Non-Linear Inflation Dynamics

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based on work with

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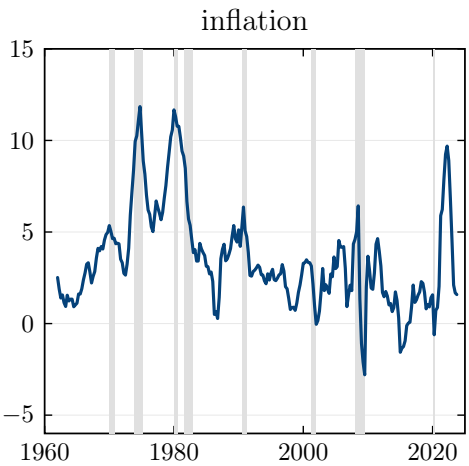
Society for Economic Dynamics

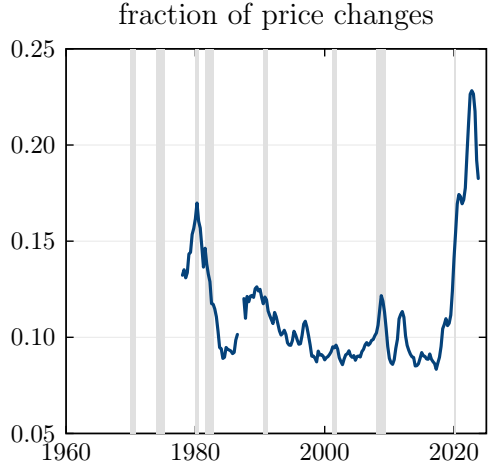
Athens, 2026

Motivation

- Large increase in inflation in many economies in 2021 – 2023
- In NK models, output cost of reducing inflation
  - slope of Phillips curve
  - depends on fraction of price changes
- Data: fraction of price changes increases with inflation

Evidence from the US





Nakamura et al. (2018), Montag–Villar (2023); fraction of regular price changes per month

Question

- How does the slope of the Phillips curve fluctuate over time?
  - in models that match comovement of inflation and fraction of price changes
- Menu cost models natural starting point
- But predictions of these models are subtle
  - slope of Phillips curve depends on more than fraction of price changes
  - don't necessarily reproduce comovement inflation and fraction
- Today: review menu cost literature, then our recent work

Aggregate Price Stickiness In Menu Cost Economies

General Formulation

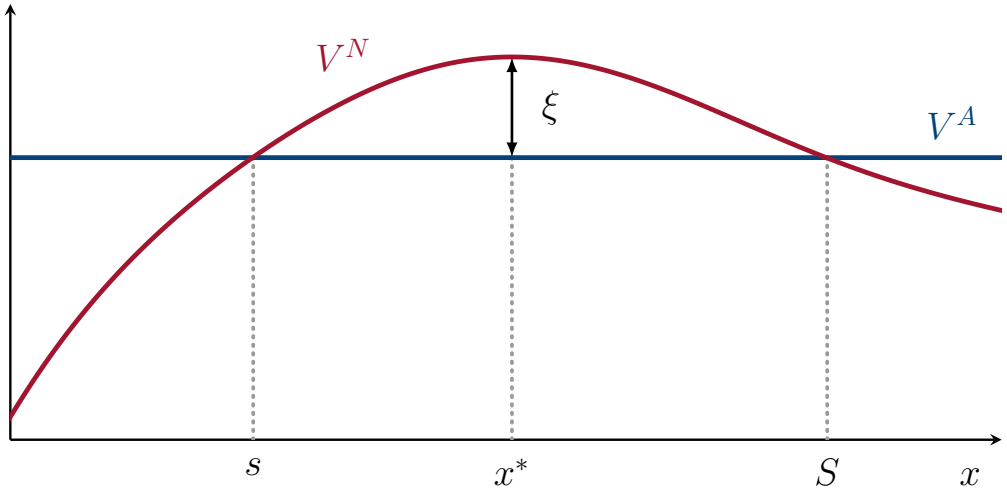
- Key ingredients
  - consumers with log-linear preferences, $P_t C_t = M_t = W_t$
  - technology $Y_{it} = Z_{it} L_{it}$, demand $Y_{it} \sim \left(\frac{P_{it}}{P_t}\right)^{-\theta}$, menu cost ξ
- Price gaps

$$X_{it} = \frac{Z_{it} P_{it}}{M_t}, \quad X_t = \frac{P_t}{M_t} = \left(\int X_{it}^{1-\theta} \mathrm{d}i \right)^{\frac{1}{1-\theta}}$$

- Flow profits depend on own and aggregate price gaps

$$\Pi_{it} = X_t^{\theta-1} \left(X_{it}^{1-\theta} - X_{it}^{-\theta} \right), \qquad x_{it} = x_{it-1} + \Delta z_{it} - \Delta m_t \quad \text{unless adjust}$$

