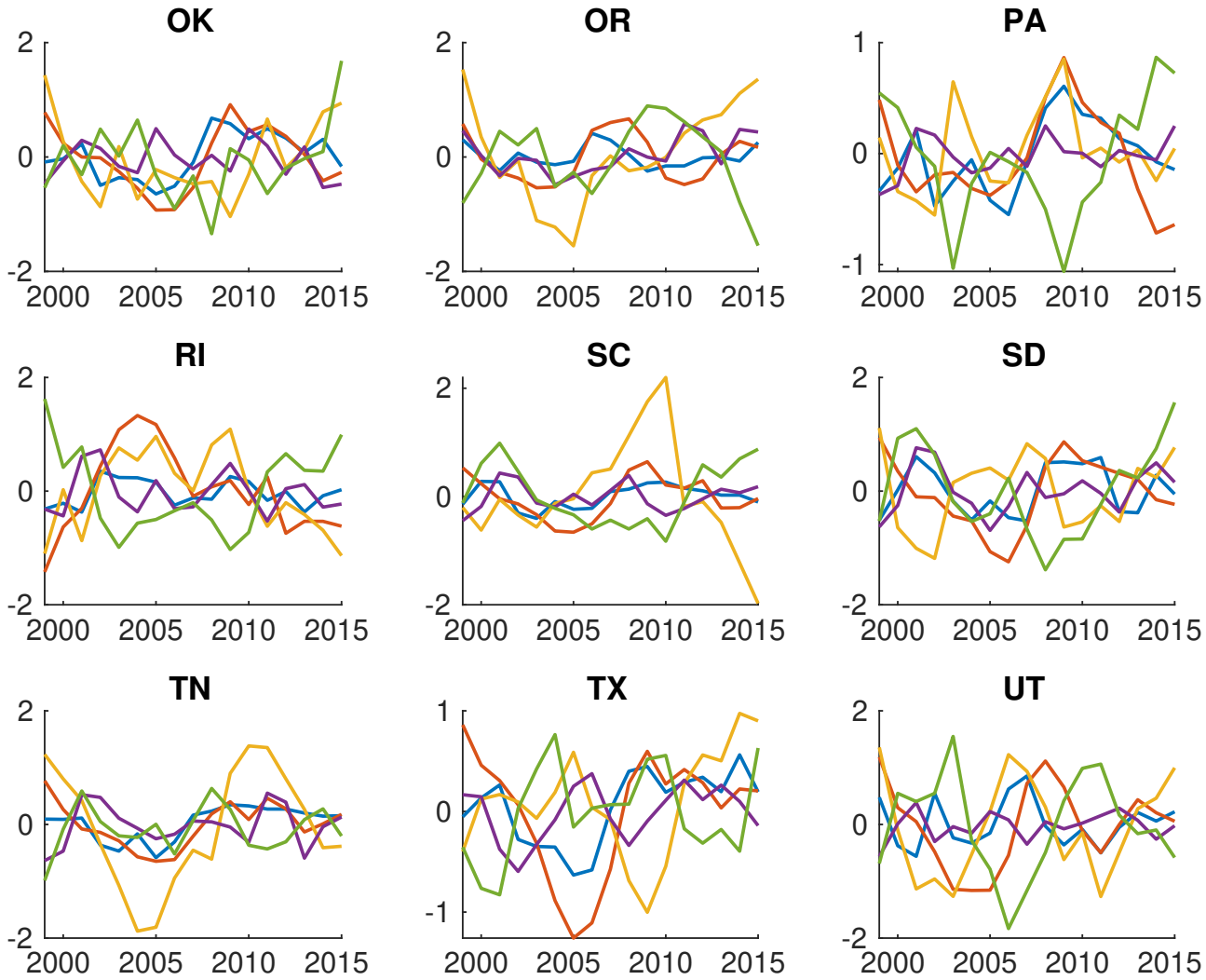


Figure 48: State-level Structural Shocks (cont)