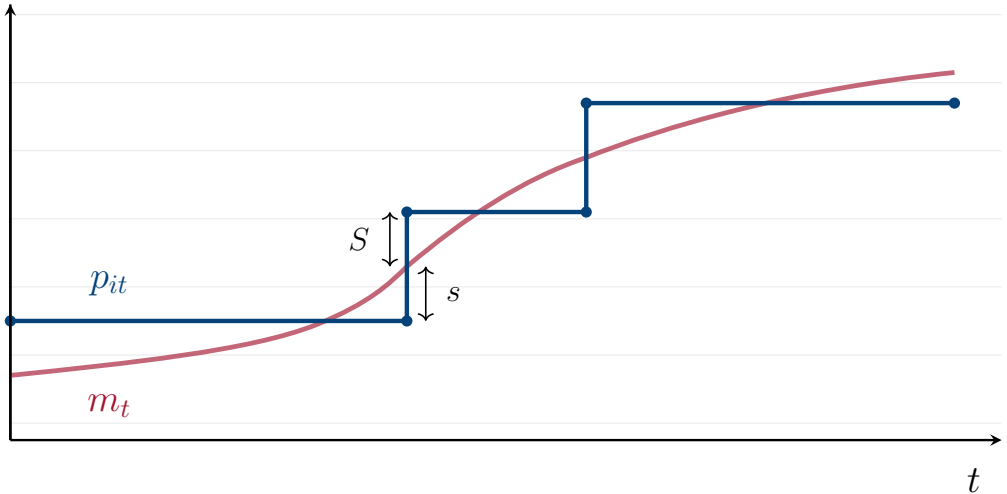
Decision Rules



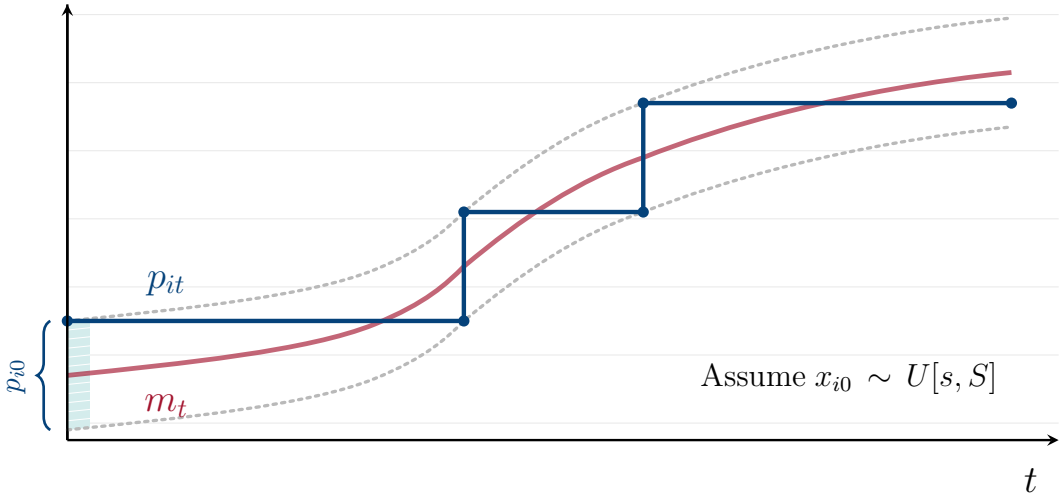
Caplin–Spulber (1987)

- No idiosyncratic shocks, only variation in growth rate of M_t
  - M_t continuous, increases over time
  - so $x^* = S$
  - faster money growth erodes price gaps faster, so more frequent adjustments
- Show P_t increases one-for-one with M_t
  - regardless of how large menu costs are

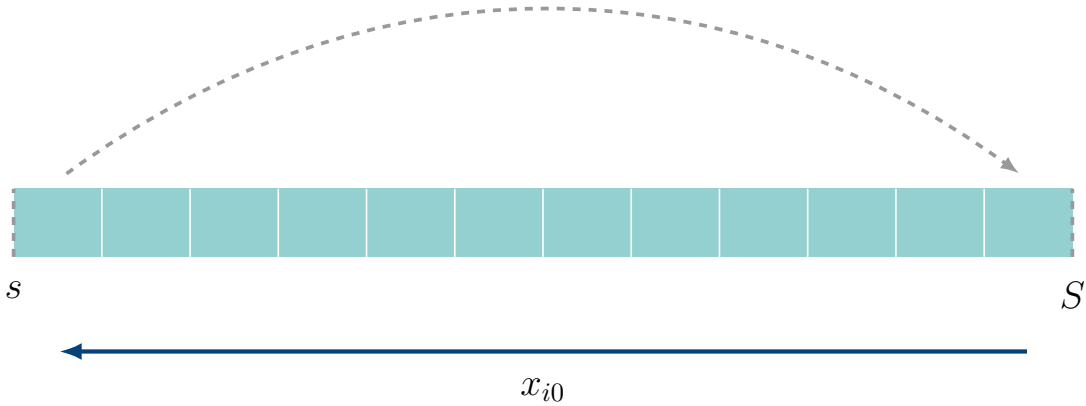
Price Paths Under (S, s) Policies



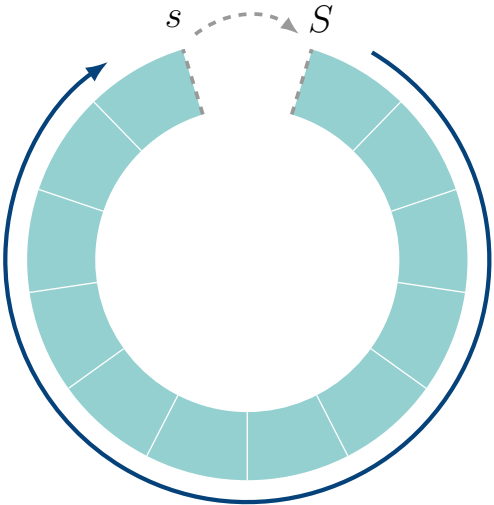
Aggregation



Initial Gaps Uniform on $[s, S]$



Uniform Gap Distribution Time-Invariant



- Faster m growth increases
  - speed at which gaps rotate
  - not relative position
- So distribution $F(x)$ invariant
- As is the price level

$$p_t = \int x \, \mathrm{d}F(x) + m_t$$

Intuition

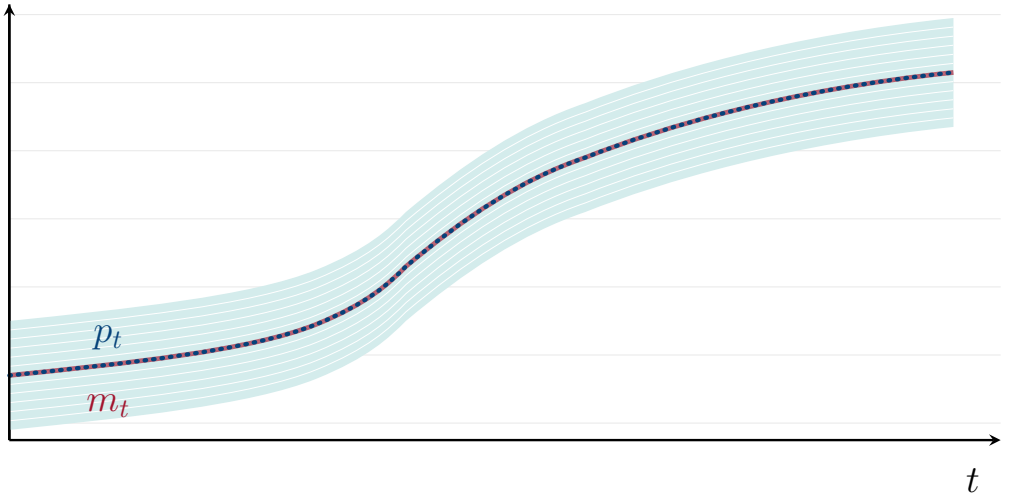
- $\text{Inflation}_t = \underbrace{\text{fraction price changes}_t}_{n_t} \times \underbrace{\text{average price change}_t}_{S-s}$

- Fraction of price changes moves one-for-one with m_t
  - only firms with $x_{it} \in [s,\, s + \Delta m_t]$ adjust from t to $t + \Delta t$, so $n_t = \frac{\Delta m_t}{S-s}$

$$\Delta p_t = \frac{\Delta m_t}{S-s} \times (S-s) = \Delta m_t$$

- In contrast to Calvo where frequency of price changes constant

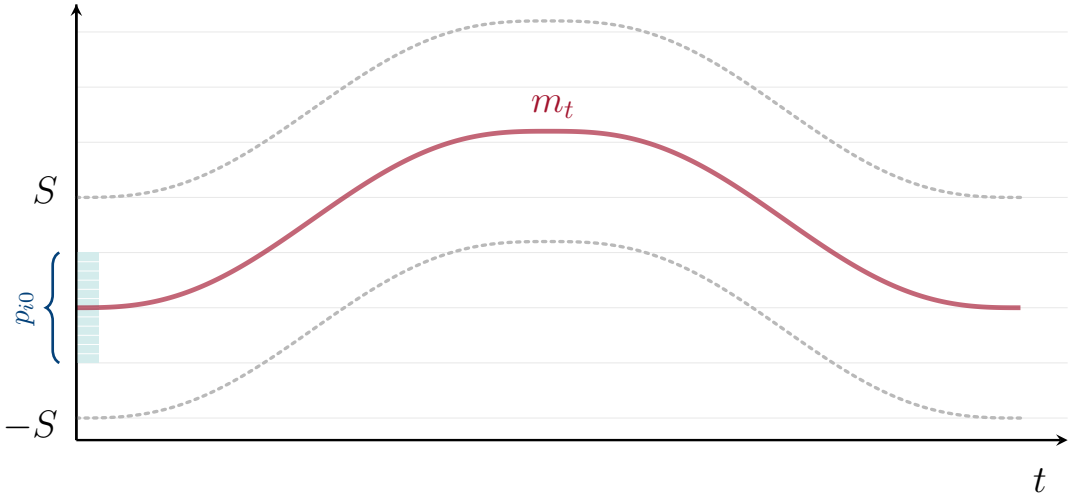
Price Paths Under (S, s) Policies



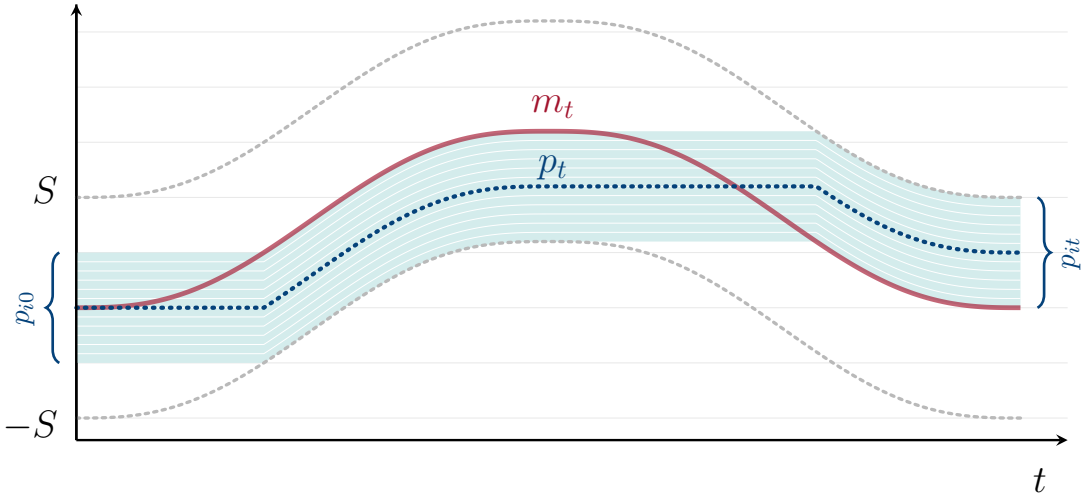
Caplin–Leahy (1991)

- Show Caplin–Spulber neutrality result is not robust
  - assume M_t follows a random walk
  - reset price to 0, adjust whenever x hits $-S$ or S
  - $F_0(x)$ is uniform and has length S
- Then $\Delta p_t = 0$, unless $F_t(x)$ reaches one of boundaries

Caplin–Leahy (1991)