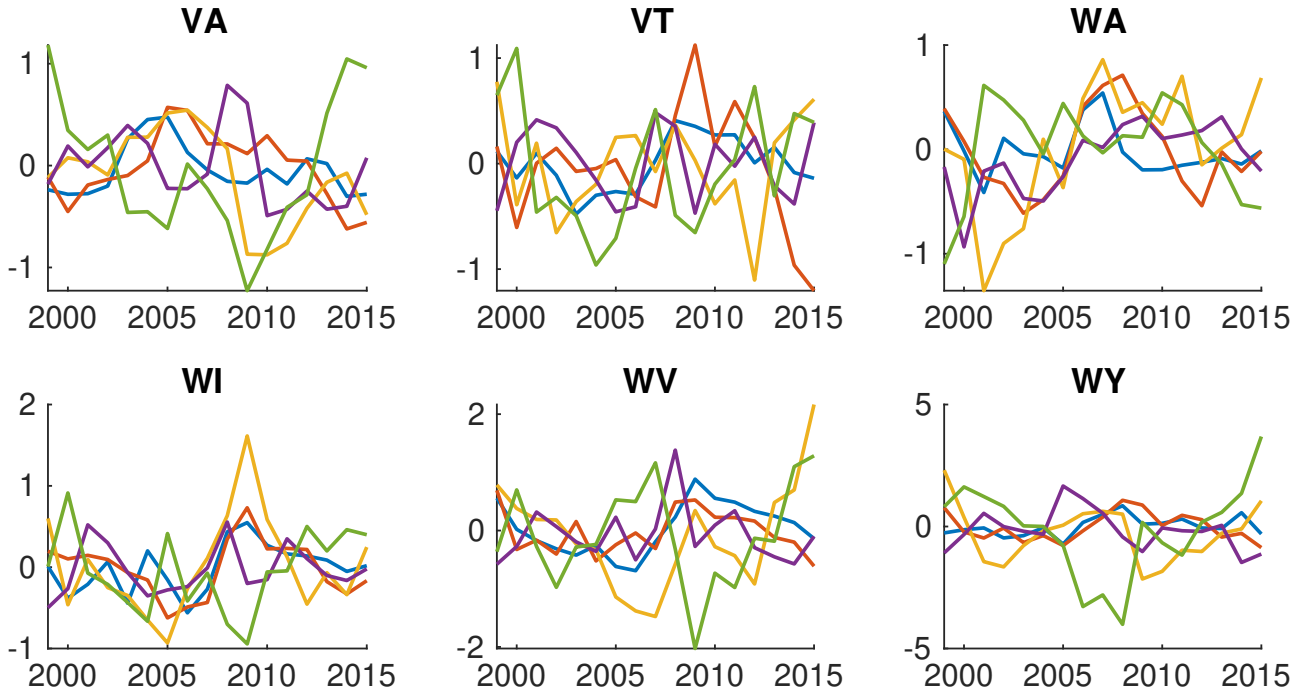
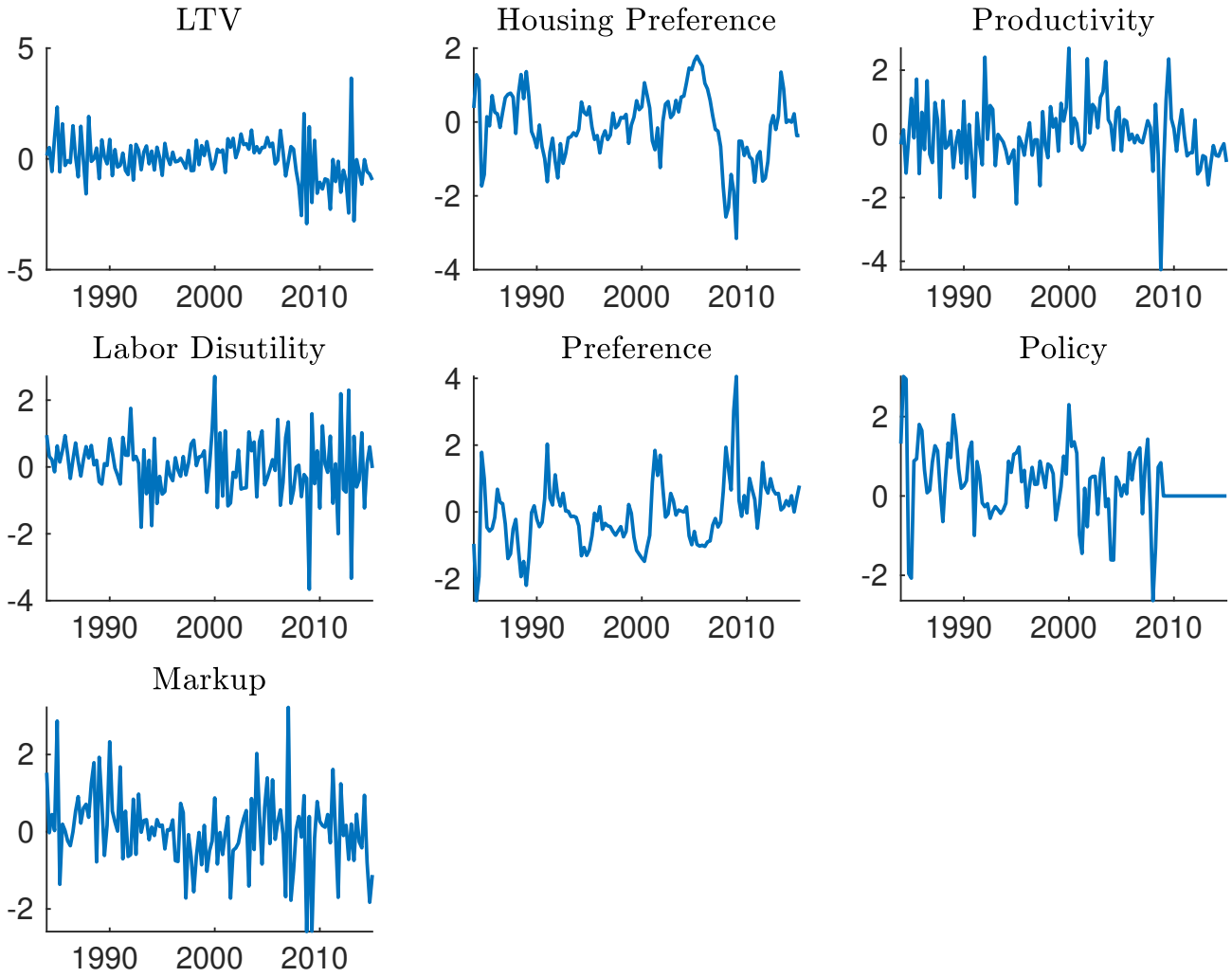


Figure 49: Aggregate-level Structural Shocks



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