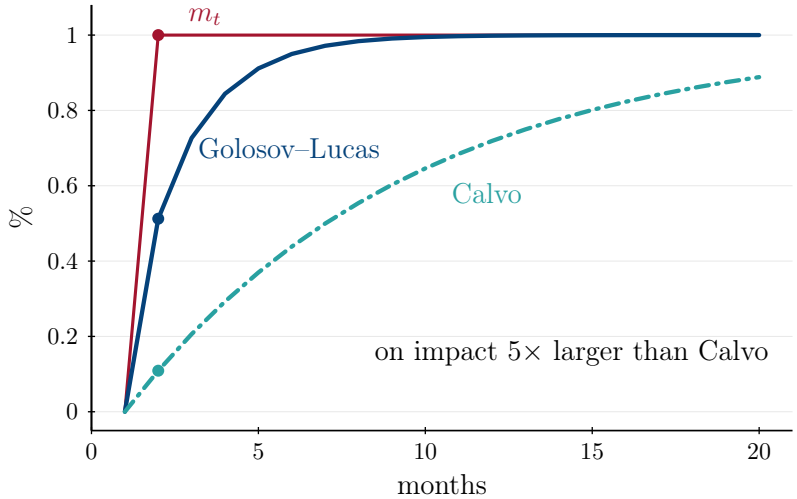
Caplin–Leahy (1991)



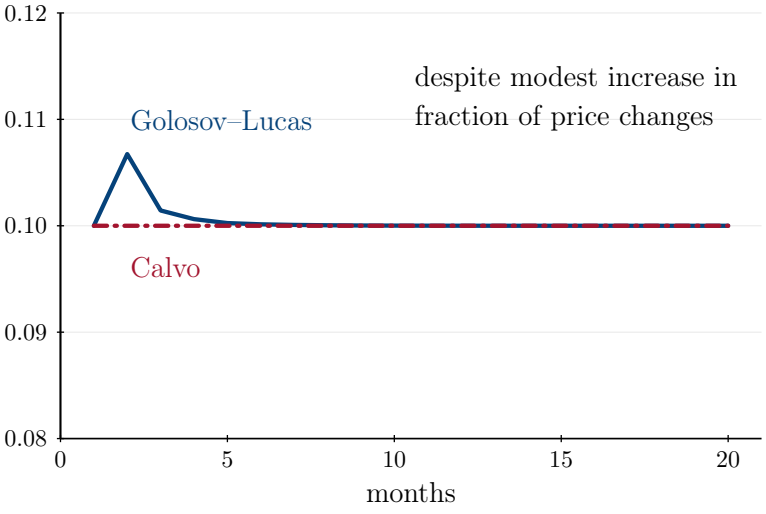
Golosov–Lucas (2007)

- Motivated by Bils–Klenow (2004), Klenow–Kryvtsov (2008) evidence
  - average price change large in absolute value, nearly 10%
  - in low-inflation 1990s
  - many price decreases
- Introduce firm-specific shocks
- Show price level responds rapidly to monetary shocks

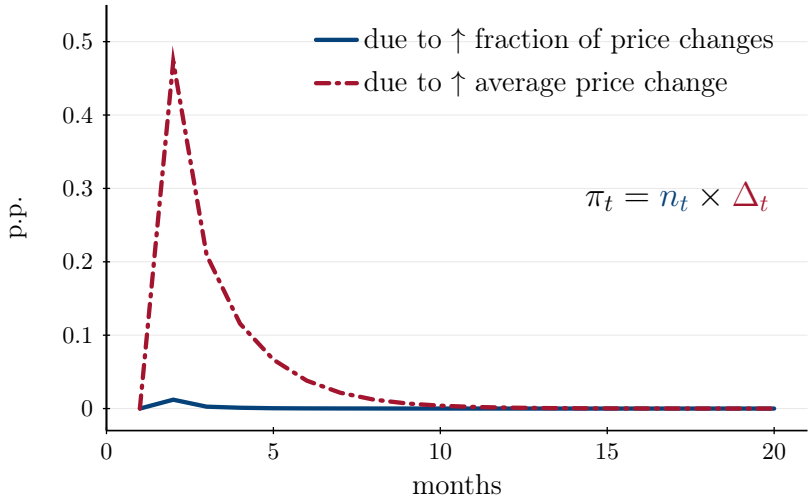
Price Response to Monetary Shock



Fraction of Price Changes



Decomposing the Inflation Response



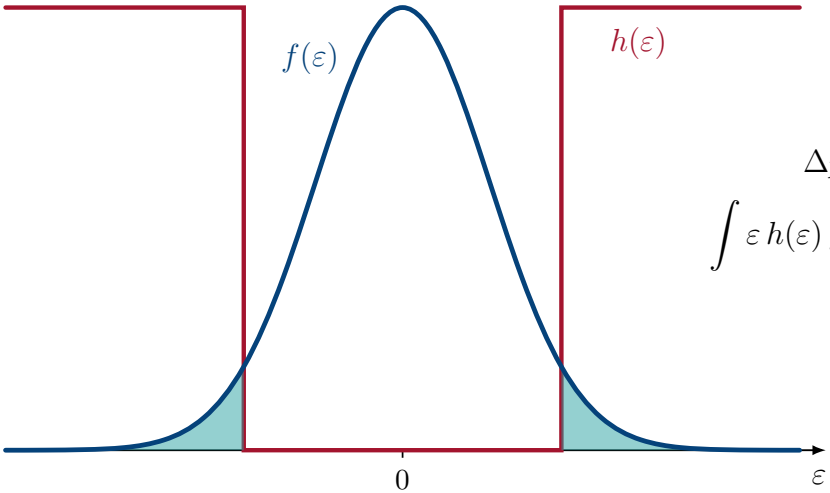
Selection Effect

- Firms that adjust exactly those that need largest price changes

- Let ε_{it} desired price change absent Δm_t
  - distribution $f(\varepsilon)$, adjustment hazard $h(\varepsilon)$

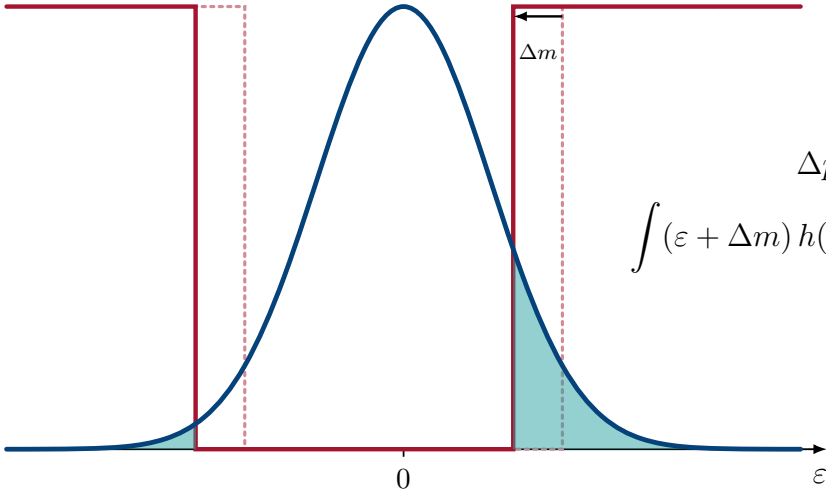
- Assume Δm_t raises desired price one-for-one
  - conditional on adjustment, $\Delta p_{it} = \varepsilon_{it} + \Delta m_t$
  - adjustment hazard changes to $h(\varepsilon_{it} + \Delta m_t)$

Absent Money Shock



$$\Delta p = \int \varepsilon \, h(\varepsilon) \, f(\varepsilon) \, \mathrm{d}\varepsilon = 0$$

With Money Shock



$$\Delta p = \int (\varepsilon + \Delta m)\, h(\varepsilon + \Delta m)\, f(\varepsilon)\, \mathrm{d}\varepsilon$$

A Decomposition

$$\Delta p = \underbrace{\Delta m \int h(\varepsilon) f(\varepsilon)\, \mathrm{d}\varepsilon}_{\text{Calvo}} + \underbrace{\Delta m \int \left[h(\varepsilon + \Delta m) - h(\varepsilon)\right] f(\varepsilon)\, \mathrm{d}\varepsilon}_{\Delta \text{ fraction}}$$

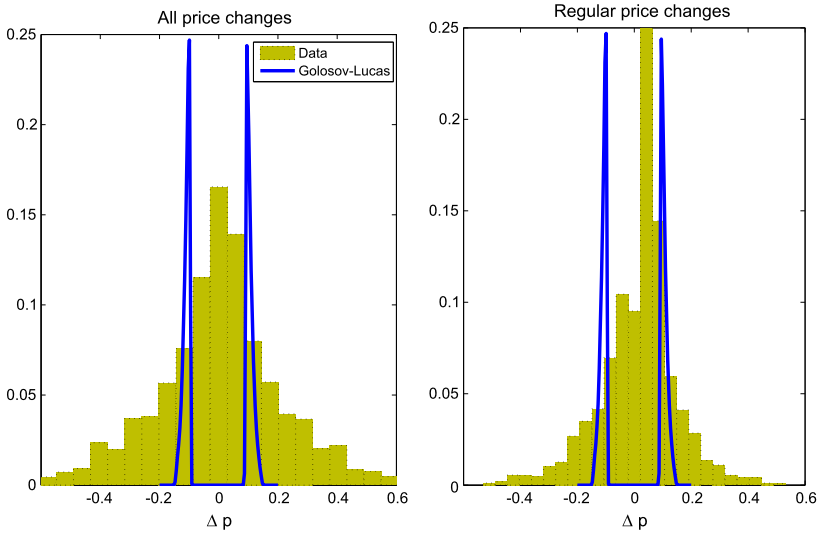
$$+ \underbrace{\int \varepsilon \left[h(\varepsilon + \Delta m) - h(\varepsilon)\right] f(\varepsilon)\, \mathrm{d}\varepsilon}_{\text{selection}}$$

- Selection strong when many marginal firms have large desired price changes

Midrigan (2011)

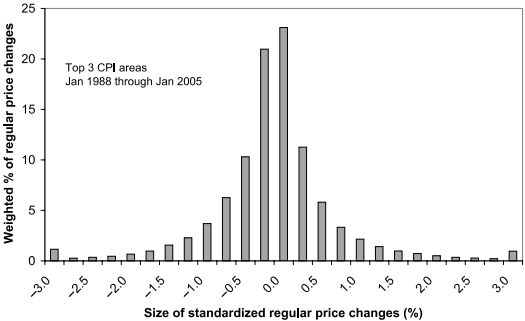
- Shows Golosov–Lucas calibration misses dispersion in size of price changes
  - many small and many large price changes
- Two ingredients to reproduce distribution of price changes
  - multi-product firms with economies of scope: flatter $h(\varepsilon)$
  - fat-tailed idiosyncratic shocks: reduce $f(\varepsilon)$ in steep hazard region
    - see also Gertler–Leahy (2008) and Karadi–Reiff (2017)
- Aggregate price rigidity 4/5th of Calvo

Distribution of Price Changes: U.S. Scanner Data



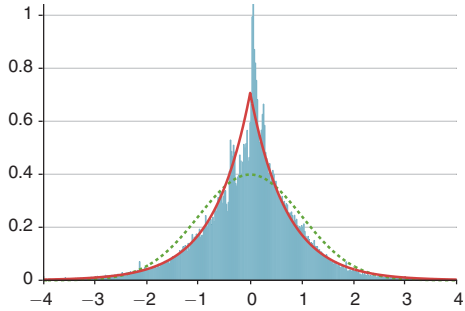
Distribution of Price Changes: U.S. and France

U.S.



Klenow–Kryvtsov (2008)

France



Alvarez–Le Bihan–Lippi (2016)

Alternative Approaches to Flattening Hazard

- Multi-product firms with economies of scope
  - Alvarez–Lippi; Bhattarai–Schoenle; Alvarez–Le Bihan–Lippi
- Random menu costs
  - Caballero–Engel; Dotsey–King–Wolman; Nakamura–Steinsson
- Information costs
  - Woodford; Stevens; Alvarez–Lippi–Paciello
- Directly postulate a generalized adjustment hazard
  - Costain–Nakov; Alvarez–Lippi–Oskolkov

Alvarez–Lippi Sufficient Statistic

- Cumulative output response to small monetary shock

$$\text{CIR} = \frac{1}{6} \times \frac{\text{Kurtosis of Price Changes}}{\text{Frequency of Price Changes}}$$

  - Alvarez–Le Bihan–Lippi; Alvarez–Lippi–Oskolkov
  - large class of models with both state- and time-dependent pricing
    - Calvo: kurtosis $= 6$
    - Golosov–Lucas: kurtosis $= 1$
    - Multi-product with free price changes: kurtosis $\in [1, 6]$

Non-Linear Inflation Dynamics

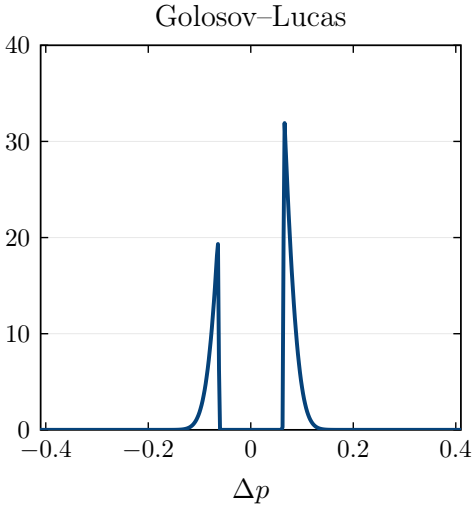
Blanco–Boar–Jones–Midrigan (2025)

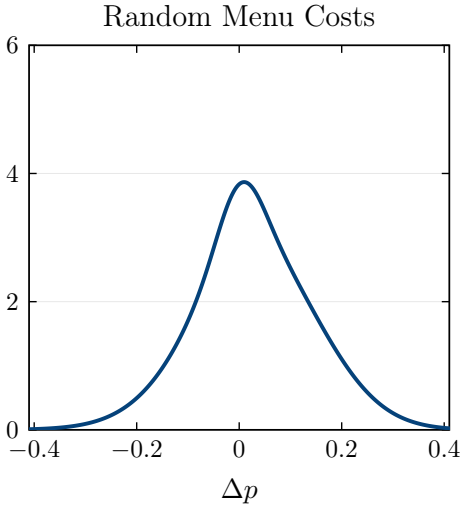
- Show that existing menu cost models
  - consistent with distribution of micro price changes
  - predict fraction of price changes nearly time invariant
  - for the size of shocks U.S. has experienced
- Therefore predict modest non-linearities in aggregate price responses
  - see also Auclert–Rigato–Rognlie–Straub (2024)

Our Experiment

- Two menu cost economies with different adjustment hazards
  - Golosov–Lucas
  - random menu costs
- Calibrate to match
  - 10.5% monthly fraction of price changes
  - 7.5% median absolute price change
- Feed each sequence of money shocks to reproduce U.S. inflation series

Distribution of Price Changes





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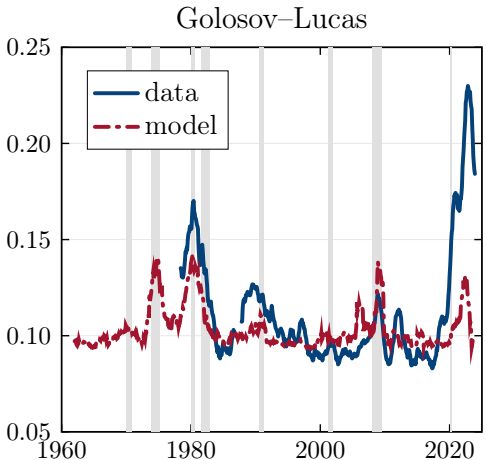
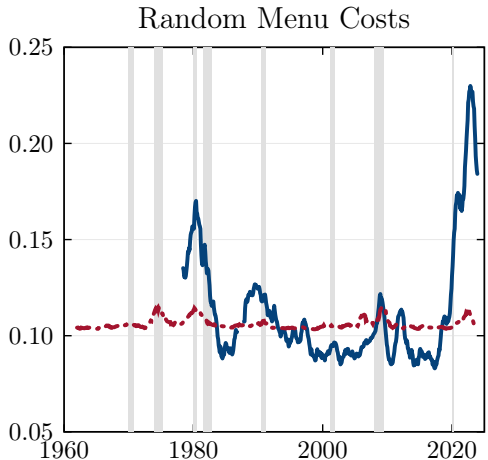
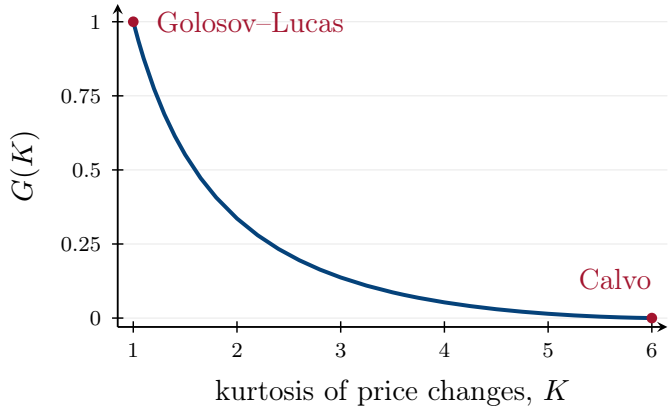




Intuition from Economy with Free Price Changes

- Alvarez–Lippi (2014): response of fraction n to money shock δ

$$\Delta n = \frac{G(K)}{2} \frac{\delta^2}{\mathrm{Var}(\Delta p)}$$



Our Resolution

- Firms sell continuum of products each, one menu cost changes all prices
- Key ingredient: products within firm poor substitutes, $\gamma < \theta$
  - consistent with low within-store demand elasticities: Simmons (2021)
  - product: closely substitutable varieties facing correlated shocks
- Product-level shocks: large price changes
  - but product-level price gaps much less costly than firm-level price gaps
  - so firms respond readily to aggregate shocks, but not to idiosyncratic shocks

Technology and Demand

- Firm i: unit mass of products k, Z_{ikt} random walk with volatility σ

- Within-firm elasticity γ, across-firm θ

$$Y_{ikt} \sim \left(\frac{P_{ikt}}{P_{it}}\right)^{-\gamma} \left(\frac{P_{it}}{P_t}\right)^{-\theta}$$

- Price gaps

$$\underbrace{X_{ikt} = \frac{Z_{ikt} P_{ikt}}{M_t}}_{\text{product}} \qquad \underbrace{X_{it} = \left(\int X_{ikt}^{1-\gamma} \, \mathrm{d}k\right)^{\frac{1}{1-\gamma}}}_{\text{firm}} \qquad \underbrace{X_t = \left(\int X_{it}^{1-\theta} \, \mathrm{d}i\right)^{\frac{1}{1-\theta}}}_{\text{aggregate}}$$

Firm Problem

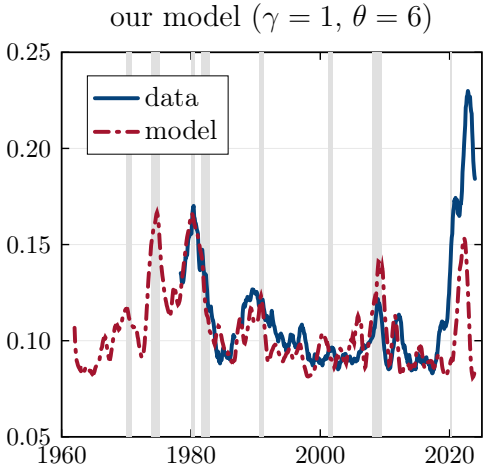
- Two state variables:
  - firm price gap: $x_{it} = x_{it-1} + (1-\gamma)\frac{\sigma^2}{2} - \Delta m_t$
  - duration of price spell d_{it} determines within-firm misallocation

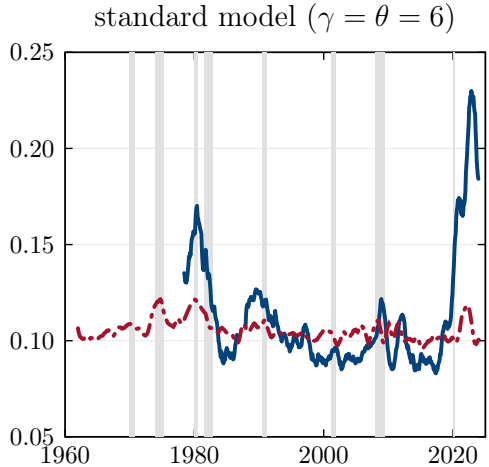
- Flow profits

$$\Pi_{it} = X_t^{\theta-1}\left[X_{it}^{1-\theta} - \exp\left(\gamma\frac{\sigma^2}{2}d_{it}\right)X_{it}^{-\theta}\right]$$

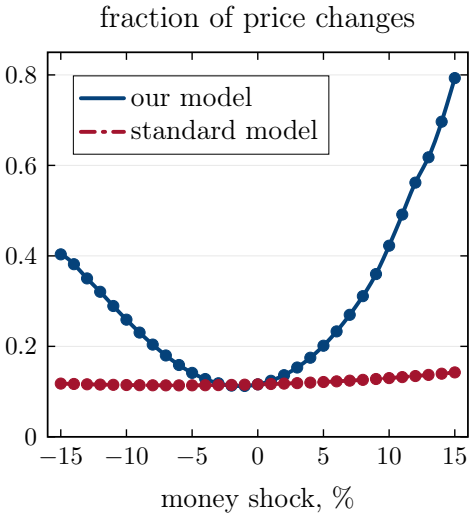
  - as $\gamma \to 0$, equivalent to single-product model without idiosyncratic shocks

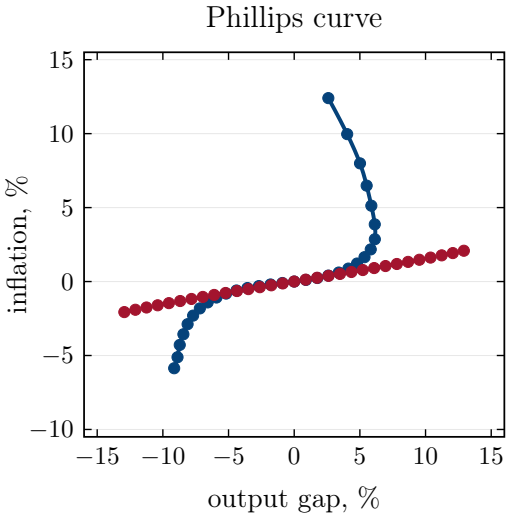
Fraction of Price Changes in Multi-Product Model





Impact Response to Monetary Shocks





The Inflation Accelerator

- Menu cost models difficult to aggregate due to curse of dimensionality

- In Blanco–Boar–Jones–Midrigan (2026) we study tractable alternative
  - multi-product firms choose fraction n_{it} to reprice
  - cost $\mathcal{C}(n_{it})$, *adjusting prices selected at random*

- Admits exact aggregation, so highly tractable. E.g.

$$P_{it} = \left(\int (P_{ikt})^{1-\gamma} \, \mathrm{d}k \right)^{\frac{1}{1-\gamma}} = \left(n_{it} \, (P_{it}^{*})^{1-\gamma} + (1 - n_{it}) \, (P_{it-1})^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

- One new equation relative to Calvo – determines fraction of price changes

Nonlinear Phillips Curve

- Log-linearize around date t equilibrium:

$$\hat{\pi}_t = \mathcal{N}_t \, \hat{p}_t^* + \mathcal{M}_t \, \hat{n}_t$$

$$\mathcal{N}_t = \frac{1 - (1 - n_t) \, \pi_t^{\theta - 1}}{(1 - n_t) \, \pi_t^{\theta - 1}} \text{ increases with } n_t$$

$$\mathcal{M}_t = \frac{1}{(1 - n_t) \, \pi_t^{\theta - 1}} \, \frac{\pi_t^{\theta - 1} - 1}{\theta - 1} \text{ increases with } \pi_t$$

- Inflation more sensitive to fraction of price changes when inflation is high
  - Caplin–Spulber (1987) intuition

Nonlinear Phillips Curve

- Fraction of price changes:

$$\hat{n}_t = \mathcal{A}_t \, \hat{\pi}_t + \mathcal{B}_t \, \hat{p}_t^* + \dots$$

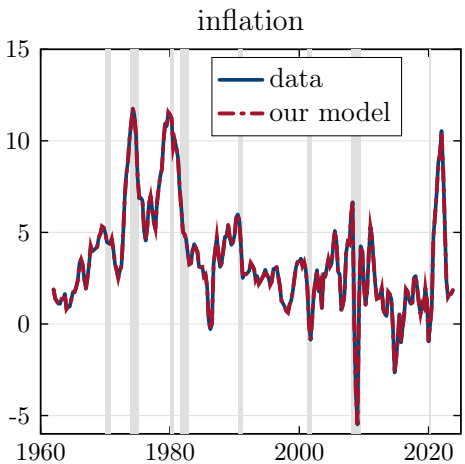
  - $\mathcal{A}_t$ and $\mathcal{B}_t$ increase with inflation

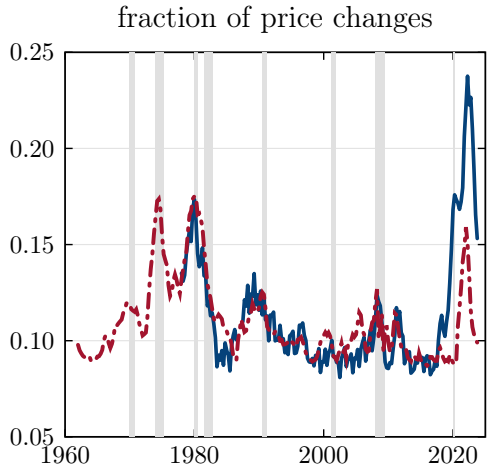
- Solving fixed point implies slope of Phillips curve proportional to

$$\frac{\mathcal{M}_t \mathcal{B}_t + \mathcal{N}_t}{1 - \mathcal{M}_t \mathcal{A}_t}$$

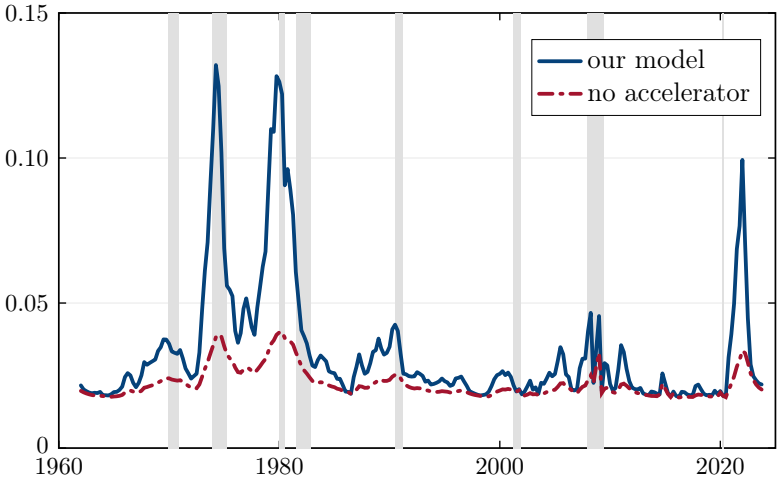
  - $\mathcal{N}_t$ mechanical effect: Calvo with time-varying n_t
  - all others capture *inflation accelerator*: $\uparrow \pi_t \longrightarrow \uparrow n_t \longrightarrow \uparrow \pi_t \longrightarrow \dots$

Inflation and the Fraction of Price Changes

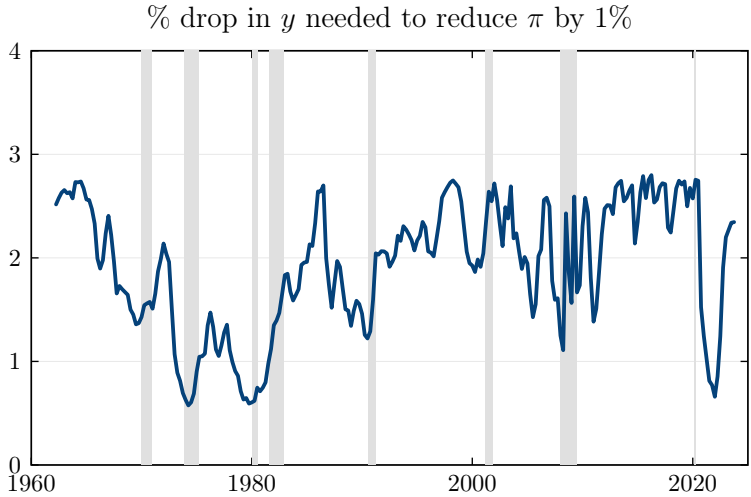




Slope of the Phillips Curve



Sacrifice Ratio



Conclusion

- Data: fraction of price changes increases with inflation
- Menu cost models consistent with this evidence predict
  - slope of Phillips curve rapidly increases with inflation
  - small changes in fraction of price changes – large effects when inflation is high
- So in high-inflation periods:
  - lower output costs of reducing inflation
  - more volatile output and inflation